CS512 - Assignment - 0

SPRING - 23

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(A)

(1) $2a - b$

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \Rightarrow 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

(2) $$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{i+2j+3k}{\sqrt{1^2+2^2+3^2}} = \frac{i+2j+3k}{\sqrt{14}} = \begin{bmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{bmatrix}$$

(3) $\|a\| = \sqrt{14}$

$\vec{a} = \vec{i + 2j + 3k}$ $\quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\vec{b} = i + 0 + 0$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a|\,|b|} = \frac{1}{\sqrt{14}\sqrt{1}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) = 74.49$$

(4) Cosine direction of a

$$\frac{\vec{a}}{|\vec{a}|} = \frac{i+2j+3k}{\sqrt{14}} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

(5) $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a||b|} = \dfrac{(i+2j+3k)\cdot(4i+5j+6k)}{\sqrt{14}\sqrt{17}}$

$= \dfrac{32}{\sqrt{14}\cdot\sqrt{17}} \Rightarrow \dfrac{32}{7\sqrt{22}}$

(6) $\vec{a}\cdot\vec{b} = (\vec{i}+2\vec{j}+3\vec{k})\cdot(4\vec{i}+5\vec{j}+6\vec{k})$

$= 32$

$\vec{b}\cdot\vec{a} = (4\vec{i}+5\vec{j}+6\vec{k})\cdot(\vec{i}+2\vec{j}+3\vec{k})$

$= 32$

$\therefore$ Commutative

(7) $\vec{a}\cdot\vec{b} = \cos\theta\,|a|\cdot|b|$

$\therefore \cos\theta = \dfrac{32}{7\sqrt{22}}$, $|a| = \sqrt{14}$, $|b| = \sqrt{77}$

$= \cancel{\sqrt{14}\cdot\sqrt{77}}\cdot\dfrac{32}{\cancel{7\sqrt{22}}}$

$= 32$

(8) Scalar projection of b onto $\hat{a}$

$= \dfrac{\vec{b}\cdot\hat{a}}{|\vec{a}|}$

$|\vec{a}| = \sqrt{14}$

$\vec{b}\cdot\vec{a} = 32$

$\therefore \dfrac{32}{\sqrt{14}} = 8.556$

(9) let x is perpendicular to $\vec{a}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{x}}{|\vec{a}|\cdot|\vec{x}|}, \quad x = [x_1, x_2, x_3].$$

Perpendicular where angle should be $90°$

$$\cos\theta = \cos(90) \Rightarrow \underline{0}$$

$$(\vec{i} \cdot 2\vec{j} + 3\vec{k}) \cdot \cancel{x_1 j + x_2 j} \, (x_1\vec{i} + x_2\hat{j} + x_3\vec{k}) = 0$$

let $x_2 = 4$

$x_3 = 5$

$x_1 + 2(4) + 3(5) = 0$

$x_1 + 8 + 15 = 0$

$\underline{x_1 = -23}$

Perpendicular Vector to Vector A is $\begin{bmatrix} -23 \\ 4 \\ 5 \end{bmatrix}$

(10) $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$= \vec{i}(2\times6 - 5\times3) - j(6-12) + k(1\times5 - 2\times4)$

$= \vec{i}(-3) + 6\vec{j} - 3\vec{k}$

$= -3\vec{i} + 6\vec{j} - 3\vec{k}$

$\vec{b} \times \vec{a}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = \vec{i}(5\times3-6\times2)-\vec{j}(4\times3-6\times1)+ \vec{k}(4\times2-5\times1)$$

$$= 3\vec{i} - 6\vec{j} + 3\vec{k}$$

~~a and~~
Both are same in magnitude but they differ in direction

(11) let $\vec{x}$ be the vector perpendicular to both a & b

$\vec{a}\cdot\vec{x} = 0$ $\quad$ $\vec{b}\cdot\vec{x} = 0$

$\vec{x} = \vec{i} + x_2\vec{j} + x_3\vec{k}$

let $x_3 = 1$

$x_1 + 2x_2 + 3(1) = 0$

$x_1 + 2x_2 + 3 = 0$

$x_1 = -2x_2 - 3$

$x_1 = -2(-2) - 3$

$= 4 - 3$

$x_1 = 1$

$4x_1 + 5x_2 = -6$

$4(-2x_2 - 3) + 5x_2 = -6$

$-8x_2 - 12 + 5x_2 = -6$

$-3x_2 - 12 = -6$

$-3x_2 = 6$

$x_2 = -\frac{6}{3} = -2$

$x_2 = -2$

(13) $a^T b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

$ab^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$= 4 + 10 + 18$

$= 32$

B)

(1) $2A - B$

$= \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$

(2) $AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 2 & -21 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

(3) $$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} \quad (AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = \quad B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

(4) $$|A| = 1(2-15) - 2(-4-0) + 3(20-0)$$
$$= -13 + 8 + 60$$
$$= 55$$

$$|C| = 0$$

⑤

$a_{*,1} = (1,2,3)$ $a_{*,2} = (4,-2,3)$, $a_{*,3} = (0,5,-1)$

$(a_{*,1})\cdot(a_{*,2}) = 4(1)+2(-2)+3(3) \neq 0$

$(a_{*,1})\cdot(a_{*,3}) = 4(0)+2(5)+3(-1) \neq 0$

$(a_{*,2})\cdot(a_{*,3}) = 4(0)+5(-2)+3(-1) \neq 0$

$b_{*,1} = (1,2,1)$ $b_{*,2} = (2,1,-4)$, $b_{*,3} = (3,-2,1)$

$(b_{*,1})(b_{*,2}) = 2+2-4=0$ $\therefore$ orthogonal set

$(b_{*,1})\cdot(b_{*,3}) = 3-4+1=0$ $\therefore$ This forms an orthogonal set

$(b_{*,2})\cdot(b_{*,3}) = 6-2-4=0$ $\therefore$ forms an orthogonal set

$C_{*,1} = (1,2,3)$, $C_{*,2} = (4,5,6)$, $C_3 = (-1,1,3)$

~~$C_{*,1} =$~~

$(C_{*,1})(C_{*,2}) = 1(4)+2(5)+6(3) \neq 0$

$(C_{*,2})(C_{*,3}) = 4(-1)+5(1)+6(3) \neq 0$

$(C_{*,1})(C_{*,3}) = 1(-1)+2(1)+3(3) \neq 0$

$\therefore$ B is the only vectors which forms orthogonal set.

(6) $A^{-1} = \frac{1}{|A|} \times Adj(A)$

$Adj(A) = \begin{bmatrix} -13 & 4 & 20 \\ 12 & -1 & -5 \\ 12 & 7 & 10 \end{bmatrix}$, $|A| = 55$

$\frac{1}{55} \times \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.236 & 0.309 & 0.218 \\ 0.072 & 0.018 & 0.163 \\ 0.363 & -0.099 & -0.181 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 0.166 & 0.095 & 0.214 \\ 0.332 & 0.047 & 0.142 \\ 0.166 & -0.190 & 0.071 \end{bmatrix}$

(7) Not possible since $|C|$ is zero.

(8) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 9 \\ 13 \end{bmatrix}$

(9) Scalar projection $\vec{a}$ on $\hat{d} = \frac{\vec{a} \cdot \vec{d}}{|\vec{d}|}$

$|\vec{d}| = \sqrt{1^2 + 2^2 + 3^2}$

$= \sqrt{14}$

$\cancel{a_{*1} = 1(1)+2(2)+3(3)}$

$(a_*, 1) * d = 1(1)+2(2)+3(3)$

$= 14$

$\frac{\vec{a} \cdot \vec{d}}{|\vec{d}|} = \frac{14}{\sqrt{14}}$

$(a_*, 2) * d = 4(1)+(-2)2+3(3)$

$= 9$

$\frac{\vec{a} \cdot \vec{d}}{|\vec{d}|} = \frac{9}{\sqrt{14}}$

$(a_*, 3) * d = 0(1)+5(2)-1(3)$

$= 7$

$= \frac{7}{\sqrt{14}}$

(10) Vector projection

$$\frac{\vec{a} \cdot \vec{d}}{(|\vec{d}|)^2} \cdot \vec{d}$$

$(\overrightarrow{a_*, 1}) \cdot \vec{d} = 14$

$= \frac{14}{\sqrt{14}} \cdot (1\ 2\ 3)$

$= 1\ 2\ 3$

$(\vec{a}_{*,2}) \cdot \vec{d} = 9$

$= \frac{9}{(\sqrt{14})^2} \cdot (1,2,3)$

$= \frac{9}{14} \quad \frac{18}{14} \quad \frac{27}{14}$

$(\vec{a}_{*,3}) \cdot \vec{d} = 7$

$= \frac{7}{(\sqrt{14})^2} \cdot (1,2,3)$

$= \frac{7}{14}, \frac{14}{14}, \frac{21}{14}$

(11) Columns of (A) $= \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$

$d = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$d A_{*,1} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$ $\quad d A_{*2} = \begin{bmatrix} 4 \\ -4 \\ 10 \end{bmatrix}$ $\quad d A_{*}3 = \begin{bmatrix} 9 \\ 9 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$

(12) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & -6 & 0 \\ 0 & -8 & -2 & 0 \end{bmatrix}$$

$R_2 \to R_2 \times (-8) \qquad R_3 \to R_3 \times 3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 24 & 48 & 0 \\ 0 & -24 & -6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 24 & 48 & 0 \\ 0 & 0 & 42 & 0 \end{bmatrix}$$

$42c = 0 \qquad\qquad 24b + 48c = 0 \qquad\qquad 1a + 2b + 1c = 1$

$c = 0 \qquad\qquad 24b + 0 = 0 \qquad\qquad a = 1$

$b = \cancel{24}\ 0$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(13) $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1 \qquad R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 3 & 6 & 4 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

C cannot be determined and vectors are not linearly independent.

c)

1) $D = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $\qquad \lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$D - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$$

$= (1-\lambda)(2-\lambda) - 3(2)$

$= -3\lambda + \lambda^2 - 4$

$= \lambda^2 - 3\lambda - 4 = 0$

$= \lambda^2 - 4\lambda - \lambda - 4 = 0$

$= \lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$(\lambda + 1)(\lambda - 4) = 0$

$\lambda = -1, \quad \lambda = 4$

if $\lambda = -1$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_1 \to 3R_1 - 2R_2$

$$\begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x_1 + 3x_2 = 0$

$x_1 = -x_2$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ eigenvectors

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+2 \\ -3+2 \end{bmatrix} = \cancel{\lambda} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \cancel{\lambda}_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} = -1 \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -2+4 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

if $\lambda = 4$

$$\begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~R_3~~ $R_2 \to R_2 - R_1$

$$\begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 2x_2 = 0$$

$$2x_2 = 3x_1$$

$$x_1 = \frac{2}{3} x_2$$

$\begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix}$ eigenvector

(2) $\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{pmatrix}$

(3) $E = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

~~$E \to$~~ $E - \lambda I = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$

$$= (2-\lambda)(5-\lambda) - 2(-2)$$

$$= 10 - 2\lambda - 5\lambda + \lambda^2 - 4$$

$$= \lambda^2 - 7\lambda + 6$$

$\lambda = 1 \quad , \lambda = 6$

if $\lambda = 1$

$$\begin{bmatrix} 2-1 & -2 \\ -2 & 5-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$1x_1 - 2x_2 = 0$

$x_1 = 2x_2$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

if $\lambda = 6$

$$\begin{bmatrix} 2-6 & -2 \\ -2 & 5-6 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

$R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 0 & 0 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x_1 - x_2 = 0$

$-2x_1 = x_2$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

⑦ $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - 4$

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_1 \to R_1 - 2R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

It is invertable & independent matrix.

D)

① $f(x) = x^2 + 3$

$= \frac{d}{dx}(x^2 + 3)$

$= \frac{d}{dx} 2x$

$= x$

② $q(x,y) = x^2 + y^2$

$$\frac{dq}{dx} = 2x$$

$$\frac{dq}{dy} = 2y$$

③ $\nabla q(x,y) = \frac{dq}{dx}, \frac{dq}{dy}$

$$\frac{d}{dx}(x^2+y^2) = 2x$$

$$\frac{d}{dy}(x^2+y^2) = 2y$$

$$\frac{dq}{dx} = 2x, \quad \frac{dq}{dy} = 2y$$

$$\nabla q(x,y) = (2x, 2y)$$

④ $\frac{d}{dx} f(g(x))$

$$g(x) = x^2$$

$$f(x^2) = x^4 + 3$$

$$\frac{d}{dx}(x^4+3)$$

$$= 4x^{4-1}$$

$$= 4x^3$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} f(x^2) \times \frac{d}{dx} g(x)$$

$$\frac{d}{dx}(x^2+3) \cdot \frac{d}{dx}(x^2)$$

$$= 2x \cdot 2x$$

$$= 4x^2$$