1. Large deviations

   (a) Intuition (10 minutes)

      i. Mention properties of characteristic/moment-generating function.

      ii. Show log-log plot comparing $p_n(\delta) = n^{-1}\sum_{k=1}^n X_k > \delta$ against appropriate CLT approximation; conclude radically different slopes for $\delta > \mathrm{E}X_1$.

      iii. In light of preceding issue, motivate estimator $\hat{p}_n(\delta) = e^{-nI(\delta)}$ with a logarithmic first-order asymptotic:
$$p_n(\delta) = e^{-nI(\delta)+o(n)},$$
to conclude that finding exponential decay $I(\delta)$ involves finding the following limit.
$$\lim_{n\to\infty} \frac{1}{n} \log p_n(\delta) = \lim_{n\to\infty} \Big(-I(\delta) + \frac{o(n)}{n} \Big) = -I(\delta)$$

      iv. With preceding intuition, we can define a large deviations principle, noting that we extend from function $I : \mathbb{R} \to [0, \infty)$ to $r : \mathcal{R} \to [0, \infty)$ by a variational expression $rA = \inf_{\delta \in A} I(\delta)$.

   (b) Useful large deviations results (10 minutes)

      i. State Gärtner-Ellis theorem; hint at gist of proof by mentioning Chebyshev and change of measure. Show how log-log plot of LDP for preceding example is consistent with $p_n(\delta) = e^{-nI(\delta)+o(n)}$.

      ii. State how stochastic processes can be viewed as measurable maps on the weak algebra generated by the time-vector projections, and that on this algebra, measures and convergence (with tightness) are determined by finite-dimensional projections. Ask loaded question of if similar trick can be done with large deviations results.

      iii. State answer: Dawson-Gärtner theorem.

      iv. State Friedlin-Wentzel theorem.

      v. State limitations of Friedlin-Wentzel ("bad" diffusion coefficients).

   (c) Large deviations applications (perhaps instead at end of talk, or beginning if we start instead with Affine processes) (8 minutes)

      Though we haven't researched in this direction, I would like to have such results, because:

      i. I enjoy computer science, so I would like to implement the simulation/statistical algorithms.

      ii. The statistical applications would be something I could leverage into a career.

      iii. The particular statistical application of affine processes having a nice inverse problem 'econometric data $\to$ affine process' seems very interesting and could be something in which the other committee members have expertise.

      iv. The paper involving LDP's in stochastic volatility problems seems similar to JP's results from my naive perspective, and could be worth investigating for similar reasons.

2. Affine processes (12 minutes).

   (a) Define Markov processes and time-homogeneity. Show how these, together with 1(a)i and 1(b)ii, are such that law is determined by the marginals' exponential moments, i.e. the function M:
$$M(t, x, u) = \mathrm{E}_x \exp\langle u, X_t\rangle.$$

   (b) State definition of an affine process.

   (c) Show example of simple process which ends up being affine.
$$\begin{aligned} X_t &= x + \sigma W_t + \sum_{k=1}^{\infty} Z_k 1_{[0,t]}(T_k) \\ \rightsquigarrow \quad \mathrm{E}_x \exp\langle u, X_t\rangle &= \exp\Big(t\frac{1}{2}\langle u, \sigma^T \sigma u\rangle + t\int_{\mathbb{R}\setminus\{0\}} \big(\exp\langle v, u\rangle - 1\big)\mu(\mathrm{d}v) + \langle u, x\rangle \Big) \end{aligned}$$

   Make observation that (ϕ, ψ) satisfy a generalized Riccati ODE.

(d) State Keller-Ressel, Mayerhofer result for affine transform formula.

3. Current results (15 minutes)

   (a) State with Kang, Kang result; recall limitations of Friedlin-Wentzel.

   (b) Mention how this result does not include our earlier example, due to the discontinuities.

   (c) Show how LDP is derived setting up $(X^\epsilon)_{\epsilon>0}$ such that following reparameterizations hold,

   $$\epsilon\phi^\epsilon(t, u/\epsilon) = \phi(t, u), \quad \epsilon\psi^\epsilon(t, u/\epsilon) = \psi(t, u),$$

   and using Gärtner-Ellis with Dawson-Gärtner.

   (d) Remark how we can do the preceding setup by setting up Riccati ODE's accordingly and using affine transform formula.

   (e) Verify it all checks out on birth process.

   (f) Verify it all checks out on Hawkes process.

4. Potential further results (5 minutes)

   (a) Apply the result for a specific model.

   (b) See if we can provide alternative proof with exponential martingales.

   (c) Experiment with reparameterization that works with compensated measures / truncation functions, since method seems to work independently of these details.