Solutions to Sheet 9.

Problem 1&2

Problem 3

Problem 4

Okay, let $c > 0$ and let q and q' be two exceptional moduli with zeroes characters χ, χ' and real zeroes β, β' satisfying the condition of the exercise. Let's compare the assumptions with the statement of (5.12).

(A) We have $1 - \frac{c}{\log q} < \beta$, and similar for q'.

(5.12) There is some small $d > 0$ (independent of q and q') such that we have $\min(\beta, \beta') \leq 1 - \frac{d}{\log(qq')}$.

If we assume $q < q'$, we certainly obtain

$$1 - \frac{c}{\log q} < 1 - \frac{d}{\log(qq')}, \quad \text{i.e.} \quad \frac{d}{c} < \frac{\log(qq')}{\log q}, \quad \text{i.e.} \quad q' > q^{d/c-1}.$$

Thus, any $c < d/3$ does the job.

This shows that there are $O(\log \log n)$ exceptional moduli up to n.

Max von Consbruch, email: s6mavonc@uni-bonn.de. Date: December 13, 2022