A Proposal for an Extension of the SIR-Macro Model: The SI4R-Macro Model

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1 Introduction. Limitations of the SIR-Macro Model

In recent months, the COVID-19 crisis has guided the efforts of many epidemiologists and economists to understand the dynamics of the pandemic and the economic and health consequences of different action paths. In the SIR-Macro model of Eichenbaum et al. (2020) the canonical SIR model of Kermack and McKendrick (1927) is extended to deal with the interplay between economic decisions and rates of infections.

The competitive equilibrium of the model is not efficient due to a classic problem of externalities: the infected do not internalize the costs that they are imposing on others (increasing the risk of infection) while deciding to consume and work. The government could impose confinement measures but these have economic costs, since the interrumption of economic activity generates an economic recession. Thus, there is a trade-off between health and economic consequences.

The solution proposed is an optimal confinement path that allows the people to acquire herd inmunity while mitigating the economic costs of the measure.

However, the SIR-Macro Model has some shortcomings that reduce the set of possible political interventions: it does not model the possibility of being asymptomaticly infected, which is an important factor to understand the reaction of both people and government, and the confinement measures cannot be targeted to apply only to the ones that the goverment know that are infected.

In addition, several prominent economists including Paul Romer (2020a; 2020b) have proposed massive state investment to develop state capacity for testing on a regular basis as a fundamental part in a "protect, test and trace" strategy. In their view, that is the best way to save lives *and* avoid the economic downturn.

In this paper we propose and extension for the baseline SIR-Macro model[1] in which we include the previous elements to create a more comprehensive framework that we will refer to as the SI4R-Model.

2 Population Health Dynamics

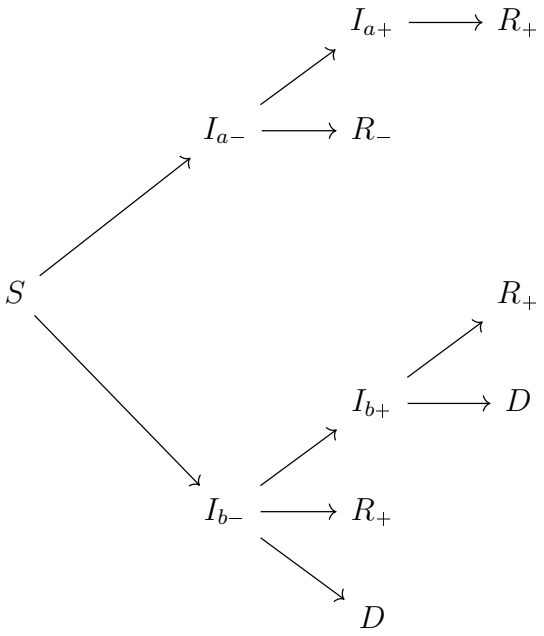

Figure 1: Health State Tree in the SI4R-Model

As in the SIR-Macro model, the transition between health states will depend on the economic decisions of individuals with respect to consumption and work. The difference will lie in the disaggregation of the infected in asymptomatically and symptomatically infected and, in each of those cases, in the possibility of being tested and thus, marked by the government as a target of the confinement measures.

We are going to use two symplifying assumptions: test are perfect, so that if an infected person is tested it will test positive and be signalled as tested, and the asymptomatic will not develop any symptoms over the course of the disease. This simplifies the model while capturing the relevant elements we want to represent.

The population is divided into seven groups, instead of three: susceptible (people who have not yet been exposed to the disease, S_t), asymptomatic infected untested (people who contracted the disease but who have no symptoms and have not been tested, I_t^{a-}),

[1]The baseline SIR-Macro the one does not include the extensions related to the colapse of the health care system or the development of vaccines.

asymptomatic infected tested (the former ones after being tested, I_t^{a+}), symptomatic infected (people who contracted the disease, present symptoms but have not been tested, I_t^{b-}), symptomatic infected tested (the former ones after being tested, I_t^{b+}), recovered without knowing they've been infected, R_t^-, recovered aware (people who survived the disease and acquired inmunity knowing that they were infected, R_t^+) and deceased (people who died from the diseas, D_t).

Note that the possibility of having agents that are infected but untested implies that they can still be unaffected by the targeted measures of the government and, with his economic decisions, increase the risk of infection for others without internalizing the cost.

Following the exposition of the original SIR-Macro Model, we are going to show the equations for the transitions between health states.

The total number of newly infected (and untested) is still given by the same equation as in the original model, but a fraction of those newly infected (π_b) develop symptoms and the rest of the newly infected (π_a) are asymptomatic:

$$T_t = \pi_{s1}(S_t C_t^s)(I_t C_t^i) + \pi_{s2}(S_t N_t^s)(I_t N_t^i) + \pi_{s3} S_t I_t \tag{1}$$

$$T_t^{a-} = \pi_a \left[\pi_{s1}(S_t C_t^s)(I_t C_t^i) + \pi_{s2}(S_t N_t^s)(I_t N_t^i) + \pi_{s3} S_t I_t\right] \tag{2}$$

$$T_t^{b-} = \pi_b \left[\pi_{s1}(S_t C_t^s)(I_t C_t^i) + \pi_{s2}(S_t N_t^s)(I_t N_t^i) + \pi_{s3} S_t I_t\right] \tag{3}$$

Note that that if $\pi_b = 1$ then there are no asymptomatic ($\pi_a = 0$) and it's the same equation as the baseline SIR-Macro model. Also, $T_t = T_t^{a-} + T_t^{b-}$ since $\pi_a + \pi_b = 1$.

The number of susceptible people at time t+1 is equal to the number of susceptible people at time t minus the number of susceptible people that got infected (regardless of whether they present symptoms or not) at time t:

$$S_{t+1} = S_t - (T_t^{a-} + T_t^{b-}) = S_t - T_t \tag{4}$$

The number of asymptomatic infected untested people at time t+1 is equal to the number of asymptomatic infected untested people at time t plus the number of newly infected without symptoms (T_t^{a-}) minus the number of asymptomatic infected untested that was tested ($\pi_t I_t^{a-}$) and the number of asymptomatic infected untested that recovered ($\pi_r I_t^{a-}$):

$$I_{t+1}^{a-} = I_t^{a-} + T_t^{a-} - (\pi_t + \pi_r) I_t^{a-} \tag{5}$$

The number of asymptomatic infected tested people at time t+1 is equal to the number of asymptomatic infected tested people at time t plus the number of asymptomatic infected

untested that was tested ($\pi_t I_t^{a-}$) minus the number of asymptomatic infected tested that recovered ($\pi_r I_t^{a+}$):

$$I_{t+1}^{a+} = I_t^{a+} + \pi_t I_t^{a-} - \pi_r I_t^{a+} \tag{6}$$

The number of symptomatic infected untested people at time t+1 is equal to the number of symptomatic infected untested people at time t plus the number of newly infected with symptoms (T_t^{b-}) minus the number of symptomatic infected untested that was tested ($\pi_t I_t^{b-}$), the number of symptomatic infected untested that recovered ($\pi_r I_t^{b-}$) and the number of symptomatic infected untested that died ($\pi_d I_t^{b-}$):

$$I_{t+1}^{b-} = I_t^{b-} + T_t^{b-} - (\pi_t + \pi_r + \pi_d) I_t^{b-} \tag{7}$$

The number of symptomatic infected tested people at time t+1 is equal to the number of symptomatic infected tested people at time t plus the number of symptomatic infected untested that was tested ($\pi_t I_t^{b-}$) minus the number of symptomatic infected tested that recovered ($\pi_r I_t^{b+}$) and the number of symptomatic infected tested that died ($\pi_d I_t^{b+}$):

$$I_{t+1}^{b+} = I_t^{b+} + \pi_t I_t^{b-} - (\pi_r + \pi_d) I_t^{b+} \tag{8}$$

To sum up, the number of total infected people[2] at time t+1 is equal to the number of total infected people within each group at t+1 (and that takes into account each of the within-infected variations at t+1):

$$I_{t+1} = I_{t+1}^{a-} + I_{t+1}^{a+} + I_{t+1}^{b-} + I_{t+1}^{b+} \tag{9}$$

The number of recovered people at time t+1 is the number of recovered people at time t plus the number of total infected people who recovered ($\pi_r I_t$):

$$R_{t+1} = R_t + \pi_r (I_t^{a-} + I_t^{a+} + I_t^{b-} + I_t^{b+}) \tag{10}$$

$$R_{t+1}^{-} = R_t^{-} + \pi_r I_t^{a-} \tag{11}$$

$$R_{t+1}^{+} = R_t^{+} + \pi_r (I_t^{a+} + I_t^{b-} + I_t^{b+}) \tag{12}$$

Note that even if in population terms the total number of recovered is important, since we are interested in "herd inmunity", the differentiation between recovered that know they are infected of who do not know will make them behave differently, since the latter do

[2]We are going to use "total infected people at time t" to refer to the total population fraction that is infected at time t, whether they present symptoms or not and whether they have been tested or not.

not know which is their real health state[3].

The number of deceased people at time t+1 is the number of deceased people at time t plus the number of symptomatic infected people who died at t:

$$D_{t+1} = D_t + \pi_d(I_t^{b-} + I_t^{b+}) \tag{13}$$

Total population at time t+1 evolves according to:

$$Pop_{t+1} = Pop_t - \pi_d(I_t^{b-} + I_t^{b+}) \tag{14}$$

As in the original model, $Pop_0 = 1$. At time zero, a fraction ϵ of the susceptibles population is infected by the virus through exposure from animals, but this initial infected are not homogeneus, since some of them are asymptomatic infected ($\pi_a * \epsilon$) and others sympotatic infected ($\pi_b * \epsilon$), both being untested. Consequently:

$S_0 = 1 - \epsilon$
$I_0^{a-} = \pi_a * \epsilon$
$I_0^{b-} = \pi_b * \epsilon$
$I_0^{a+} = 0$; $I_0^{b+} = 0$
$I_0 = I_0^{a-} + I_0^{a+} + I_0^{b-} + I_0^{b+} = \epsilon$
$R_0 = 0$; $R_0^- = 0$; $R_0^+ = 0$;
$D_0 = 0$

3 Rational Expectations and Uncertainty About the Agents' Health States

The key assumption on this model is the role played by susceptibles, infected asymptomatic untested and recovered after being asymptomatic untested. Since they do not have symptoms, they all share the same information set without knowing their actual health state. Consequently, they try to make their best-response under rational expectations and uncertainty. Since all three face the same problem, we will refer to their maximization problem with the unoriginal name of "doubters" (d).

Doubters do not know who individually is infected, not themselves nor others. But they are aware of the initial infection and, critically, as they understand "how their world works" (the laws of motion governing the population health dynamics, the model parameters and the other agent's best responses under equilibrium) they are able to infer the

[3]This is explained in more depth in the section devoted to the equilibrium strategies of each type of agent.

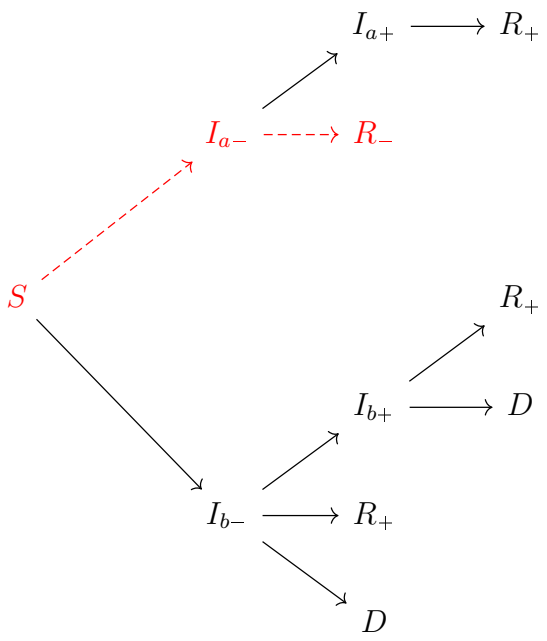

Figure 2: Health State Tree in the SI4R-Model. Doubters information set in red

evolution of the asymptomatic and untested at any point in time even if this is "hidden information" in a sense.

Thus, they are able to infer the conditional probabilities of being susceptible, asymptomatic untested or recovered unaware at any point in time, given that they can perceive those aggregates.

Note that, in general, the probabilities for each health state at any time t coincide with the fraction of the population that have that health state, but in this conditional setting they coincide with the fraction of doubters that have that particular health state in which their information sets coincide:

$$P_t(S|D) = \frac{P_t(D|S)P_t(S)}{P_t(D)} = \frac{P_t(S)}{P_t(D)} = \frac{S_t/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{S_t}{S_t + I_t^{a-} + R_t^-} \tag{15}$$

$$P_t(A-|D) = \frac{P_t(D|A-)P_t(A-)}{P_t(D)} = \frac{P_t(A-)}{P_t(D)} = \frac{I_t^{a-}/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{I_t^{a-}}{S_t + I_t^{a-} + R_t^-} \tag{16}$$

$$P_t(R-|D) = \frac{P_t(D|R-)P_t(R-)}{P_t(D)} = \frac{P_t(R-)}{P_t(D)} = \frac{R_t^-/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{R_t^-}{S_t + I_t^{a-} + R_t^-} \tag{17}$$

Consequently, we can model their behavior as if they were doing a learning in bayesian terms of the real probabilities of being in each health state given that they are doubters

and their perceived probabilities coincide also in equilibrium with the "real model" of the world.

4 Agents Strategies, Government Measures and Equilibrium Conditions

As in the original SIR-Macro model, the agents take as given the aggregates and probabilities and try to maximize their utiliy subject to uncertainty, their budget constraint and -on susceptibles- the risk of getting infected that they want to assume. The SI4R-Macro Model adds several elements to the interplay between economic decisions, health outcomes and government measures: the government now can do tests and the agents transition between states if they get tested (assuming perfect tests). Besides, there can be a targeted confinement that applies differently to those who have been tested (and thus tested positive) and those who have been not. This is reflected in the different taxes added on the budget constraints of the agents.

Budget constraint of a type-j agent ($j = d, a+, b-, b+, r+$):

$$(1 + \mu_{ct}^k)c_t^j = w_t \phi^j n_t^j + \Gamma_t \tag{18}$$

There can be different μ_{ct} parameters: μ_{ct}^k with $k = -, +$ and $\mu_{ct}^+ > \mu_{ct}^-$. Also note that $\phi^j < 1$ for the symptomatic infected (tested or untested) and $\phi^j = 1$ for the rest of types.

Individual risk of infection of a type-j agent:

$$\tau_t^j = \pi_{s1} c_t^j \sum_{i \in I} I_t^i C_t^i + \pi_{s2} n_t^j \sum_{i \in I} I_t^i N_t^i + \pi_{s3} \sum_{i \in I} I_t^i \tag{19}$$

for $I = \{a-, a+, b-, b+\}$. Note that in this model, doubters could be already infected or recovered but they still perceive a risk of infection since they do not know what they really are.

Doubters The lifetime utility of a "doubter" person, U_t^d, is the weighted sum of the discounted expected utilities of the agents depending on what they could be:

$$\begin{aligned} U_t^d = u(c_t^d, n_t^d) + \\ P_t(S|D)\beta \left[(1 - \tau_t^d \pi_b) U_{t+1}^d + \tau_t^d \pi_b U_{t+1}^{b-}\right] + \\ P_t(A - |D)\beta \left[(1 - \pi_t) U_{t+1}^d + \pi_t U_{t+1}^{a+}\right] + \\ P_t(R - |D)\beta \left[U_{t+1}^d\right] \end{aligned} \tag{20}$$

The First-order conditions for consumption and hours worked are given by:

$$u_c(c_t^d, n_t^d) - (1 + \mu_{c_t^-})\lambda_{bt}^d + \lambda_{rt}^d \pi_{s1}(\sum_{i \in I} I_t^i C_t^i) = 0 \tag{21}$$

$$u_n(c_t^d, n_t^d) + w_t \lambda_{bt}^d + \lambda_{rt}^d \pi_{s2}(\sum_{i \in I} I_t^i N_t^i) = 0 \tag{22}$$

The Multiplier on the constraint for τ_t^d is obtained as:

$$\beta \left[\pi_b (U_{t+1}^{b-} - U_{t+1}^d)\right] - \lambda_{rt}^d = 0 \tag{23}$$

for $I = \{a-, a+, b-, b+\}$.
Note that the doubters problem is faced by:

- **Susceptibles**, choosing (c_t^s, n_t^s, τ_t^s);
- **Asymptomatic infected untested**, choosing $(c_t^{a-}, n_t^{a-}, \tau_t^{a-})$; and
- **Recovered from Asymptotic Infected Untested**, choosing $(c_t^{r-}, n_t^{r-}, \tau_t^{r-})$.

Regardless, it is only in the case of the asymptomatic infected that they increase the risk of infection for others.

Asymptomatic Infected Tested The lifetime utility of an asymptomatic infected tested person, U_t^{a+}, is:

$$U_t^{a+} = u(c_t^{a+}, n_t^{a+}) + \beta \left[(1 - \pi_r) U_{t+1}^{a+} + \pi_r U_{t+1}^{r+}\right] \tag{24}$$

The First-order conditions for consumption and hours worked are given by:

$$u_c(c_t^{a+}, n_t^{a+}) = \lambda_{bt}^{a+}(1 + \mu_{c_t^+}) \tag{25}$$

$$u_n(c_t^{a+}, n_t^{a+}) = -w_t \lambda_{bt}^{a+} \tag{26}$$

Symptomatic Infected Untested The lifetime utility of an symptomatic infected untested person, U_t^{b-}, is:

$$U_t^{b-} = u(c_t^{b-}, n_t^{b-}) + \beta \left[(1 - \pi_t - \pi_r - \pi_d) U_{t+1}^{b-} + \pi_t U_{t+1}^{b+} + \pi_r U_{t+1}^{r+}\right] \tag{27}$$

The First-order conditions for consumption and hours worked are given by:

$$u_c(c_t^{b-}, n_t^{b-}) = \lambda_{bt}^{b-}(1 + \mu_{c_t^-}) \tag{28}$$

$$u_n(c_t^{b-}, n_t^{b-}) = -w_t \phi^{b-} \lambda_{bt}^{b-} \tag{29}$$

Symptomatic Infected Tested The lifetime utility of an symptomatic infected tested person, U_t^{b+}, is:

$$U_t^{b+} = u(c_t^{b+}, n_t^{b+}) + \beta \left[(1 - \pi_r - \pi_d) U_{t+1}^{b+} + \pi_r U_{t+1}^{r+} \right] \tag{30}$$

The First-order conditions for consumption and hours worked are given by:

$$u_c(c_t^{b+}, n_t^{b+}) = \lambda_{bt}^{b+}(1 + \mu_{c_t^+}) \tag{31}$$

$$u_n(c_t^{b+}, n_t^{b+}) = -w_t \phi^{b+} \lambda_{bt}^{b+} \tag{32}$$

Recovered Aware The lifetime utility of a recovered person that knew he was infected, U_t^{r+}, is:

$$U_t^{r+} = u(c_t^{r+}, n_t^{r+}) + \beta \left[U_{t+1}^{r+} \right] \tag{33}$$

The First-order conditions for consumption and hours worked are given by:

$$u_c(c_t^{r+}, n_t^{r+}) = \lambda_{bt}^{r+}(1 + \mu_{c_t^-}) \tag{34}$$

$$u_n(c_t^{r+}, n_t^{r+}) = -w_t \lambda_{bt}^{b-} \tag{35}$$

Government Budget Constraint:

$$\begin{aligned} \mu_{c_t^-} \left[S_t c_t^s + I_t^{a-} c_t^{a-} + I_t^{b-} c_t^{b-} + R_t^- c_t^{r-} + R_t^+ c_t^{r+} \right] + \\ \mu_{c_t^+} \left[I_t^{a+} c_t^{a+} + I_t^{b+} c_t^{b+} \right] = \\ \Gamma_t (S_t + I_t^{a-} + I_t^{a+} + I_t^{b-} + I_t^{b+} + R_t) \end{aligned} \tag{36}$$

Equilibrium:

$$(S_t C_t^s + I_t^{a-} C_t^{a-} + I_t^{a+} C_t^{a+} + I_t^{b-} C_t^{b-} + I_t^{b+} C_t^{b+} + R_t^- C_t^{r-}) + R_t^+ C_t^{r+}) = C_t \tag{37}$$

$$(S_t N_t^s + I_t^{a-} N_t^{a-} + I_t^{a+} N_t^{a+} + I_t^{b-} N_t^{b-} + I_t^{b+} N_t^{b+} + R_t^- N_t^{r-}) + R_t^+ N_t^{r+}) = N_t \tag{38}$$

5 Quantitative Results

In this section, we are going to show and discuss the results of our extended model.

We are going to center our discussion in the novelties of our extended model and, thus, we will simply

5.1 Parameters

The previous parameters of the baseline SIR-Macro Model have not been changed. Specifically, the values for rates of mortality, recovery and the productivity for the infected with symptoms have not been changed, they behave as the infected of the original model.

Notwithstanding, the asymptomatic infected in the extension do not die and their productivity is not affected by the epidemic.

In the SI4R extension, three new parameters have been added. First of all, the parameters π_b and π_a are the fraction of infected that develop symptoms or not. Updated research on the question still has not found a definitive answer, and there are estimates of the asymptomatic fraction of the infected that range from 20 percent to 80 percent (Daniel P. Oran, 2020; He et al., 2020; Nishiura et al., 2020). In our exercises, our baseline model will adopt a guess within the confidence intervals of the estimations and that is useful for our numerical exercises: 50 percent of the infected will develop symptoms and 50 percent will not.

The other new parameter that has been added in the extension is π_t, that is, the probability of being tested in a week. This parameter could be calibrated by using estimates of the testing capacity of actual countries, but the interest in our model is the scenario where state capacity has been developed enough so that massive testing is a reality, in line with the proposals of Paul Romer and others. As a quantative exercise, we can assume a 50 percent probability of being tested in a week for the first time (since tested people are not going to be tested again).

As a background check for our model, we added an option in the code to set $\pi_a = 0$, $\pi_b = 1$, and $\pi_t = 0$ and, with this parameters, the model that is being simulated is the SIR-Macro Model since there are no asymptomatic infected nor testing. The results replicate exactly the results observed with the original model and new elements, like the conditional probabilities for doubters behave as expected ($P_t(S|D) = 0$).

The calibration of the parameters $\pi_{s1}, \pi_{s2}, \pi_{s3}$ is done with the same procedure as in the original model but iterating on the extended equations for the new types of agents in SI4R. As a result, in the calibration (see Figure 3) we obtain the values $\pi_{s1} = 7.8215 \cdot 10^(-8)$, $\pi_{s2} = 1.2411 \cdot 10^(-4)$, $\pi_{s3} = 0.38920$ and a basic reproduction number of $R_0 = 1.4998$, which is close to the 1.45 of the original model and within the bounds of confidence on previous research about R_0 for COVID-19.

5.2 SI4R-Macro: No Policy Scenario

First of all we are going to compare the equilibrium results of the extended SI4R-Macro with respect to the original SIR-Macro model under the scenario of no confinement policy (see Figure 1). The equilibrium population dynamics of SI4R-Macro are shown in Figure 4 and are similar in shape of to those of the SIR-Macro. But but since now half of the infected are asymptomatic and do not die as a consequence of coronavirus, the percentage of population deceased in SI4R-Macro is 14.17 which is is lower to the original value of 26.79 in the original model.

On the other hand the spread of the infection is similar but the peak of infected in the SI4R model is higher, 5.9271 compared to the value of 5.2329 in the original model. Eventually, 56.66 percent of the population become infected, again higher than 53.58 in the original model. This quantity implies that for the U.S., aproximately 467000 people die from coronavirus (almost half of the deceased in the original model).

In terms of the economic consequences of the epidemic we find that there is a decrease in economic activity but the fall in consumption in the SI4R-Macro Model is lower than in the SIR-Macro, both in the first year (-2.4969 versus -4.6662 percent) and during the epidemic (-5.5036 versus -9.7656 in percentage terms). This decrease has an U-shape in aggregate but depending on the type of agent works differently. Doubters do not know what is their type, but their consumption decisions show an interesting pattern: susceptibles decrease their consumption in u-shape but asympotamic infected and recovered unaware do not change they consumption patterns. On the other hand, those infected with symptoms decrease their consumption proportionally with the decrease in their productivity but the asymptomatic tested do not change their behavion with respect to consumption. Recovered do not change their consumption patterns.

With respect to work hours, we see also a decrease in U-shape which is primarily caused by the decrease in the workforce due to the deceased population. But this decrease is different for each type of agents. With respect to susceptibles (doubters) they are the ones that decrease more their work hours with respect to their steady state (-6 percent in the peak), but asymptomatic untested and recovered unaware do not change their work decisions in a perceptible manner. With respect to infected, we see marginal changes that in the Figure 5 are magnified due to the scale of the plot. But there are no big changes in their work decisions, which is what should be expected considering that they cause an externality for others but they do not internalize the consequences of the epidemic for themselves (specially, the asymptomatic that do not have a reduction in their productivity as a consequence of the epidemic).

5.3 SI4R-Macro: SIR-Macro Optimal Policy Scenario

In this section we are going to evaluate the response of the agents in the SI4R-Macro Environment to the Optimal Confinement of SIR-Macro Model[4]. This is a second-best optimal path for the confinement policy for the SI4R agents, but it is the best guess we

[4]Our code should allow us to compute an optimal path for the confinement policy that minimizes the economic harms of it while avoiding deaths. Unfortunately, after thousands of iterations and hours of computations we still have not converged to a solution but the code still iterates. We tried to alter the numerical solver procedure, but we don't have enough computation power to solve it for this deadline. In the appendix we will describe the code and the changes done in that sense.

have of an optimal policy and it is coherent with our model population dynamics and the calibrated parameters (see Figures 1 and 7).

The confinement policy increases over time as the epidemic unfolds and its results in terms of saving lifes are not as dramatic as in the original SIR-Macro. The percentage of population deceased is reduced from 14.17 to 11.5 under confinement. This implies in US Population numbers that 87800 lives are saved (in contrast to 180000 in the original model), which is a lower number but that it is explained, partly, because although the mortality rate for the symptomatic infected, the asymptomatic do not die so there are less excess deaths that can be avoided with the confinement.

Notwithstanding, the economic costs of the confinement are similar to the original model and we see an aggravated economic recession with a decrease of consumption of -15.2415 percent in the first year and -24.9163 during the epidemic (this quantities are similar to -27.61 and -9.76, respectively) of the original model. With respect to the how the consumption of different types is affected (figure 8), we can observe an u-shaped decrease in their consumption. susceptibles (doubters) have a drop of consumption of more than -20 percent with respect to their steady state; asymptomatic infected untested (doubters) reduce their consumption in the peak of the confinement by -15 percent and recovered unaware (doubters) by -10 percent. The Asymptomatic Infected Tested reduce their consumption by -20 percent, and, specifically, both symptomatic infected reduce their consumption by more than -30 percent. Interestingly, the recovered aware reduce their consumption aproximately by -25 percent.

The aggregate labor hours in the economy have a larger decrease due to the confinement, decreasing from -6 percent in the peak of the epidemic without confinement to -25 percent. Interestingly, this reduction in work hours affects each type of agents differently. As in the reductions in consumption, susceptibles are the most affected of the doubters with a drop of almost -30 percent in their work hours. Besides, sympotatic infected (tested and untested) reduce their work hours in similar quantities to the susceptibles and to the recovered aware. Overall, we see that the confinement hurts the economy gravely and the return in terms of lives saved is worse than in the original SIR-Macro model.

5.4 SI4R-Macro: SIR-Macro Optimal Policy Scenario

In this third scenario we are going to use the information provided by testing so that confinement measures can be targeted only to those that have been tested positive. The other types of agents have no limitations to consume or work, but stil can be infected, tested, recover or die.

First of all, we can observe that the targeted confinement has saved some lives (a reduction of 1 percent of death population), but not as much as the general confinement

(3 percent). In terms of US Population, only 30000 lives have been saved in contrast with the 87800 lives saved in the general confinement. In contrast, the size of the economic recession created is not as big as with the general confinement. We can see a decrease in aggregate consumption for the first year of -12 percent, half of the decrease under general confinement. The total decrease in aggregate consumption during the pandemic with targeted confinement is -14.61 percent, which is lower than -26.16 percent under general confinement.

The response of different types of agents to this policy is interesting. First of all, the aggregate consumption has an asymetrical U-shaped decrease and in the peak of the infection and confinement, the decrease is of -10 percent with respect to the steady-state. With respect to the different types of doubters, we observe an increase in consumption for asymptomatic infected untested and for recovered unaware, but a decrease in the regular u-shaped form for susceptibles. The most notorious change in the reduction of consumption for the infected happens for the Asymptomatic Infected Tested. Since they have been targeted the consumption is reduced dramatically due to the confinement measures. Similarly the symptomatic infected tested reduce even more than the habitual reduction due to the loss of productivity from the epidemic. Finally, we see that the recovered aware reduce their consumption by -20 percent in the U-shaped form even though the confinement measures do not apply to them.

The change in aggregate work hours shows first an U-shaped decrease but around week 30 it creates an inverted U-shaped increase pattern. For each type of agents, this changes are not as dramatic as in consumption, but they are still relevant. For doubters, susceptibles decrease their work hours in the u-shaped form with a decrease of -10 percent close to the peak of the infection and confinement; asymptomatic infected untested and recovered unaware do not present a big change of their work decisions from their steady state values. In the case of the infected we see a decrease in the work hours following a u-shape but their size is very modest -0.6 percent with respect to the steady-state. Finally, we see a notorious increase (+30 percent) in work hours for those recovered aware.

6 Conclusions

As in the original SIR-Macro model, the interaction between economic decisions and rates of infection is modeled in this extended model. The introduction of asymptomatic infected has changed the baseline policy scenario to one in which less people die and the confinement measures have a lower return in terms of saving lives. The introduction of a combination of bayesian learning and rational expectations for the best-responses of doubters is an interesting mechanism that tries to model the decision procedure that real

people would do in a similar environment.

Considering all the responses to the general confinement and targeted confinement policy it can be said that the targeted measures that follows from our parameters and numbers is not very promising in terms of lives saved, although it alleviates slightly the economic costs of a confinement measures. We have done some tests to study how this scenarios would apply keeping our testing capacity constant, but changing the fraction of infected that become asymptomatic (π_a). Our general conclussions do not change with different fractions of asymptomatic infected: only the size of the economic recession is lower with more asymptomatic, but that is due as well due to the productivity that is not lost during the epidemic if more people do not develop symptoms. Even if our parameter choices are within the confidence intervals of previous evidence, we assume a testing probability in an idealized scenario and our results are still quite modest.

Our assumptions with respect to testing or the spread of the infection might underestimate the returns of a testing policy or the value of confinement measures. They also provide a controlled conterfactual for the option of considering a policy of zero-confinement. Further research on the model could find new ways to calibrate this parameters or change parts of the model so that new dynamics can be represented and studied. In any case, they are a useful first approach to the question that adds new elements and can be used as a basis for further numerical exercises.

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Appendix I - Equilibrium Computation for SI4R-Macro

For a given sequence of containment rates, μ_{ct}, for some large horizon, H, guess sequences for $n_t^s, n_t^{a-}, n_t^{a+}, n_t^{b-}, n_t^{b+}, n_t^{r-}, n_t^{r+}$ from $t = 0$ to $t = H-1$. In practice, we solve the model for $H = 250$ weeks. Compute the sequence of the remaining unknowns variables in each of the following equilibrium equations.

$$\theta n_t^{r+} = A\lambda_{bt}^{r+} \tag{39}$$

$$(c_t^{r+})^{-1} = (1+\mu_{ct})\lambda_{bt}^{r+} \tag{40}$$

$$u_t^{r+} = \ln c_t^{r+} - \frac{\theta}{2}(n_t^{r+}) \tag{41}$$

Iterate backwards from the post-epidemic steady-states values of U_t^{r+}:

$$U_t^{r+} = u(c_t^{r+}, n_t^{r+}) + \beta\left[U_{t+1}^{r+}\right] \tag{42}$$

Calculate the sequence for remaining unknowns in the following equations:

$$\theta n_t^{b-} = \phi^{b-} A\lambda_{bt}^{b-} \tag{43}$$

$$(c_t^{b-})^{-1} = \lambda_{bt}^{b-} \tag{44}$$

$$u_t^{b-} = \ln c_t^{b-} - \frac{\theta}{2}(n_t^{b-}) \tag{45}$$

$$\theta n_t^{b+} = \phi^{b+} A \lambda_{bt}^{b+} \tag{46}$$

$$(c_t^{b+})^{-1} = \lambda_{bt}^{b+} \tag{47}$$

$$u_t^{b+} = \ln c_t^{b+} - \frac{\theta}{2}(n_t^{b+}) \tag{48}$$

$$\theta n_t^{a+} = A \lambda_{bt}^{a+} \tag{49}$$

$$(c_t^{a+})^{-1} = \lambda_{bt}^{a+} \tag{50}$$

$$u_t^{a+} = \ln c_t^{a+} - \frac{\theta}{2}(n_t^{a+}) \tag{51}$$

$$(1 + \mu_{ct}) c_t^s = A n_t^s + \Gamma \tag{52}$$

$$u_t^s = \ln c_t^s - \frac{\theta}{2}(n_t^s) \tag{53}$$

$$(1 + \mu_{ct}) c_t^{a-} = A n_t^{a-} + \Gamma \tag{54}$$

$$u_t^{a-} = \ln c_t^{a-} - \frac{\theta}{2}(n_t^{a-}) \tag{55}$$

$$(1 + \mu_{ct}) c_t^{r-} = A n_t^{r-} + \Gamma \tag{56}$$

$$u_t^{r-} = \ln c_t^{r-} - \frac{\theta}{2}(n_t^{r-}) \tag{57}$$

Given initial values for Pop_0, $S_0, I_0^{a-}, I_0^{a+}, I_0^{b-}, I_0^{b+}, R_0^-, R_0^+$, iterate forward using the following equations for $t = 0, ..., H-1$:

$$T_t^{a-} = \pi_a \left[\pi_{s1} (S_t C_t^s)(I_t C_t^i) + \pi_{s2} (S_t N_t^s)(I_t N_t^i) + \pi_{s3} S_t I_t \right] \tag{58}$$

$$T_t^{b-} = \pi_b \left[\pi_{s1} (S_t C_t^s)(I_t C_t^i) + \pi_{s2} (S_t N_t^s)(I_t N_t^i) + \pi_{s3} S_t I_t \right] \tag{59}$$

$$S_{t+1} = S_t - (T_t^{a-} + T_t^{b-}) \tag{60}$$

$$I_{t+1}^{a-} = I_t^{a-} + T_t^{a-} - (\pi_t + \pi_r) I_t^{a-} \tag{61}$$

$$I_{t+1}^{a+} = I_t^{a+} + \pi_t I_t^{a-} - \pi_r I_t^{a+} \tag{62}$$

$$I_{t+1}^{b-} = I_t^{b-} + T_t^{b-} - (\pi_t + \pi_r + \pi_d) I_t^{b-} \tag{63}$$

$$I_{t+1}^{b+} = I_t^{b+} + \pi_t I_t^{b-} - (\pi_r + \pi_d) I_t^{b+} \tag{64}$$

$$I_{t+1} = I_{t+1}^{a-} + I_{t+1}^{a+} + I_{t+1}^{b-} + I_{t+1}^{b+} \tag{65}$$

$$R_{t+1} = R_t + \pi_r (I_t^{a-} + I_t^{a+} + I_t^{b-} + I_t^{b+}) \tag{66}$$

$$R_{t+1}^- = R_t^- + \pi_r I_t^{a-} \tag{67}$$

$$R_{t+1}^+ = R_t^+ + \pi_r (I_t^{a+} + I_t^{b-} + I_t^{b+}) \tag{68}$$

$$D_{t+1} = D_t + \pi_d(I_t^{b-} + I_t^{b+}) \tag{69}$$

$$Pop_{t+1} = Pop_t - \pi_d(I_t^{b-} + I_t^{b+}) \tag{70}$$

Iterate backwards from the post-epidemic steady-states values of U_t^{b-}, U_t^{b+}, U_t^{a+}:

$$U_t^{b-} = u(c_t^{b-}, n_t^{b-}) + \beta\left[(1 - \pi_t - \pi_r - \pi_d)U_{t+1}^{b-} + \pi_t U_{t+1}^{b+} + \pi_r U_{t+1}^{r+}\right] \tag{71}$$

$$U_t^{b+} = u(c_t^{b+}, n_t^{b+}) + \beta\left[(1 - \pi_r - \pi_d)U_{t+1}^{b+} + \pi_r U_{t+1}^{r+}\right] \tag{72}$$

$$U_t^{a+} = u(c_t^{a+}, n_t^{a+}) + \beta\left[(1 - \pi_r)U_{t+1}^{a+} + \pi_r U_{t+1}^{r+}\right] \tag{73}$$

Calculate the sequence for the conditional probabilities for the doubters:

$$P_t(S|D) = \frac{P_t(D|S)P_t(S)}{P_t(D)} = \frac{P_t(S)}{P_t(D)} = \frac{S_t/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{S_t}{S_t + I_t^{a-} + R_t^-} \tag{74}$$

$$P_t(A-|D) = \frac{P_t(D|A-)P_t(A-)}{P_t(D)} = \frac{P_t(A-)}{P_t(D)} = \frac{I_t^{a-}/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{I_t^{a-}}{S_t + I_t^{a-} + R_t^-} \tag{75}$$

$$P_t(R-|D) = \frac{P_t(D|R-)P_t(R-)}{P_t(D)} = \frac{P_t(R-)}{P_t(D)} = \frac{R_t^-/Pop_t}{(S_t + I_t^{a-} + R_t^-)/Pop_t} = \frac{R_t^-}{S_t + I_t^{a-} + R_t^-} \tag{76}$$

Iterate backwards from the post-epidemic steady-states values of each type of doubters, U_t^d:

$$\begin{aligned} U_t^d = u(c_t^d, n_t^d) + \\ P_t(S|D)\beta\left[(1 - \tau_t^d\pi_b)U_{t+1}^d + \tau_t^d\pi_b U_{t+1}^{b-}\right] + \\ P_t(A-|D)\beta\left[(1 - \pi_t)U_{t+1}^d + \pi_t U_{t+1}^{a+}\right] + \\ P_t(R-|D)\beta\left[U_{t+1}^d\right] \end{aligned} \tag{77}$$

Calculate the sequences of remaining lambdas for budget and risk on the FOCs for each type of doubter and adjust the result for arbitrary precision using a gradient method.

Appendix II - Explanation on Code

The original code uses fsolve with options set to display the iterations and a tolerance for the optimization. Since we have transformed the vector of guesses for the values of n, we had to change the option in the solver for the Levenberg-Marquardt algorithm.

Furthermore, we tried to compute an optimal policy for the SI4R-Macro Model but we need more computing power to do so. We tried to accelerate the code by using a limit on the number of iterations or changing the tolerance, but we were not able to compute

the optimal policy.

The original SIR-Macro model uses several files to compute the model:

- sir_macro.m The original model, where the parameters and options are set.
- go_calibrate_pis.m and calibrate_pis. Used to calibrate the parameters.
- geterr.m. It computes the equilibrium and the residuals used to adjust the result using a gradient method.
- getU.m. It computes the optimal policy by using fmincon and iterating the guesses of n for a given policy and obtaining the policy that maximizes welfare.

This work includes the code in Matlab to replicate this model and the quantitative exercises that have been performed. Besides, it includes some extra-models that are used as a mean of testing that the extension coded is still compatible with the original SIR-Macro model as a particular case. The Models included in the zip folder are:

- Baseline SIR-Macro. It is the original baseline code.
- Simplified Baseline SIR-Macro. It is a simplified version of the former, without the need for setting the parameters of the vaccination and treatment models.
- SI4R-Macro with parameters set to simulate SIR-Macro.
- SI4R-Macro with parameters set to simulate SIR-Macro with Optimal Policy.
- **SI4R-Macro without Confinement.**
- **SI4R-Macro with Optimal Policy.**
- **SI4R-Macro with Optimal Policy only for the treated.**

	SIR: No Confinement	SIR: Optimal Confinement	SI4R: No Confinement	SI4R: Optimal Confinement
Aggregate Consumption	-9.7656	-27.6173	-5.5036	-24.9163
Agg Consumption 1st Year	-4.6662	-16.8352	-2.4969	-15.2415
Terminal	53.5795	43.0267	56.6604	46.0149
Peak Infection	5.2329	3.1848	5.9271	3.8385
Terminal Death Share Percent	0.2679	0.2151	0.1417	0.115
Terminal Death US Millions	0.8841	0.7099	0.4674	0.3796

Table 1: Figure 1. Results of SIR-Macro and SI4R-Macro Model.

		SI4R: No Confinement	SI4R: Optimal Confinement	SI4R: Optimal Confinement for Tested
$\pi_a = 0.3$	Aggregate Consumption	-7.3383	-26.1226	-14.6172
$\pi_a = 0.5$	Aggregate Consumption	-5.5036	-24.9163	-12.0319
$\pi_a = 0.7$	Aggregate Consumption	-3.4674	-23.5588	-9.3576
$\pi_a = 0.3$	Agg Consumption 1st Year	-3.4092	-15.9475	-7.5817
$\pi_a = 0.5$	Agg Consumption 1st Year	-2.4969	-15.2415	-6.4776
$\pi_a = 0.7$	Agg Consumption 1st Year	-1.532	-14.4442	-4.9881
$\pi_a = 0.3$	Terminal	55.3982	44.7769	51.8276
$\pi_a = 0.5$	Terminal	56.6604	46.0149	53.0941
$\pi_a = 0.7$	Terminal	57.965	47.3118	54.4469
$\pi_a = 0.3$	Peak Infection	5.6226	3.5655	4.8707
$\pi_a = 0.5$	Peak Infection	5.9271	3.8385	5.1683
$\pi_a = 0.7$	Peak Infection	6.2572	4.1412	5.5146
$\pi_a = 0.3$	Terminal Death Share Percent	0.1939	0.1567	0.1814
$\pi_a = 0.5$	Terminal Death Share Percent	0.1417	0.115	0.1327
$\pi_a = 0.7$	Terminal Death Share Percent	0.0869	0.071	0.0817
$\pi_a = 0.3$	Terminal Death US Millions	0.6398	0.5172	0.5986
$\pi_a = 0.5$	Terminal Death US Millions	0.4674	0.3796	0.438
$\pi_a = 0.7$	Terminal Death US Millions	0.2869	0.2342	0.2695

Table 2: Figure 2. Robustness of Results to various values for π_a. SI4R-Macro Model.

Figure 3: SI4R Model, Calibration

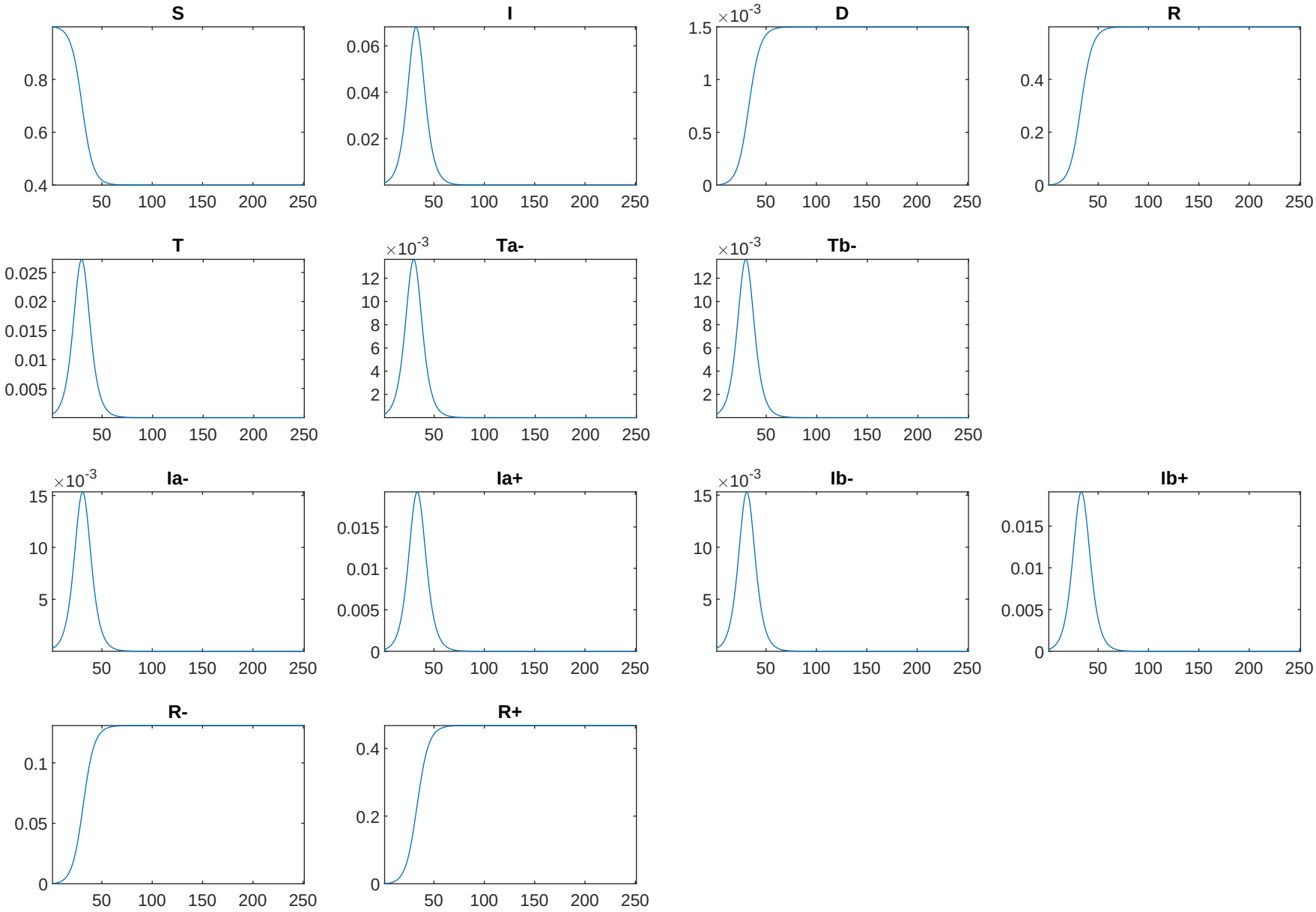

Figure 4: The Evolution of an Epidemic - SI4R, No Policy.

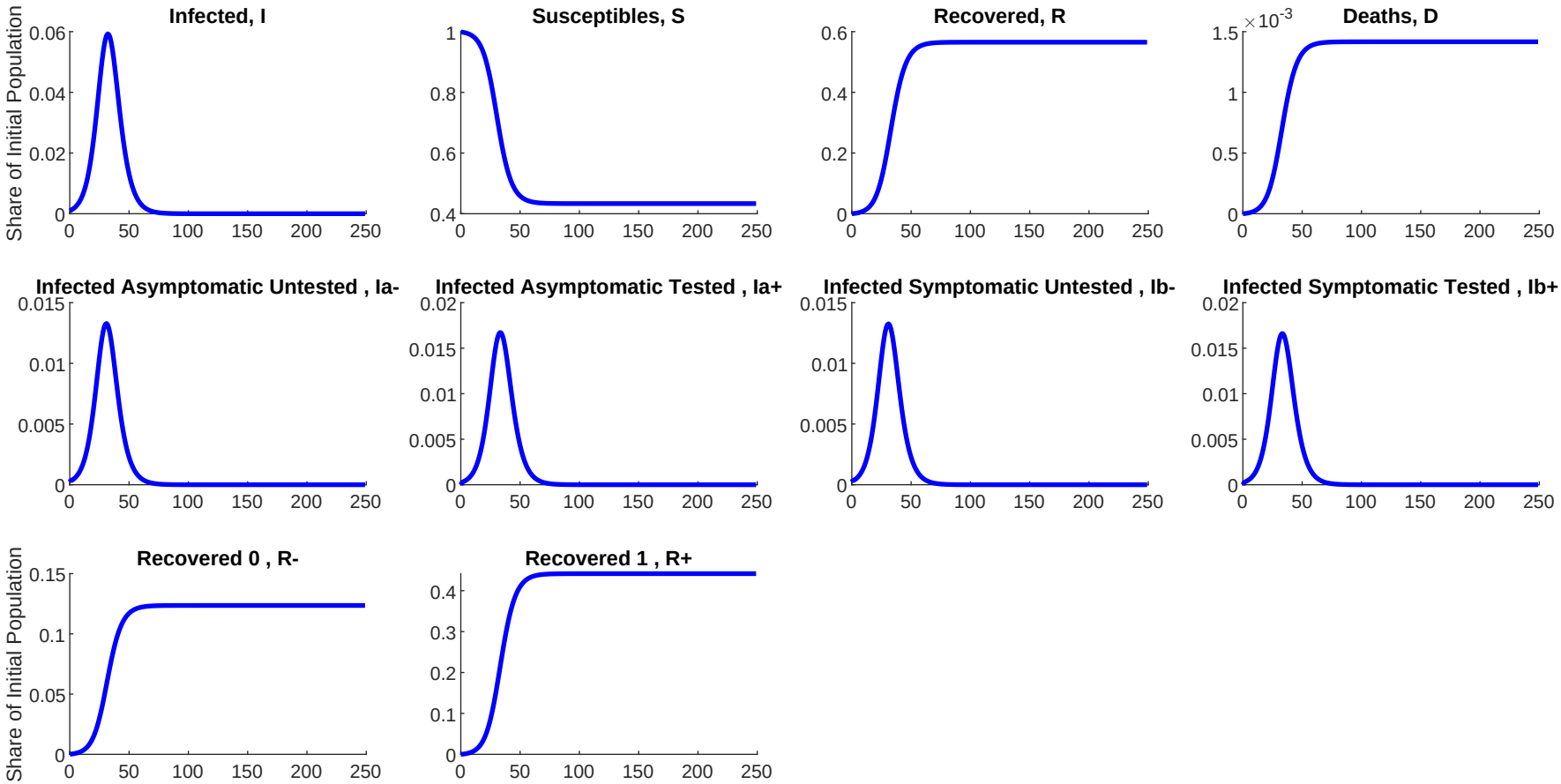

Figure 5: The Evolution of an Epidemic - SI4R, No Policy.

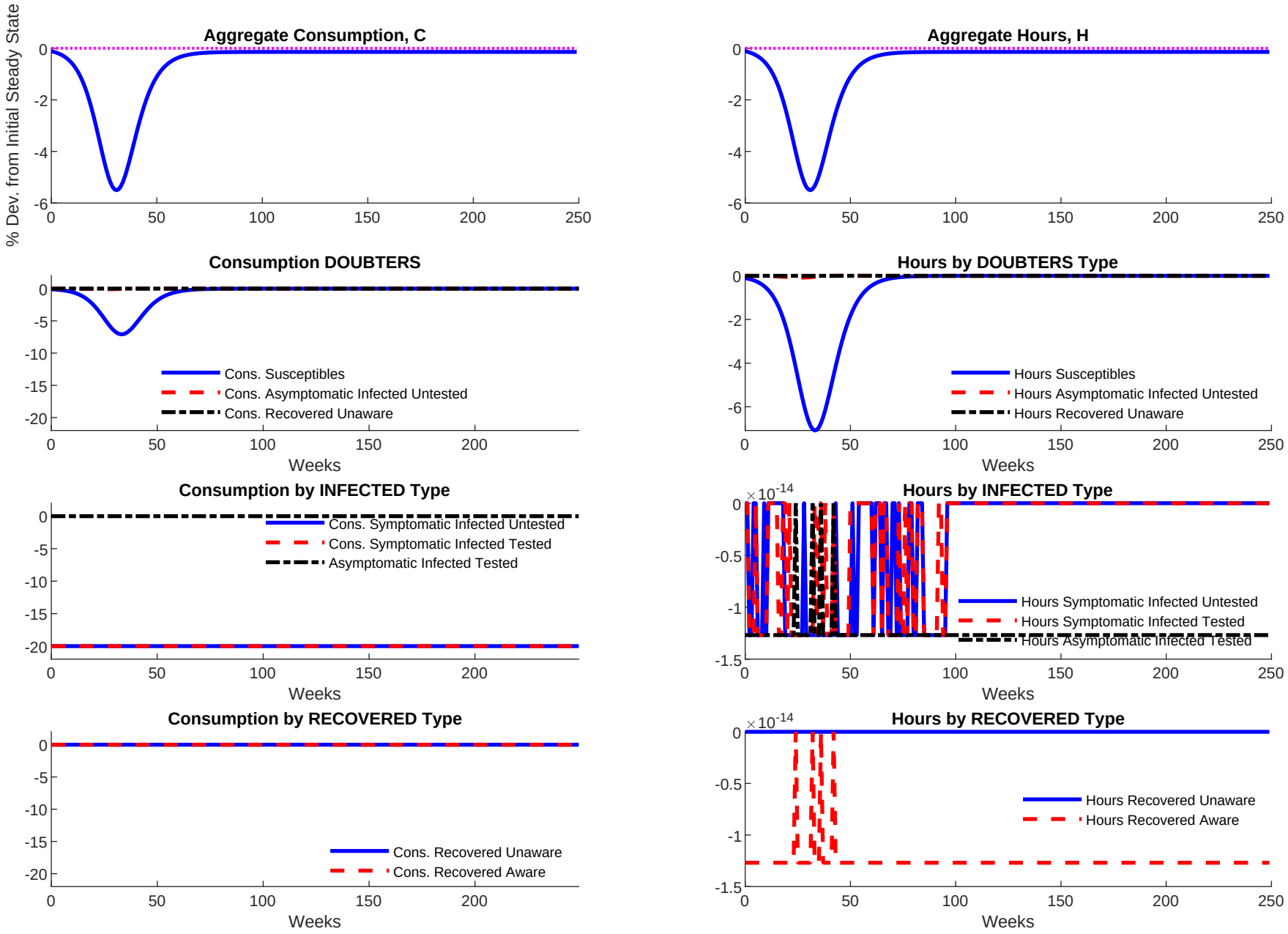

Figure 6: The Evolution of an Epidemic - SI4R, No Policy.

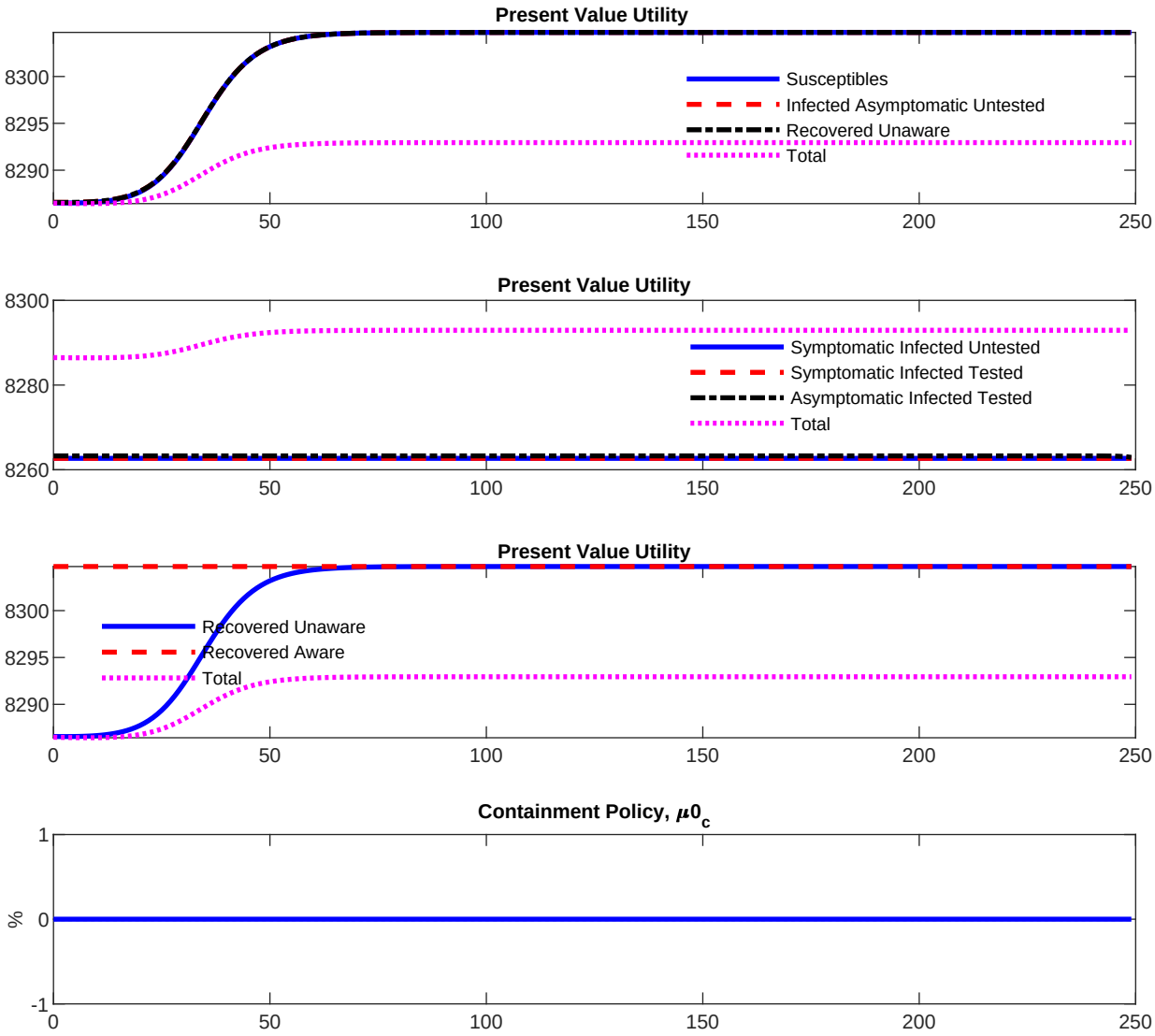

Figure 7: The Evolution of an Epidemic - SI4R, Optimal Policy.

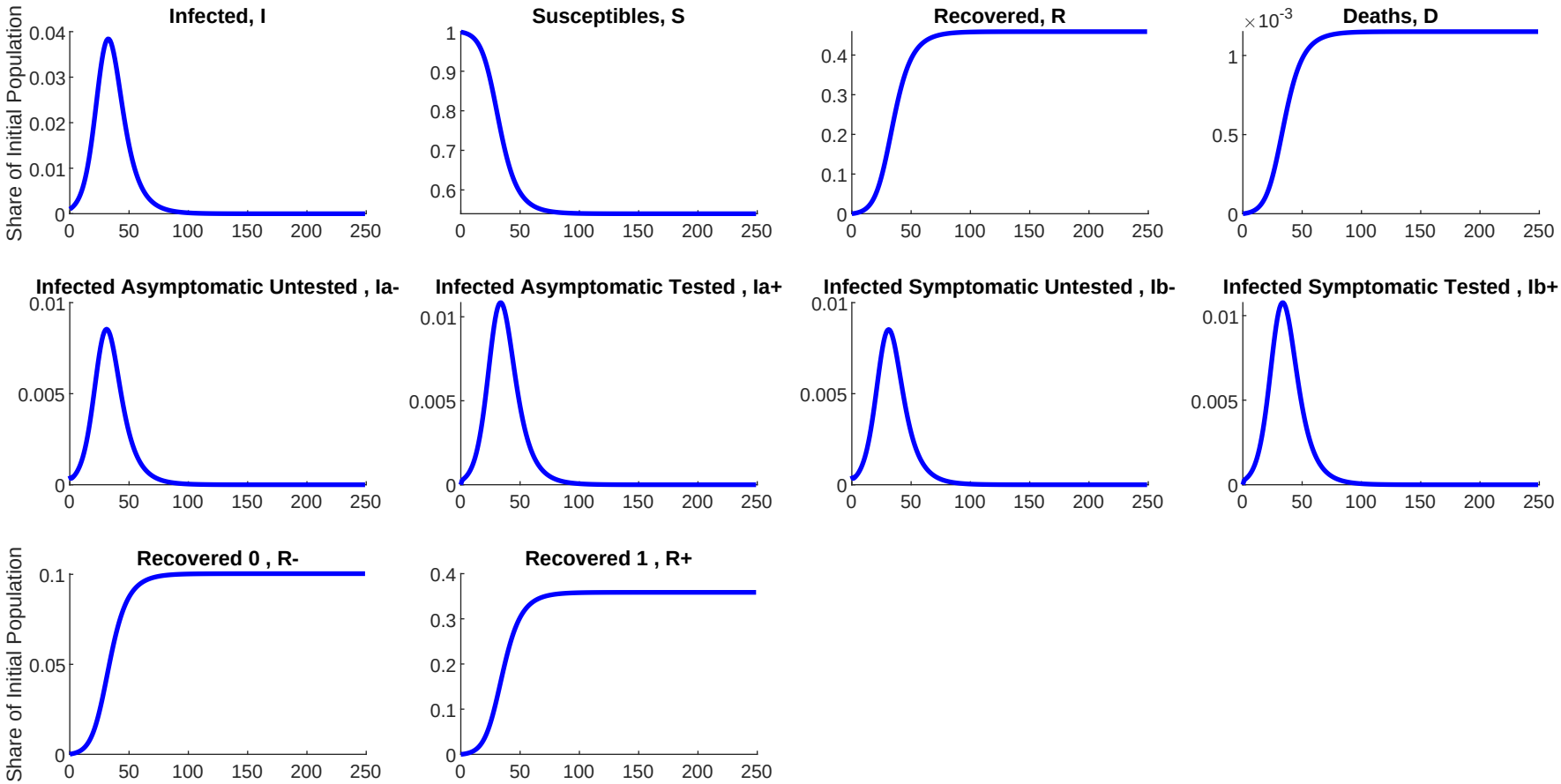

Figure 8: The Evolution of an Epidemic - SI4R, Optimal Policy.

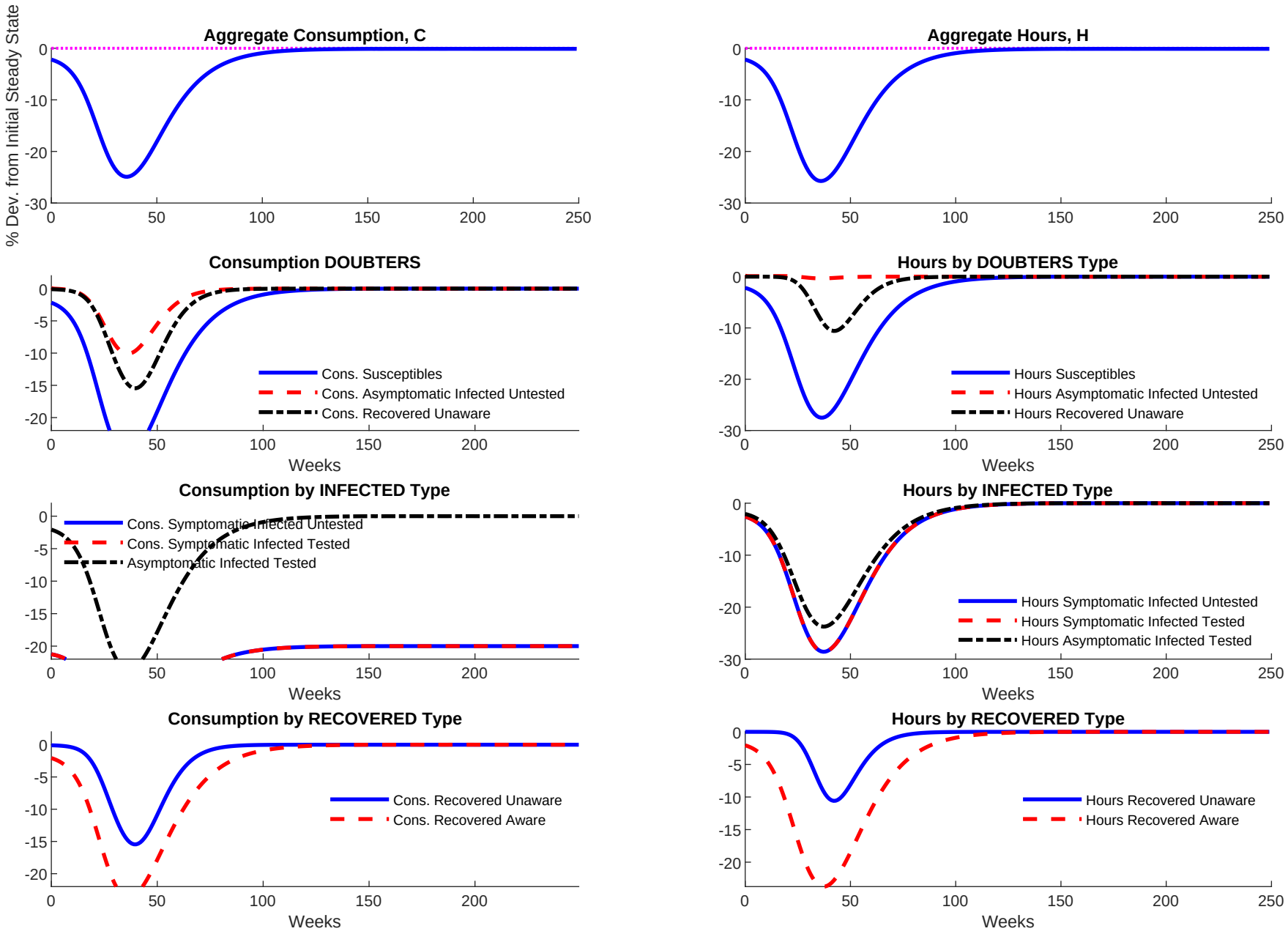

Figure 9: The Evolution of an Epidemic - SI4R, Optimal Policy.

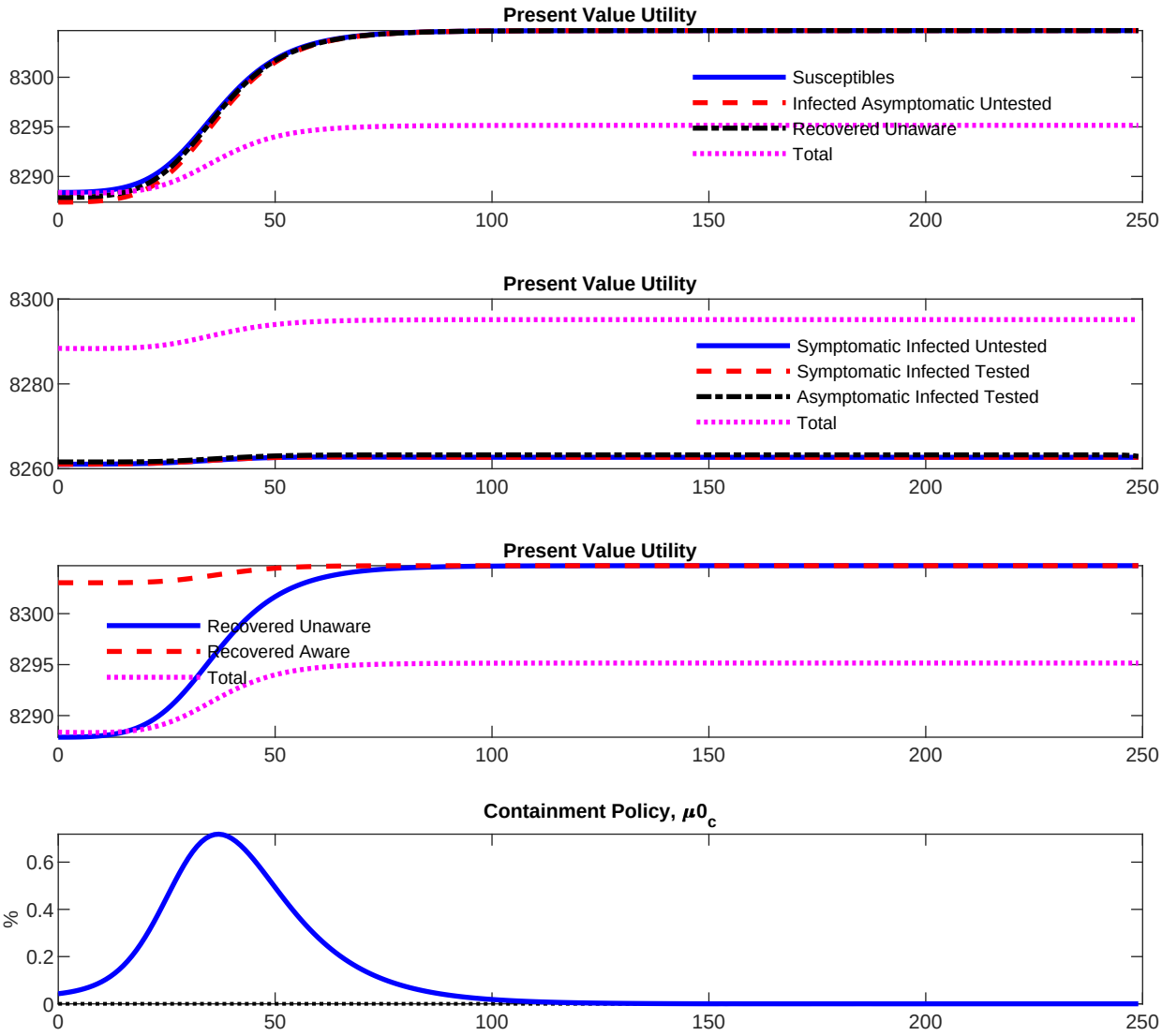

Figure 10: The Evolution of an Epidemic - SI4R, Optimal Policy for Tested.

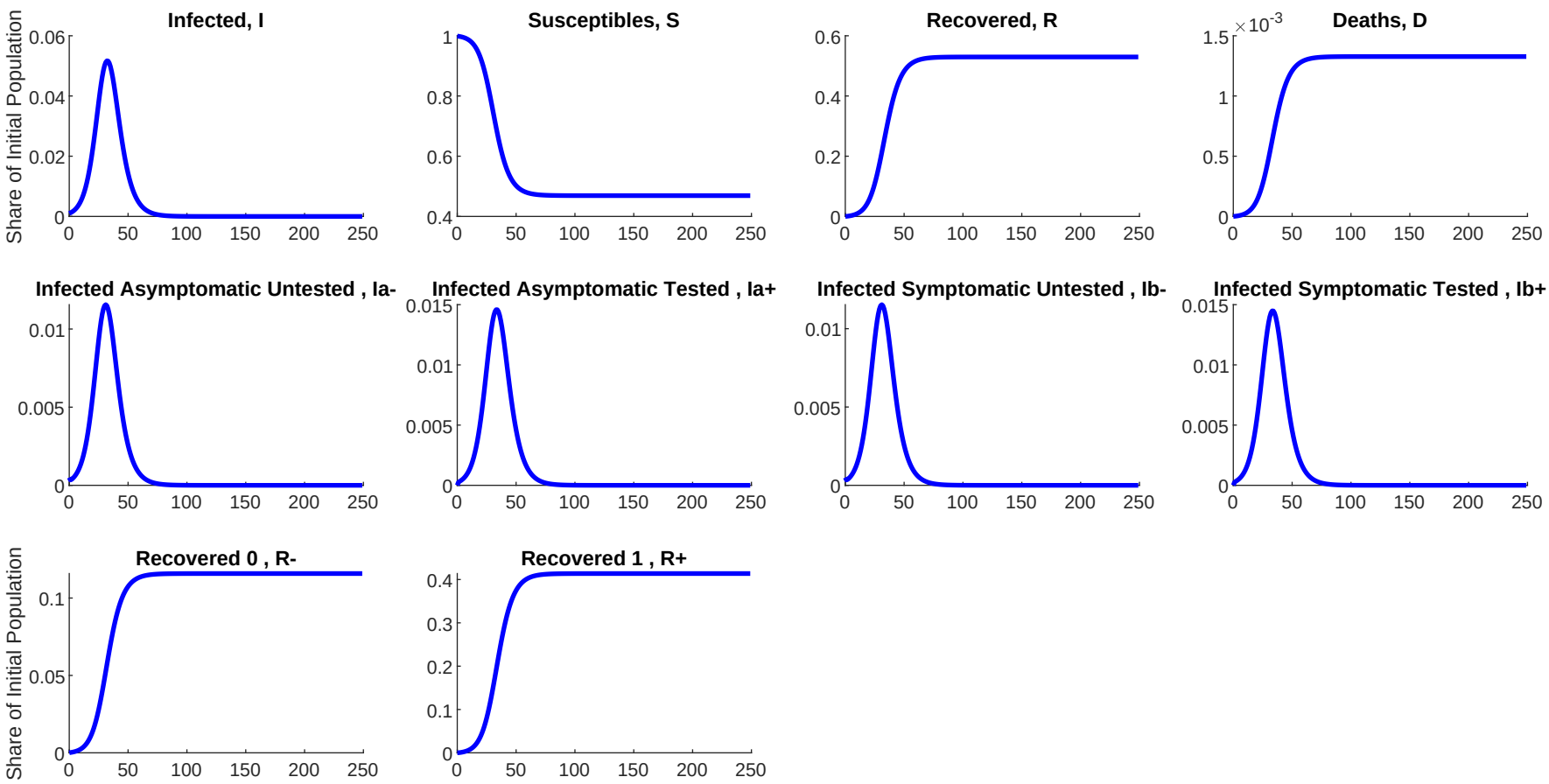

Figure 11: The Evolution of an Epidemic - SI4R, Optimal Policy for Tested.

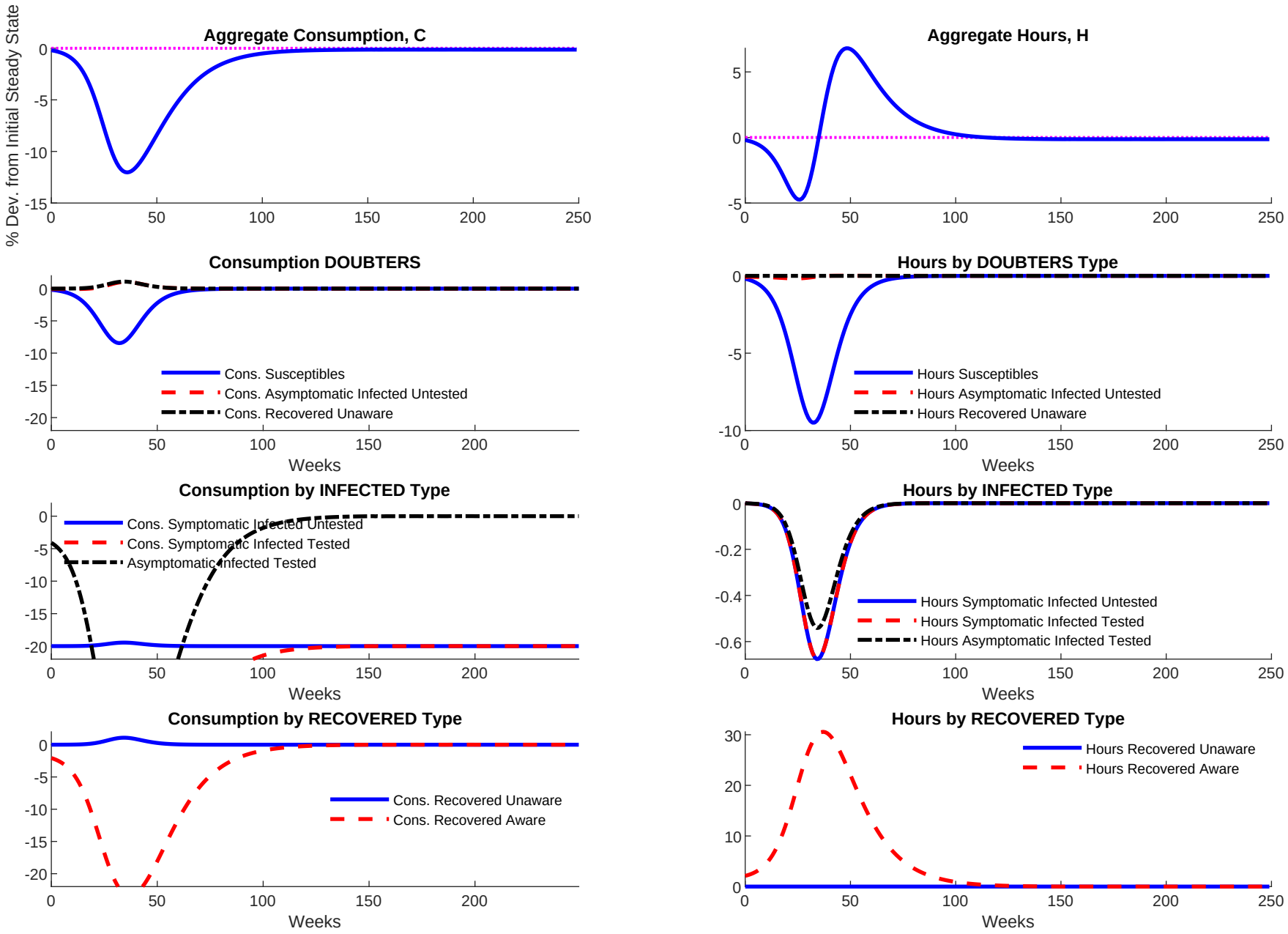

Figure 11: The Evolution of an Epidemic - SI4R, Optimal Policy for Tested.