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AMS 326: Numerical Analysis

Test 3 Report

Problem T3-2

1 Problem Description

9 trees tops have 3D coordinates x, y, z, where x and y are drawn randomly from a uniform distribution $\in$ (0,100) and z is drawn randomly from a normal distribution with $\mu = 10$ and $\sigma = 1.777$. A monkey hops through the tree tops for the shortest total travel distance), starting from the tree whose coordinates (x, y, z) satisfy $x + y + z$ is max. Design a path for the monkey (who does not return to its starting tree). You need to report: The 3d coordinates of all treetops, Optimal-distance path, and The optimal distance.

2 Algorithm Description

2.1 Uniform Random Number Generation

To generate uniformely distributed random numbers, I used the following pseudo random number generator:

$$x_{n+1} = (x_n + x_{n-1}) \ mod \ 1.0 = \begin{cases} x_n + x_{n-1} - 1.0 & if \ (x_n + x_{n-1}) > 1.0 \\ x_n + x_{n-1} & Otherwise \end{cases}$$

Source: Professor Deng's lecture notes (Lecture 2)

2.2 Generating Normal Distribution

I used the Box-Muller transform on each pair of numbers x_1 and x_2 to generate a normal distribution:

$$x_1 = \sqrt{-2\ln x_1}\cos 2\pi x_2$$

$$x_2 = \sqrt{-2\ln x_1}\sin 2\pi x_2$$

Then, to achieve the required μ and σ, I transformed x_1 and x_2 by the following:

$$z_1 = \mu + x_1 * \sigma$$

$$z_2 = \mu + x_2 * \sigma$$

2.3 Finding Shortest Path

Using the distance formula the distance between 2 three dimensional points can be found by $((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)^{\frac{1}{2}}$. Using this I check the total distance of each possible permutation of the randomly generated coordinates such that the first coordinate in the path is max (i.e. coordinates $x + y + z$ are the max among the 9 coordinates).

3 Results

3.1 3D Coordinates of Treetops (x, y, z)

(47.4379613045044, 2.971243100891119, 9.53763597959801)

(82.55217810900879, 53.260480348632825, 7.5572070603014865)

(29.990139413513184, 56.23172344952394, 9.457561363946272)

(12.542317522521973, 9.492203798156762, 11.331446398993005)

(42.53245693603516, 65.7239272476807, 9.75050478381763)

(55.07477445855713, 75.21613104583747, 9.4517502268753)

(97.60723139459229, 40.94005829351816, 10.276347716903915)

(52.68200585314942, 16.15618933935563, 9.067764692144415)

(50.2892372477417, 57.0962476328738, 8.811972415783842)

3.2 Optimal Distance Path

Trees: $7 \to 2 \to 6 \to 9 \to 5 \to 3 \to 8 \to 1 \to 4$

Coordinates: (97.60723139459229, 40.94005829351816, 10.276347716903915), (82.55217810900879, 53.260480348632825, 7.5572070603014865), (55.07477445855713, 75.21613104583747, 9.4517502268753), (50.2892372477417, 57.0962476328738, 8.811972415783842), (42.53245693603516, 65.7239272476807, 9.75050478381763), (29.990139413513184, 56.23172344952394, 9.457561363946272), (52.68200585314942, 16.15618933935563, 9.067764692144415), (47.4379613045044, 2.971243100891119, 9.53763597959801), (12.542317522521973, 9.492203798156762, 11.331446398993005)

3.3 Optimal Distance

Optimal distance = 196.7875617415878

4 Brief comments on performance & other questions

This problem is a variant of the Traveling Salesman Problem (TSP). TSP is an NP-Hard problem, with a time complexity of $O(N!)$. Due to this, when N gets even slightly big some sort of heuristic may be required to speed things up, or the exact optimal path may be impossible to even calculate. However, since for this problem $N = 9$, we're only looking at $9! = 362880$, which is fairly easy for a modern computer to calculate in a few seconds. Therefore, the brute-force method of checking all possible permutations is acceptable.