Solutions to Sheet 10.

Problem 1

One might think that this problem is incredibly complicated, but in reality it is terribly simple. Let $P(q,a)$ be the smallest prime congruent to a mod q. The idea is now to plug this into to use the that

$$\psi(P(q,a)-1,q,a)=0.$$

Siegel-Walfisz says that $\psi(x,q,a)\approx\frac{x}{\varphi(q)}$ if x is large, so we'd expect to find that $P(q,a)$ cannot be too large if the equality above holds.

To apply Siegel-Walfisz we first need to make sure that $x\geq\exp(q^{\varepsilon})$ for some $\varepsilon>0$. We then have $\sqrt{\log x}\gg q^{\varepsilon/2}\gg(\log 2)^{\varepsilon}$ so that certainly

Problem 2

Problem 3

Problem 4

Max von Consbruch, email: s6mavonc@uni-bonn.de. Date: December 17, 2022