Dark Matter in Compact Objects (TBD)

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Abstract

DM in COs Heat up Maybe See

Publications

Refs. [2, 3, 5, 4, 1] below are the journal publications, and preprints authored or co-authored during my PhD candidature. The authors are listed alphabetically in all of the titles.

Journal papers and preprints

[1] Papers

Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD except where indicated in the preface;
2. due acknowledgement has been made in the text to all other material used;
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Michael Virgato, XXX XXX

Preface

We don't know what DM is. Can NSs constrain it?

Acknowledgements

Why did I do this?

Contents

List of Figures **xiv**

List of Tables **xvi**

1 Introduction **1**

2 Compact Objects for Particle Physics **3**

3 Dark Matter Capture in Celestion Bodies **5**
3.1 Capture in the Sun 5
3.2 Capture in Compact Objects 5
3.3 White Dwarfs 5
3.4 Neutron Stars 5

4 Thermalisation in Compact Objects **7**

5 Dark Matter Induced Heating **9**

6 Conclusion **11**

Appendix A Kinematics **13**
A.1 Kinetic Heating 13
A.1.1 Keeping Angular Dependence 15
A.1.2 Checking Newtonian/Non-Relativistic Limit 15
A.1.3 Procedure for calculating kinetic heating time 16

Definition of Symbols and Abbreviations **17**

List of Figures

List of Tables

1 Introduction

Background on DM and its current status

This is the intro. About DM and its current status

2
Compact Objects for Particle Physics

Introduce COs, formation, structure etc...

3
Dark Matter Capture in Celestion Bodies

Review capture in the Sun, move to what's needed for COs in general, then specify to WDs (ions + electrons) and NS (interacting baryons)

3.1 Capture in the Sun

3.2 Capture in Compact Objects

3.3 White Dwarfs

3.4 Neutron Stars

4
Thermalisation in Compact Objects

Go Over the full thermalisation process for WDs and Neutron Stars

5
Dark Matter Induced Heating

DM kinetic and annihilation heating applied to NSs and WDs

6 Conclusion

Concluding remarks

A
Kinematics

Derivation of $E_f^{'}$ as needed for capture and other kinematics

A.1 Kinetic Heating

The metric at any point inside or outside the NS can be written as

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\phi + \sin\theta d\theta^2) \tag{A.1}$$

Along an orbit, the conserved conjugate momenta are the angular momentum per unit mass, $p_\phi = -L$ and the energy per unit mass $p_t = E_\chi$, and taking the orbit to lie in the $\theta = \pi/2$ plane leads to $p_\theta = 0$.

The equation which describes the orbit can be obtained from the square of the energy-momentum 4-vector,

$$g_{\alpha\beta}p^\alpha p^\beta - m_\chi^2 = 0 \tag{A.2}$$
$$\implies g^{\alpha\beta}p_\alpha p_\beta - m_\chi^2 = 0 \tag{A.3}$$

with

$$g^{tt} = 1/B(r), \quad g^{rr} = -1/A(r), \quad g^{\phi\phi} = -1/r^2 \tag{A.4}$$

$$\implies 0 = g^{tt}p_t p_t + g^{rr}p_r p_r + g^{\phi\phi}p_\phi p_\phi - m_\chi^2 \tag{A.5}$$

$$= \frac{E_\chi^2}{B(r)} - \frac{1}{A(r)}\left(g_{rr'}p^{r'}\right)\left(g_{rr'}p^{r'}\right) - \frac{L^2}{r^2} - m_\chi^2 \tag{A.6}$$

$$= \frac{E_\chi^2}{B(r)} - m_\chi^2 A(r)\left(\frac{dr}{d\tau}\right)^2 - \frac{L^2}{r^2} - m_\chi^2 \tag{A.7}$$

To find $dt/d\tau$, we use

$$p^t = m_\chi \frac{dt}{d\tau} = g^{tt}p_t = \frac{E_\chi}{B(r)} \tag{A.8}$$

$$\implies \frac{dt}{d\tau} = \frac{1}{B(r)}\frac{E_\chi}{m_\chi} \tag{A.9}$$

This gives

$$\left(\frac{dr}{dt}\right)^2 = \frac{B}{\tilde{E}_\chi^2 A}\left[\tilde{E}_\chi^2 - B(r)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right] \tag{A.10}$$

For simplicity, consider orbits that are a straight line ($\tilde{L} = 0$), which has a radial extent R. This is related to $\tilde{E}_\chi$ through

$$\tilde{E}_\chi^2 = B(R) \tag{A.11}$$

$$\implies R = \frac{2GM_\star}{1 - \tilde{E}_\chi^2}, \quad R > R_\star \tag{A.12}$$

using $B(r > R_\star) = 1 - 2GM_\star/r$.

It is important to note that E_χ so far has been the *conserved* energy along the orbit, which for the initial approach is $E_\chi = m_\chi + \frac{1}{2}m_\chi u^2 \sim m_\chi$. We now call this energy E_χ^{orbit}, which is related to the DM energy as seen by a distant observer, E_χ^{int}, and is the energy used in calculating the interaction rates, through

$$E_\chi^{\text{orbit}} = \sqrt{g_{tt}}E_\chi^{\text{int}} = \sqrt{B(r)}E_\chi^{\text{int}} \tag{A.13}$$

and as $E_\chi^{\text{orbit}} < m_\chi$ for all subsequent scatters after capture, eq. A.12 is always positive.

These "orbits" are straight lines that pass through the star's centre and extend an amount $R - R_\star$ on either side. Due to the symmetry of the motion, the period of the orbit is then

$$T_{\text{orbit}} = 4\int_0^R \frac{1}{dr/dt}dr \tag{A.14}$$

More relevant to this application is the time spent inside and outside the star, which is given by

$$T_{\text{inside}} = 4\int_0^{R_\star} \frac{1}{dr/dt} dr \tag{A.15}$$

$$T_{\text{inside}} = 4\int_{R_\star}^{R} \frac{1}{dr/dt} dr \tag{A.16}$$

A.1.1 Keeping Angular Dependence

For $\tilde{L} \neq 0$, we need the equation of motion for the angular coordinate ϕ,

$$\frac{d\phi}{d\tau} = g^{\phi\phi} p_\phi = \frac{\tilde{L}}{r^2} \tag{A.17}$$

$$\implies \frac{d\phi}{dt} = \frac{B(r)}{\tilde{E}_\chi} \frac{\tilde{L}^2}{r^2} \tag{A.18}$$

The maximum value of $\tilde{L}^2$ is given by

$$\tilde{L}^2_{\text{MAX}} = \frac{\tilde{E}_\chi^2 - B(r)}{B(r)} r^2 \tag{A.19}$$

and we parameterise the possible angular momentum along the orbit as

$$\tilde{L}^2 = y\tilde{L}^2_{\text{MAX}}, \quad 0 < y < 1. \tag{A.20}$$

leading to the equation for dr/dt simplifying to

$$\frac{dr}{dt} = \frac{B(r)}{\tilde{E}_\chi^2 A(r)} \left(\tilde{E}_\chi^2 - B(r)\right)(1 - y) \tag{A.21}$$

showing that the maximum radius of the orbit does not change from the $\tilde{L} = 0$ case, only the time spent outside the star changes.

A.1.2 Checking Newtonian/Non-Relativistic Limit

In the Newtonian limit, we take

$$B - 1 \approx 2\phi \ll 1, \tag{A.22}$$

$$A - 1 \approx -2GM(r)/r \equiv -2V(r) \ll 1, \tag{A.23}$$

$$\tilde{L}^2/r^2 \ll 1, \tag{A.24}$$

$$\tilde{E} - 1 = \varepsilon \ll 1, \tag{A.25}$$

with ε the non-relativistic energy per unit mass. Then expanding Eq. A.10 we get

$$\left(\frac{dr}{dt}\right)^2 = (1+2\phi)(1+2V) - (1+2\phi)^2(1+2V)(1-2\varepsilon)\left(1+\frac{\tilde{L}^2}{r^2}\right) \tag{A.26}$$

$$= 1 + 2\phi + 2V - \left(1 + 4\phi + 2V + \frac{\tilde{L}^2}{r^2} - 2\varepsilon\right) \tag{A.27}$$

$$= -2\phi - \frac{\tilde{L}^2}{r^2} + 2\varepsilon \tag{A.28}$$

$$\implies \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{\tilde{L}^2}{2r^2} + \phi = \varepsilon \tag{A.29}$$

which is the standard result for a Newtonian orbit.

A.1.3 Procedure for calculating kinetic heating time

- Select a point in the star for the DM to scatter off, $r_{\text{scatter},0}$.
- DM comes in from infinity with initial energy $E_\chi \approx m_\chi$
- Boost DM to local energy of $m_\chi/\sqrt{B(r_{\text{scatter}})}$
- Scatter the DM and calculate initial ΔE_χ
- Set local DM energy to $E_\chi \equiv p^t = m_\chi/\sqrt{B(r_{\text{scatter}})} - \Delta E_\chi$
- Calculate the new conserved energy per unit mass along the orbit as

$$\tilde{E}_\chi^{\text{orbit}} = \sqrt{B(r_{\text{scatter}})}E_\chi/m_\chi = \frac{\sqrt{B(r_{\text{scatter}})}}{m_\chi}(m_\chi/\sqrt{B(r_{\text{scatter},0})} - \Delta E_\chi) \tag{A.30}$$

- Use Equation A.11 to solve for the maximum radius of the orbit, R_{orbit}.
- Use equations A.15 and A.16 to calculate $T_{\text{in}}/(T_{\text{in}} + T_{\text{out}})$
- Adjust the time interval between scatter by $dt \to dt(T_{\text{in}}/(T_{\text{in}} + T_{\text{out}}))^{-1}$
- Iterate until $R_{\text{orbit}} < R_\star$

Definition of Symbols and Abbreviations

$\boldsymbol{C}_{\mathrm{geo}}$ Geometric Capture Rate

DM Dark Matter
$\boldsymbol{K}_{\chi}$ Dark Matter Kinetic Energy
$\boldsymbol{\rho}_{\chi}$ DM halo density
$\boldsymbol{m}_{\chi}$ Dark Matter Mass

EFT Effective Field Theory
EoS Equation of State

$\boldsymbol{f}_{\mathrm{FD}}$ Fermi-Dirac Distribution
$\boldsymbol{\varepsilon}_{F,i}$ Fermi kinetic energy of target species

$|\overline{\boldsymbol{\mathcal{M}}}|^{\mathbf{2}}$ Spin-averaged squared matrix element
$\boldsymbol{\mu}$ DM-Target mass ratio, m_χ/m_i

NS Neutron Star

PB Pauli Blocking

QMC Quark-Meson-Coupling EoS

$\boldsymbol{\sigma}_{\mathbf{th}}$ Threshold Cross Section

$\boldsymbol{T}_{\mathrm{eq}}$ Equilibrium Temperature
$\boldsymbol{t}_{\mathrm{eq}}$ Capture-Annihilation equilibrium time
$\boldsymbol{T}_{\star}$ Temperature of the star
$\boldsymbol{t}_{\mathrm{therm}}$ Thermalisation time

$\boldsymbol{v}_{d}$ DM halo dispersion velocity
$\boldsymbol{v}_{\star}$ Star velocity

Bibliography

[1] Filippo Anzuini et al. "Improved Treatment of Dark Matter Capture in Neutron Stars III: Nucleon and Exotic Targets". In: *JCAP* 11.11 (Nov. 29, 2021), p. 056. DOI: 10.1088/1475-7516/2021/11/056. arXiv: 2108.02525 [hep-ph].

[2] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in Neutron Stars". In: *JCAP* 09 (Sept. 15, 2020), p. 028. DOI: 10.1088/1475-7516/2020/09/028. arXiv: 2004.14888 [hep-ph].

[3] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in Neutron Stars II: Leptonic Targets". In: *JCAP* 03 (Mar. 26, 2021), p. 086. DOI: 10.1088/1475-7516/2021/03/086. arXiv: 2010.13257 [hep-ph].

[4] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in White Dwarfs". In: *JCAP* 10 (Oct. 29, 2021), p. 083. DOI: 10.1088/1475-7516/2021/10/083. arXiv: 2104.14367 [hep-ph].

[5] Nicole F. Bell et al. "Nucleon Structure and Strong Interactions in Dark Matter Capture in Neutron Stars". In: *Phys. Rev. Lett.* 127.11 (Sept. 10, 2021), p. 111803. DOI: 10.1103/PhysRevLett.127.111803. arXiv: 2012.08918 [hep-ph].