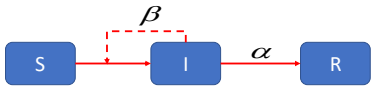
Lecture 2

Modelling Exercises

Classical Models:
Epidemiology – SIR Model



$$S' = -\beta SI$$
$$I' = \beta SI - \alpha I$$
$$R' = \alpha I$$

Classical Models:
Population Dynamics – Interacting Species

Neutralism
Amensalism
Commensalism
Competition
Mutualism
Predation
Parasitism

A general two-species interaction model

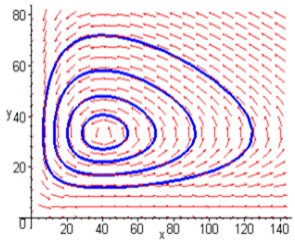
$$\frac{dx}{dt} = \alpha x + \beta xy$$
$$\frac{dy}{dt} = \gamma y + \delta xy$$

$\alpha, \beta, \gamma > 0 \quad \delta < 0$ $\alpha, \gamma > 0 \quad \beta, \delta < 0$

$\alpha, \beta, \delta > 0 \quad \gamma < 0$

Lotka-Volterra Model

$$\frac{dN}{dt} = N(a - bP)$$
$$\frac{dP}{dt} = P(cN - d)$$



Questions

Lotka-Volterra Model

$$\frac{dN}{dt} = N(a - bP)$$
$$\frac{dP}{dt} = P(cN - d)$$

Which species is the prey? Predator?

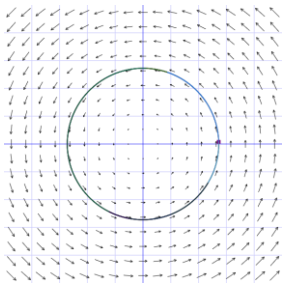
Changing which parameter will make it more likely for species 1 to go extinct?
What about species
2?

Juliet is in love with Romeo, who is a fickle lover. The more Juliet loves him, the more he begins to dislike her. But when she loses interest, his feelings for her warm up. She, on the other hand, tends to echo him: her love grows when he loves her, and turns to hate when he hates her.

$r(t)$ = Romeo's love/hate for Juliet at time t
$j(t)$ = Juliet's love/hate for Romeo at time t

$$\frac{dj}{dt} = br \qquad \frac{dr}{dt} = -aj \qquad a, b \in \mathbb{R}^{\oplus}$$

The solutions of this system is a never-ending cycle of love and hate



Make your own love story

$$\frac{dj}{dt} = br \qquad \frac{dr}{dt} = aj$$