CS-512 : COMPUTER VISION

ASSIGNMENT-1

(SPRING-2023)

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a) matrix form

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$f = 10$, $(x, y, z) = (3, 2, 1)$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

b) In traditional pinhole camera model, the light rays passes through a single point and converge to form an inverted image on the image plane whereas in alternative pinhole camera model, the image won't be inverted. The choice of which model to choose depends on specific needs of the application and both can be used effectively in different situations.

c) If focal length gets bigger, then the objects in the scene appear smaller due to smaller projection of the object onto the image plane. Similarly, when distance of the object increases, the projection of the object becomes smaller and may also change in shape.

d) Given the 2D point (1,1)

$(1,1) \rightarrow (1,1,1)$

2D 2DH

Another 2DH point that corresponds to same 2D point is

We can find it by scaling the original homogeneous coordinates with non-zero value

$= (x,y,1) * w$

$= (1,1,1) * 4$

$= 4,4,4$

Bringing back

$\left(\frac{x}{w}, \frac{y}{w}, \frac{w}{w}\right) = \left(\frac{4}{4}, \frac{4}{4}, \frac{4}{4}\right)$

$= (1,1,1)$

e) Given 2DH point (1,1,2)

2D point $= \left(\frac{1}{2}, \frac{1}{2}\right)$

f) 2DH point (1,1,0) represents the infinity which represents the direction

g) The use of homogeneous coordinates makes it possible to write non-linear projections as linear equations because it allows divide by w operation.

* Convert the points to homogeneous coordinates by adding 1 in last dimension.

h) $M = K[I|0]$ $M \to 3\times4,\ K = 3\times3,\ I \to 3\times3,\ 0 \to 3\times3$

3×4 3×3 3×3 3×3

i) $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ $P = [1, 2, 3]$

$$\underset{\text{2DH}}{\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}} = \underset{\substack{\text{projection} \\ \text{matrix}}}{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{\text{3D points}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

To represent 2DH point in 2D, we divide ~~that~~ $\frac{u}{w}$ and $\frac{v}{w}$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{18}{10} \\ \frac{46}{10} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1.8 \\ 4.6 \end{bmatrix}$$

2)

a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

b)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

c)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

d)

Rotation about abitrary point

$$R_{P,G}(\theta) = T(P)\, R(\theta)\, T(-P)$$

$$= \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -tx \\ 0 & 1 & -ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -2\sqrt{2}+2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ \sqrt{2}-(2\sqrt{2})+2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0.58 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0.58 \end{bmatrix}$$

e) $M' = T \cdot R$

f) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(transformation matrix)

This follows $\begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix}$ i.e. Scaling matrix

g) $M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ This is 2D "translation"

$$\begin{bmatrix} I & T \\ 0 & 1 \end{bmatrix} \cdot (P)$$

h) If we want to reverse the effect of M

Just Multiply it with M^{-1} - We get Identity matrix I

$= M \cdot M^{-1}$ $\Rightarrow \underline{(M \cdot P) \cdot M^{-1}}$ $\because P$ is 2D points

i) $M = R(45)\, T(1,2)$

$M^{-1} = T^{-1}(1,2) \cdot R^{-1}(45)$

j) $M = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$(i + 3j) \cdot (x_1 i + x_2 j) = 0$

let x_2 be 4

$(i + 3j) \cdot (x_1 i + 4j) = 0$

$x_1 + 3(4) = 0$

$x_1 = -12$

A vector which is perpendicular to $\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -12 \\ 4 \end{bmatrix}$

k) Vector projection

projecting vector $(1,3)$ onto the direction of vector $(2,5)$

let vector $a = (1,3)$ and $b = (2,5)$

$$\frac{\vec{a} \cdot \vec{b}}{(|\vec{b}|)^2} \cdot \vec{b}$$

$$|\vec{b}| = \sqrt{29}$$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}$$

$$\Rightarrow \frac{17}{(\sqrt{29})^2} \cdot (2,5)$$

$$= \frac{17}{29} \cdot (2,5)$$

$$= \begin{bmatrix} 1.172 \\ 2.931 \end{bmatrix}$$

3)

a) We use projection matrix because it is easy to convert 3D points from the world coordinates to 2D points on the image plane.

b) let $p^{(c)}$ be the points in the World Coordinates and $p^{(w)}$ be the points in the camera coordinates.

$$p^{(c)} = M_{c \leftarrow w} \, p^{(w)}$$

Where,

M is transformation Matrix

If we had to apply Rotation and translation then the projection matrix will be

$$M_{c \leftarrow w} = (T(t) R)^{-1}$$

$$= R^{-1} T^{-1}(t)$$

$$= R^T T(-t)$$

c)

$$R = \begin{bmatrix} \hat{x}_0 & \hat{y}_0 & \hat{z}_0 \end{bmatrix}$$

d)

$$M = \begin{bmatrix} R^* & T^* \\ 0 & I \end{bmatrix} \qquad \begin{aligned} R^* &= R^T \\ T^* &= -R^T t \end{aligned}$$

Where R^* is the Rotation matrix and T^* is the translation matrix

e)

$$M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$u_0, v_0 = (512, 512)$

$$M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & 512 \\ 0 & k_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

f)

$$p^{(i)} = \underbrace{K^*}_{\text{Intrinsic}} \underbrace{\left[R^* \mid T^* \right]}_{\text{Extrinsic}} p^{(W)}$$

K^* is ~~Responsible for~~ Intrinsic parameter having k_u, k_v, u_0, v_0, and f (focal length).

R^* and T^* are responsible for Rotation and translation and are extrinsic parameters from World to Camera Coordinate Systems.

g) We add skew parameter in the Camera model so that x & y axis are not perpendicular and necessary for Construction of 3D ~~image~~ Scenes ~~from~~.

h) If we take the wide angle lens, we may see that the straight lines which are far from the Center may appear as a Curve. (bend).

$$p^{(i)} \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} k^* [R|T^*] \; p^{(w)}$$

λ will change when you go further from the Center of the image

i) A weak perspective Camera model is a simple camera model which is used in Computer vision where parallel lines in the scene appear parallel ~~and~~ in the image and scale of the object is dependent on the depth. The objects which are closer to camera appear larger in the image while objects which are far from the Camera appear small in the image.

An affine Camera model considers basic shape of the objects in the image even if their size changes. It takes ~~into~~ lens distortion into Consideration.

4)

a) <u>Surface Radiance</u> :- It is the amount of light emitted from a Surface in the Scene and expressed in units of radiance

<u>Image Radiance</u> :- It is the amount of light Captured by Camera and applied it on the image.

b) Radiosity equations relating Surface radiance and image radiance

$$E(P) = L(P)\ \frac{\pi}{4}\left(\frac{d}{f}\right)^2 (\cos\alpha)^4$$

Where,

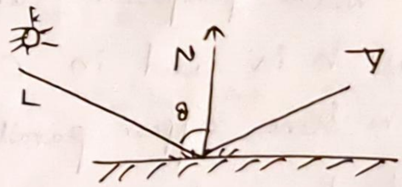
$E(P)$ = light at image

$L(P)$ = light at Surface

d = diameter of lens

f = focal length

α = angle b/w principal axis and Surface normal



c) <u>Albedo of a Surface</u> :-

Albedo of a Surface refers to the fraction of light that is reflected by the Surface, It is the measure of Surface's reflectivity and brightness and it is represented by a value b/w 0 and 1.

0 - being Completely black that absorbs all light

1 - White Surface that reflects all light.

g) We add Skew parameter in the Camera model so that x & y axis are not perpendicular and necessary for Construction of 3D ~~image~~ Scenes ~~from~~.

h) If we take the wide angle lens, we may see that the straight lines which are far from the Center may appear as a curve (bend).

$$p^{(i)} \quad \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} \quad k^* \left[R \mid T^* \right] \quad p^{(w)}$$

λ will change when you go further from the Center of the image

i) A Weak perspective Camera model is a simple camera model which is used in Computer vision where parallel lines in the Scene appear parallel ~~and~~ in the image and Scale of the object is dependent on the depth. The objects which are closer to Camera appear larger in the image while objects which are far from the Camera appear smaller in the image.

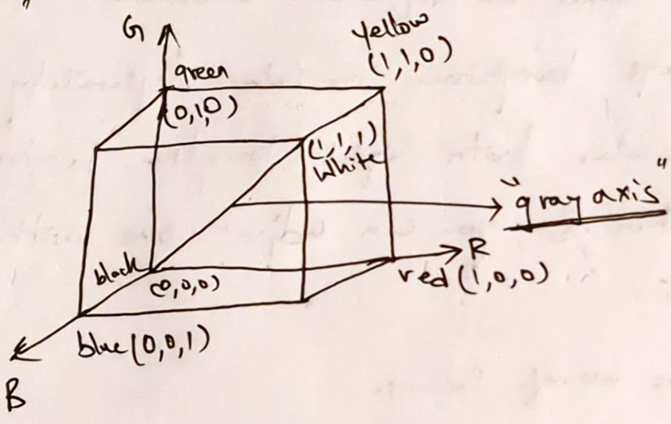
An affine Camera model considers basic shape of the objects in the image even if their size changes. It takes ~~into~~ lens distortion into Consideration.

d) We use RGB to represent colors because :-

i) It is based on the way human eye process color and can create a large range of colors

ii) By adding more light in red, green and blue we can make brighter colors say "white" for example.

e)



The diagonal line that is connecting (0,0,0) and (1,1,1) is gray color between white and black and it is called as gray axis.

f) Using CIE table we can map RGB colors to real world scenes by tuning different values of RGB.

g) Y in the CIE XYZ color model represents the brightness of a color that describes how bright ~~a color~~ or dark a color appears to human eye and also carries yellow-green information. The X component represents red-green axis and Z component represents blue-yellow axis.

h) Advantages of LAB Color Space :-

LAB Color Space represents Colors in three dimensional Space, Where one dimension represent the lightness (L) of the Color and the other two represent the Color itself.

Advantages :-

1. It keeps the color same on different devices.
2. It lets you change brightness or Color Separately because it Separates both light (brightness) information and Color information. So, you can adjust one without affecting the other.
3. It Can Show more no. of Colors.