Dark Matter in Compact Objects (TBD)

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Abstract

DM in COs Heat up Maybe See

Publications

Refs. [2, 3, 5, 4, 1] below are the journal publications, and preprints authored or co-authored during my PhD candidature. The authors are listed alphabetically in all of the titles.

Journal papers and preprints

[1] Papers

Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD except where indicated in the preface;
2. due acknowledgement has been made in the text to all other material used;
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Michael Virgato, XXX XXX

Preface

We don't know what DM is. Can NSs constrain it?

Acknowledgements

Why did I do this?

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1
Introduction

Background on DM and its current status

A common remark during my time as a PhD candidate was that every possible introductory statement about the nature of Dark Matter (DM) has already been writen. While this is somewhat of an exhageration, one often finds that they have writen what they think is a new and interesting introductory statement, only to find it was exactly what you had writen a few papers ago! Therefore, as is the current custom, I defer this task to chat-GPT:

> “In the pursuit of understanding the fundamental constituents of the universe, the nature of dark matter stands as a persistent enigma. Evading direct detection and eluding conventional characterization, dark matter’s elusive presence exerts a profound influence on the large-scale structure and dynamics of the cosmos, compelling a rigorous exploration into its inherent mysteries.”

While this is not at all a unique introduction, it is sufficient for us to move on to the interesting parts of the thesis: the physics.

1.1 Evidence for Dark Matter

1.2 Potential Models of Dark Matter

1.3 Current Status of Dark Matter Constraints

1.3.1 Collider Bounds

1.3.2 Direct Detection Searches

1.3.3 Indirect Detection

Compact Objects

2
Compact Objects for Particle Physics

Introduce COs, formation, structure etc...

2.1 Production of COs

2.2 Internal structure

2.2.1 White Dwarfs

2.2.2 Neutron Stars

2.3 Observational Status

2.3.1 White Dwarfs

2.3.2 Neutron Stars

3

Introductio to Capture in Compact Objects: Point Like Targets

Review capture in the Sun, move to what's needed for COs in general, then specify to WDs (ions + electrons) and NS (interacting baryons)

3.1 Dark Matter Capture in the Sun

3.2 Capture in Compact Objects

3.3 White Dwarfs: Electron Targets

3.4 Neutron Stars: Leptoinc Targets

4
Capture Rate for Baryons

Go Over the full thermalisation process for WDs and Neutron Stars

5 Dark Matter Induced Heating of Neutron Stars

DM kinetic and annihilation heating applied to NSs and WDs

5.1 Thermalisation Time

5.2 Capture-Annihilation Equilibrium

5.3 Dark Matter Heating

5.3.1 Kinetic Heating

5.3.2 Annihilation Heating

6
Conclusion

Concluding remarks

A
Kinematics

Derivation of E_f' as needed for capture and other kinematics

B
Kinetic Heating

B.1 DM Orbits in General Isometric Metric

The metric at any point inside or outside the NS can be written as

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\phi + \sin\theta d\theta^2) \tag{B.1}$$

Along an orbit, the conserved conjugate momenta are the angular momentum per unit mass, $p_\phi = -L$ and the energy per unit mass $p_t = E_\chi$, and taking the orbit to lie in the $\theta = \pi/2$ plane leads to $p_\theta = 0$.

The equation which describes the orbit can be obtained from the square of the energy-momentum 4-vector,

$$g_{\alpha\beta}p^\alpha p^\beta - m_\chi^2 = 0 \tag{B.2}$$

$$\implies g^{\alpha\beta}p_\alpha p_\beta - m_\chi^2 = 0 \tag{B.3}$$

with

$$g^{tt} = 1/B(r), \quad g^{rr} = -1/A(r), \quad g^{\phi\phi} = -1/r^2 \tag{B.4}$$

$$\implies 0 = g^{tt}p_t p_t + g^{rr}p_r p_r + g^{\phi\phi}p_\phi p_\phi - m_\chi^2 \tag{B.5}$$

$$= \frac{E_\chi^2}{B(r)} - \frac{1}{A(r)}\left(g_{rr'}p^{r'}\right)\left(g_{rr'}p^{r'}\right) - \frac{L^2}{r^2} - m_\chi^2 \tag{B.6}$$

$$= \frac{E_\chi^2}{B(r)} - m_\chi^2 A(r)\left(\frac{dr}{d\tau}\right)^2 - \frac{L^2}{r^2} - m_\chi^2 \tag{B.7}$$

To find $dt/d\tau$, we use

$$p^t = m_\chi \frac{dt}{d\tau} = g^{tt} p_t = \frac{E_\chi}{B(r)} \tag{B.8}$$

$$\implies \frac{dt}{d\tau} = \frac{1}{B(r)} \frac{E_\chi}{m_\chi} \tag{B.9}$$

This gives

$$\left(\frac{dr}{dt}\right)^2 = \frac{B}{\tilde{E}_\chi^2 A} \left[\tilde{E}_\chi^2 - B(r)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right] \tag{B.10}$$

For simplicity, consider orbits that are a straight line ($\tilde{L} = 0$), which has a radial extent R. This is related to $\tilde{E}_\chi$ through

$$\tilde{E}_\chi^2 = B(R) \tag{B.11}$$

$$\implies R = \frac{2GM_\star}{1 - \tilde{E}_\chi^2}, \quad R > R_\star \tag{B.12}$$

using $B(r > R_\star) = 1 - 2GM_\star/r$.

It is important to note that E_χ so far has been the *conserved* energy along the orbit, which for the initial approach is $E_\chi = m_\chi + \frac{1}{2} m_\chi u^2 \sim m_\chi$. We now call this energy E_χ^{orbit}, which is related to the DM energy as seen by a distant observer, E_χ^{int}, and is the energy used in calculating the interaction rates, through

$$E_\chi^{\text{orbit}} = \sqrt{g_{tt}} E_\chi^{\text{int}} = \sqrt{B(r)} E_\chi^{\text{int}} \tag{B.13}$$

and as $E_\chi^{\text{orbit}} < m_\chi$ for all subsequent scatters after capture, eq. B.12 is always positive.

These "orbits" are straight lines that pass through the star's centre and extend an amount $R - R_\star$ on either side. Due to the symmetry of the motion, the period of the orbit is then

$$T_{\text{orbit}} = 4 \int_0^R \frac{1}{dr/dt} dr \tag{B.14}$$

More relevant to this application is the time spent inside and outside the star, which is given by

$$T_{\text{inside}} = 4 \int_0^{R_\star} \frac{1}{dr/dt} dr \tag{B.15}$$

$$T_{\text{inside}} = 4 \int_{R_\star}^{R} \frac{1}{dr/dt} dr \tag{B.16}$$

B.2 Checking Newtonian/Non-Relativistic Limit

In the Newtonian limit, we take

$$B - 1 \approx 2\phi \ll 1, \tag{B.17}$$
$$A - 1 \approx -2GM(r)/r \equiv -2V(r) \ll 1, \tag{B.18}$$
$$\tilde{L}^2/r^2 \ll 1, \tag{B.19}$$
$$\tilde{E} - 1 = \varepsilon \ll 1, \tag{B.20}$$

with ε the non-relativistic energy per unit mass. Then expanding Eq. B.10 we get

$$\left(\frac{dr}{dt}\right)^2 = (1+2\phi)(1+2V) - (1+2\phi)^2(1+2V)(1-2\varepsilon)\left(1+\frac{\tilde{L}^2}{r^2}\right) \tag{B.21}$$
$$= 1 + 2\phi + 2V - \left(1 + 4\phi + 2V + \frac{\tilde{L}^2}{r^2} - 2\varepsilon\right) \tag{B.22}$$
$$= -2\phi - \frac{\tilde{L}^2}{r^2} + 2\varepsilon \tag{B.23}$$
$$\implies \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{\tilde{L}^2}{2r^2} + \phi = \varepsilon \tag{B.24}$$

which is the standard result for a Newtonian orbit.

B.3 Procedure for calculating kinetic heating time

- Select a point in the star for the DM to scatter off, $r_{\text{scatter},0}$.
- DM comes in from infinity with initial energy $E_\chi \approx m_\chi$
- Boost DM to local energy of $m_\chi/\sqrt{B(r_{\text{scatter}})}$
- Scatter the DM and calculate initial ΔE_χ
- Set local DM energy to $E_\chi \equiv p^t = m_\chi/\sqrt{B(r_{\text{scatter}})} - \Delta E_\chi$
- Calculate the new conserved energy per unit mass along the orbit as

$$\tilde{E}_\chi^{\text{orbit}} = \sqrt{B(r_{\text{scatter}})}E_\chi/m_\chi = \frac{\sqrt{B(r_{\text{scatter}})}}{m_\chi}\left(m_\chi/\sqrt{B(r_{\text{scatter},0})} - \Delta E_\chi\right) \tag{B.25}$$

- Use Equation B.11 to solve for the maximum radius of the orbit, R_{orbit}.
- Use equations B.15 and B.16 to calculate $T_{\mathrm{in}}/(T_{\mathrm{in}}+T_{\mathrm{out}})$
- Adjust the time interval between scatter by $dt \rightarrow dt(T_{\mathrm{in}}/(T_{\mathrm{in}}+T_{\mathrm{out}}))^{-1}$
- Iterate until $R_{\mathrm{orbit}} < R_{\star}$

Definition of Symbols and Abbreviations

$\boldsymbol{C}_{\text{geo}}$ Geometric Capture Rate

DM Dark Matter

$\boldsymbol{K}_{\chi}$ Dark Matter Kinetic Energy

$\boldsymbol{\rho}_{\chi}$ DM halo density

$\boldsymbol{m}_{\chi}$ Dark Matter Mass

EFT Effective Field Theory

EoS Equation of State

$\boldsymbol{f}_{\text{FD}}$ Fermi-Dirac Distribution

$\boldsymbol{\varepsilon}_{F,i}$ Fermi kinetic energy of target species

$|\overline{\boldsymbol{\mathcal{M}}}|^{\mathbf{2}}$ Spin-averaged squared matrix element

$\boldsymbol{\mu}$ DM-Target mass ratio, m_χ/m_i

NS Neutron Star

PB Pauli Blocking

QMC Quark-Meson-Coupling EoS

$\boldsymbol{\sigma}_{\mathbf{th}}$ Threshold Cross Section

$\boldsymbol{T}_{\text{eq}}$ Equilibrium Temperature

$\boldsymbol{t}_{\text{eq}}$ Capture-Annihilation equilibrium time

$\boldsymbol{T}_{\star}$ Temperature of the star

$\boldsymbol{t}_{\text{therm}}$ Thermalisation time

$\boldsymbol{v}_{d}$ DM halo dispersion velocity

$\boldsymbol{v}_{\star}$ Star velocity

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