Punto 2

$q = 1$

$$\frac{dv}{dt} = v \rightarrow \int \frac{dv}{v} = \int dt$$

$$\boxed{v = e^{t}}$$

$q > 1$

$$\frac{dv}{dt} = v^{q} \rightarrow \frac{v^{-q}\,dv}{dt} = v^{-q}v^{q}$$

$$\int_{v_0}^{v} v^{-q}\,dv = \int_{t_0}^{t} dt$$

$$\left.\frac{v^{(-q+1)}}{(-q+1)}\right|_{v_0}^{v} = t$$

$$\frac{1}{(-q+1)}\left[v^{(1-q)} - v_0^{(1-q)}\right] = t - t_0$$

$t_0 = 0$ y $U_0 = 1$

$$\frac{1}{(1-q)}\left[(U)^{(1-q)} - \cancel{y_0}^{\,1}\right] = t$$

$$U^{(1-q)} = t(1-q) + 1$$

$$\boxed{U = \sqrt[(1-q)]{t(1-q)+1}}$$

$q = 2$. $$U = \sqrt[(-1)]{t(1-2)+1}$$

$$= (t(-1)+1)^{(-1)}$$

$$\boxed{U_2 = \frac{1}{1-t}}$$

$q = 5$ $$U = (t(1-q)+1)^{\frac{1}{1-q}}$$

$$= (t(1-5)+1)^{\frac{1}{1-5}} = (1-4t)^{-\frac{1}{4}}$$

$$= \frac{1}{(1-4t)^{1/4}}$$

(2) B →

$$(x^{-3})\,x^3 y' = [x^4 y^2 - 2x^2 y - 1]\,x^{-3}$$

$$y' = \frac{dy}{dx} = x y^2 - \frac{2}{x} y - \frac{1}{x^3} \quad (A)$$

R.E.

$$\frac{dy}{dx} = A(x) y^2 + B(x)\, y + C(x) \quad (B)$$

por comparacion $A(x) = x$ $C(x) = -\frac{1}{x^3}$

$B(x) = \frac{-2}{x}$

$y_1 = \frac{1}{x^2}$ $\boxed{y = u(x) + y_1(x)}$ (3)

Tomando y

organizando en (B)

$$u' + [-2(A)\,y_1 - B]\,u = A\,u^2$$

$$v' + \left[\cancel{-2\left(x\right)\left(\frac{1}{x^2}\right)} - \left(\frac{-2}{x}\right)\right] v = xv^2$$

$$\frac{dv}{dx} = xv^2$$

$$\int \frac{dv}{v^2} = \int x\,dx$$

$$\frac{v^{-1}}{-1} = \frac{x^2}{2}$$

$$\frac{-1}{v} = \frac{x^2}{2} \rightarrow \boxed{v = \frac{-2}{x^2}}$$

$$y(x) = \frac{1}{x^2} - \frac{2}{x^2}$$

$$\boxed{y(x) = \frac{-1}{x^2}}$$

$$y(\sqrt{2}) = 0$$