# Dark Matter in Compact Objects (TBD)

Michael Virgato
0000-0002-8396-0896

Submitted in total fulfilment
of the requirements of the degree of

Doctor of Philosophy

School of Physics
The University of Melbourne

XXX XXX

Produced on archival quality paper.

# Abstract

DM in COs Heat up Maybe See

# Publications

Refs. [2, 3, 5, 4, 1] below are the journal publications, and preprints authored or co-authored during my PhD candidature. The authors are listed alphabetically in all of the titles.

## Journal papers and preprints


[1] Papers

# Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD except where indicated in the preface;
2. due acknowledgement has been made in the text to all other material used;
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Michael Virgato, XXX XXX

# Preface

We don't know what DM is. Can NSs constrain it?

# Acknowledgements

Why did I do this?

# Contents

# List of Figures

# List of Tables

# 1 Introduction

*Background on DM and its current status*

This is the intro. About DM and its current status

# 2 Compact Objects for Particle Physics

*Introduce COs, formation, structure etc...*

# 3

# Dark Matter Capture in Celestion Bodies

*Review capture in the Sun, move to what's needed for COs in general, then specify to WDs (ions + electrons) and NS (interacting baryons)*

## 3.1 Capture in the Sun

## 3.2 Capture in Compact Objects

## 3.3 White Dwarfs

## 3.4 Neutron Stars

# 4
# Thermalisation in Compact Objects

*Go Over the full thermalisation process for WDs and Neutron Stars*

# 5
# Dark Matter Induced Heating

*DM kinetic and annihilation heating applied to NSs and WDs*

# 6 Conclusion

*Concluding remarks*

# A
# Kinematics

*Derivation of $E_f^{'}$ as needed for capture and other kinematics*

# B
# Kinetic Heating

## B.1 DM Orbits in General Isometric Metric

The metric at any point inside or outside the NS can be written as

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\phi + \sin\theta d\theta^2) \tag{B.1}$$

Along an orbit, the conserved conjugate momenta are the angular momentum per unit mass, $p_\phi = -L$ and the energy per unit mass $p_t = E_\chi$, and taking the orbit to lie in the $\theta = \pi/2$ plane leads to $p_\theta = 0$.

The equation which describes the orbit can be obtained from the square of the energy-momentum 4-vector,

$$g_{\alpha\beta}p^\alpha p^\beta - m_\chi^2 = 0 \tag{B.2}$$

$$\implies g^{\alpha\beta}p_\alpha p_\beta - m_\chi^2 = 0 \tag{B.3}$$

with

$$g^{tt} = 1/B(r), \quad g^{rr} = -1/A(r), \quad g^{\phi\phi} = -1/r^2 \tag{B.4}$$

$$\implies 0 = g^{tt}p_t p_t + g^{rr}p_r p_r + g^{\phi\phi}p_\phi p_\phi - m_\chi^2 \tag{B.5}$$

$$= \frac{E_\chi^2}{B(r)} - \frac{1}{A(r)}\left(g_{rr'}p^{r'}\right)\left(g_{rr'}p^{r'}\right) - \frac{L^2}{r^2} - m_\chi^2 \tag{B.6}$$

$$= \frac{E_\chi^2}{B(r)} - m_\chi^2 A(r)\left(\frac{dr}{d\tau}\right)^2 - \frac{L^2}{r^2} - m_\chi^2 \tag{B.7}$$

To find $dt/d\tau$, we use

$$p^t = m_\chi \frac{dt}{d\tau} = g^{tt} p_t = \frac{E_\chi}{B(r)} \tag{B.8}$$

$$\implies \frac{dt}{d\tau} = \frac{1}{B(r)} \frac{E_\chi}{m_\chi} \tag{B.9}$$

This gives

$$\left(\frac{dr}{dt}\right)^2 = \frac{B}{\tilde{E}_\chi^2 A} \left[\tilde{E}_\chi^2 - B(r)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right] \tag{B.10}$$

For simplicity, consider orbits that are a straight line ($\tilde{L} = 0$), which has a radial extent $R$. This is related to $\tilde{E}_\chi$ through

$$\tilde{E}_\chi^2 = B(R) \tag{B.11}$$

$$\implies R = \frac{2GM_\star}{1 - \tilde{E}_\chi^2}, \quad R > R_\star \tag{B.12}$$

using $B(r > R_\star) = 1 - 2GM_\star/r$.

It is important to note that $E_\chi$ so far has been the *conserved* energy along the orbit, which for the initial approach is $E_\chi = m_\chi + \frac{1}{2} m_\chi u^2 \sim m_\chi$. We now call this energy $E_\chi^{\text{orbit}}$, which is related to the DM energy as seen by a distant observer, $E_\chi^{\text{int}}$, and is the energy used in calculating the interaction rates, through

$$E_\chi^{\text{orbit}} = \sqrt{g_{tt}} E_\chi^{\text{int}} = \sqrt{B(r)} E_\chi^{\text{int}} \tag{B.13}$$

and as $E_\chi^{\text{orbit}} < m_\chi$ for all subsequent scatters after capture, eq. B.12 is always positive.

These "orbits" are straight lines that pass through the star's centre and extend an amount $R - R_\star$ on either side. Due to the symmetry of the motion, the period of the orbit is then

$$T_{\text{orbit}} = 4 \int_0^R \frac{1}{dr/dt} dr \tag{B.14}$$

More relevant to this application is the time spent inside and outside the star, which is given by

$$T_{\text{inside}} = 4 \int_0^{R_\star} \frac{1}{dr/dt} dr \tag{B.15}$$

$$T_{\text{inside}} = 4 \int_{R_\star}^{R} \frac{1}{dr/dt} dr \tag{B.16}$$

## B.2 Checking Newtonian/Non-Relativistic Limit

In the Newtonian limit, we take

$$B - 1 \approx 2\phi \ll 1, \tag{B.17}$$
$$A - 1 \approx -2GM(r)/r \equiv -2V(r) \ll 1, \tag{B.18}$$
$$\tilde{L}^2/r^2 \ll 1, \tag{B.19}$$
$$\tilde{E} - 1 = \varepsilon \ll 1, \tag{B.20}$$

with $\varepsilon$ the non-relativistic energy per unit mass. Then expanding Eq. B.10 we get

$$\left(\frac{dr}{dt}\right)^2 = (1+2\phi)(1+2V) - (1+2\phi)^2(1+2V)(1-2\varepsilon)\left(1+\frac{\tilde{L}^2}{r^2}\right) \tag{B.21}$$
$$= 1 + 2\phi + 2V - \left(1 + 4\phi + 2V + \frac{\tilde{L}^2}{r^2} - 2\varepsilon\right) \tag{B.22}$$
$$= -2\phi - \frac{\tilde{L}^2}{r^2} + 2\varepsilon \tag{B.23}$$
$$\implies \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{\tilde{L}^2}{2r^2} + \phi = \varepsilon \tag{B.24}$$

which is the standard result for a Newtonian orbit.

## B.3 Procedure for calculating kinetic heating time

- Select a point in the star for the DM to scatter off, $r_{\mathrm{scatter},0}$.
- DM comes in from infinity with initial energy $E_\chi \approx m_\chi$
- Boost DM to local energy of $m_\chi/\sqrt{B(r_{\mathrm{scatter}})}$
- Scatter the DM and calculate initial $\Delta E_\chi$
- Set local DM energy to $E_\chi \equiv p^t = m_\chi/\sqrt{B(r_{\mathrm{scatter}})} - \Delta E_\chi$
- Calculate the new conserved energy per unit mass along the orbit as

$$\tilde{E}_\chi^{\mathrm{orbit}} = \sqrt{B(r_{\mathrm{scatter}})}E_\chi/m_\chi = \frac{\sqrt{B(r_{\mathrm{scatter}})}}{m_\chi}\left(m_\chi/\sqrt{B(r_{\mathrm{scatter},0})} - \Delta E_\chi\right) \tag{B.25}$$

- Use Equation B.11 to solve for the maximum radius of the orbit, $R_{\mathrm{orbit}}$.
- Use equations B.15 and B.16 to calculate $T_{\mathrm{in}}/(T_{\mathrm{in}} + T_{\mathrm{out}})$
- Adjust the time interval between scatter by $dt \to dt(T_{\mathrm{in}}/(T_{\mathrm{in}} + T_{\mathrm{out}}))^{-1}$
- Iterate until $R_{\mathrm{orbit}} < R_{\star}$

# Definition of Symbols and Abbreviations

**$C_{\text{geo}}$** Geometric Capture Rate

**DM** Dark Matter
**$K_\chi$** Dark Matter Kinetic Energy
**$\rho_\chi$** DM halo density
**$m_\chi$** Dark Matter Mass

**EFT** Effective Field Theory
**EoS** Equation of State

**$f_{\text{FD}}$** Fermi-Dirac Distribution
**$\varepsilon_{F,i}$** Fermi kinetic energy of target species

**$|\overline{\mathcal{M}}|^2$** Spin-averaged squared matrix element
**$\mu$** DM-Target mass ratio, $m_\chi/m_i$

**NS** Neutron Star

**PB** Pauli Blocking

**QMC** Quark-Meson-Coupling EoS

**$\sigma_{\mathbf{th}}$** Threshold Cross Section

**$T_{\text{eq}}$** Equilibrium Temperature
**$t_{\text{eq}}$** Capture-Annihilation equilibrium time
**$T_\star$** Temperature of the star
**$t_{\text{therm}}$** Thermalisation time

**$v_d$** DM halo dispersion velocity
**$v_\star$** Star velocity

# Bibliography

[1] Filippo Anzuini et al. "Improved Treatment of Dark Matter Capture in Neutron Stars III: Nucleon and Exotic Targets". In: *JCAP* 11.11 (Nov. 29, 2021), p. 056. DOI: 10.1088/1475-7516/2021/11/056. arXiv: 2108.02525 [hep-ph].

[2] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in Neutron Stars". In: *JCAP* 09 (Sept. 15, 2020), p. 028. DOI: 10.1088/1475-7516/2020/09/028. arXiv: 2004.14888 [hep-ph].

[3] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in Neutron Stars II: Leptonic Targets". In: *JCAP* 03 (Mar. 26, 2021), p. 086. DOI: 10.1088/1475-7516/2021/03/086. arXiv: 2010.13257 [hep-ph].

[4] Nicole F. Bell et al. "Improved Treatment of Dark Matter Capture in White Dwarfs". In: *JCAP* 10 (Oct. 29, 2021), p. 083. DOI: 10.1088/1475-7516/2021/10/083. arXiv: 2104.14367 [hep-ph].

[5] Nicole F. Bell et al. "Nucleon Structure and Strong Interactions in Dark Matter Capture in Neutron Stars". In: *Phys. Rev. Lett.* 127.11 (Sept. 10, 2021), p. 111803. DOI: 10.1103/PhysRevLett.127.111803. arXiv: 2012.08918 [hep-ph].