Homework 1

1. Perform the calculations and express the result in the form $a + ib$.

$$a)\quad (3-2i)^2-(3+2i)^2,\qquad b)\quad (1+2i)^6,\qquad c)\quad (1+i+i^2+i^3)^{100},$$

$$d)\quad \frac{4-3i}{1+i},\qquad e)\quad \frac{(1+2i)^2-(1-i)^3}{(3+2i)^3-(2+i)^2},\qquad f)\quad \left(\frac{1-i}{i+1}\right)^8.$$

2. In each part solve for z:

$$a)\quad (i-z)+(2z-3i)=-2+7i,\qquad b)\quad (4-3i)\bar{z}=i.$$

3. In each part plot the point and sketch the vector that corresponds to the given complex number.

$$a)\quad 2+3i,\qquad b)\quad -3-2i,\qquad c)\quad -5i,\quad d)\qquad -2-2i$$

4. In each part express the complex number in polar form using its principal argument

$$a)\,2i,\quad b)\,-4,\quad c)\,5+5i,\quad d)\,-3-3i,\quad e)\,2\sqrt{3}-2i,\quad f)\,-6-6\sqrt{3}i.$$

5. Given that $z_1 = 2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ and $z_2 = 3(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$, find the polar form of

$$a)\quad z_1z_2\quad ,\qquad b)\quad \frac{z_1}{z_2}\quad ,\qquad c)\quad \frac{z_2^3}{z_1^2}$$

6. Express $z_1 = i$, $z_2 = 1 - i\sqrt{3}$ and $z_3 = \sqrt{3} + i$ in polar form and use these results to find $\frac{z_1z_2}{z_3}$. Check your results by performing the calculations without using polar forms.

7. Find the modulus and the principal value of the argument of the complex number

$$z = (1-i)^4(3+3i)^2.$$

8. Express the given complex numbers in algebraic form, in polar form and as a point or a vector in a complex plane.

$$z_1 = \frac{1}{1-i\sqrt{3}},\qquad z_2 = \frac{1}{2(cos\frac{\pi}{6} - isin\frac{\pi}{6})}$$