Dark Matter in Compact Objects (TBD)

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Produced on archival quality paper.

Abstract

DM in COs Heat up Maybe See

Publications

Refs. [2, 3, 5, 4, 1] below are the journal publications, and preprints authored or co-authored during my PhD candidature. The authors are listed alphabetically in all of the titles.

Journal papers and preprints

[1] Papers

Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD except where indicated in the preface;
2. due acknowledgement has been made in the text to all other material used;
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Michael Virgato, XXX XXX

Preface

We don't know what DM is. Can NSs constrain it?

Acknowledgements

Why did I do this?

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1 Introduction

Background on DM and its current status

Dark Matter is an enigma in modern physics. Ever since it was first proposed by Fritz Zwicky nearly 90 years ago, significant scientific effort has gone into trying to discern its nature. However, despite the best efforts of generations of physicists, a definitive detection proving its existence eludes us. Nevertheless, dark matter's influence on our universe is undeniable, with a large assortment of evidence supporting its existence.

1.1 Evidence for Dark Matter

Dark matter reveals itself to us through its gravitational interactions. The first sign that there was an additional component to the mass-energy component of the universe came from observations of galaxy rotation curves. It was noted by Zwicky CITE

1.2 Potential Models of Dark Matter

1.3 Current Status of Dark Matter Constraints

1.3.1 Collider Bounds

1.3.2 Direct Detection Searches

1.3.3 Indirect Detection

It is this route that we will follow to explore dark matter EFTs.

1.4 Compact Objects as Dark Matter Probes

Compact objects, namely Neutron Stars and White Dwarfs, offer a unique laboratory for studying dark matter interactions. Their extreme environments offer many benefits in comparison to direct detection experiments. These include:

- **Gravitational focusing of the DM flux.** In the NS case, the infalling DM will be boosted to semi-relativistic velocities ($\sim 0.2 - 0.7c$ depending on the NS mass).
-

2

Compact Objects for Particle Physics

Introduce COs, formation, structure etc...

Check masses in Shapiro

The lifecycle of main sequence stars can end in various ways depending on the progenitor star's mass. Lighter stars ($0.6 - 10 M_\odot$ at the onset of hydrogen burning) will eventually end their lives as a White Dwarf. Meanwhile,

2.1 Internal structure

The internal structure of

For non-relativistic stars, such as our Sun, the structure equations are rather simple, them being

$$\frac{dM}{dr} = 4\pi r^2, \tag{2.1}$$

$$\frac{dP}{dr} = -\rho(r) G M(r) r^2, \tag{2.2}$$

where M is the mass of the star at radius r, P is the pressure, and $\rho(r)$ is the density. The first equation is known as the mass equation, and the second is the condition for hydrostatic equilibrium.

We are instead interested in compact objects, where the extreme densities of these stars

In order to solve this system, The equation of state describes the relation between the pressure and energy of the constituent matter.

2.1.1 White Dwarfs

FMT equation of state

2.1.2 Neutron Stars

Beta Equlibrium

2.2 Observational Status

2.2.1 White Dwarfs

2.2.2 Neutron Stars

3

Introduction to Capture in Compact Objects: Point Like Targets

Review capture in the Sun, move to what's needed for COs in general, then specify to WDs (ions + electrons) and NS (interacting baryons)

One of the most important quantities we will be interested in is the rate at which darm matter is captured with the star. In this chpater, we focus on building up the formalism of dark matter capture within compact objects, outlining how this differs from the standard calculation for capture in the Sun. We restrict our analysis to scattering off of point-like targets, relevent for leptonic species, i.e. electrons in White Dwarfs and electrons and muons in Neutron Stars. The complications that arise due to the finite size of hadronic targets will be discussed in the next chapter.

3.1 Dark Matter Capture in the Sun

3.2 Capture in Compact Objects

3.3 White Dwarfs: Electron Targets

3.4 Neutron Stars: Leptoinc Targets

4
Capture Rate for Baryons

Go Over the full thermalisation process for WDs and Neutron Stars

5

Dark Matter Induced Heating of Neutron Stars

DM kinetic and annihilation heating applied to NSs and WDs

5.1 Thermalisation Time

5.2 Capture-Annihilation Equilibrium

5.3 Dark Matter Heating

5.3.1 Kinetic Heating

5.3.2 Annihilation Heating

6 Conclusion

Concluding remarks

A
Kinematics

Derivation of $E_f^{'}$ as needed for capture and other kinematics

B
Kinetic Heating

B.1 DM Orbits in General Isometric Metric

The metric at any point inside or outside the NS can be written as

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\phi + \sin\theta d\theta^2) \tag{B.1}$$

Along an orbit, the conserved conjugate momenta are the angular momentum per unit mass, $p_\phi = -L$ and the energy per unit mass $p_t = E_\chi$, and taking the orbit to lie in the $\theta = \pi/2$ plane leads to $p_\theta = 0$.

The equation which describes the orbit can be obtained from the square of the energy-momentum 4-vector,

$$g_{\alpha\beta}p^\alpha p^\beta - m_\chi^2 = 0 \tag{B.2}$$

$$\implies g^{\alpha\beta}p_\alpha p_\beta - m_\chi^2 = 0 \tag{B.3}$$

with

$$g^{tt} = 1/B(r), \quad g^{rr} = -1/A(r), \quad g^{\phi\phi} = -1/r^2 \tag{B.4}$$

$$\implies 0 = g^{tt}p_t p_t + g^{rr}p_r p_r + g^{\phi\phi}p_\phi p_\phi - m_\chi^2 \tag{B.5}$$

$$= \frac{E_\chi^2}{B(r)} - \frac{1}{A(r)}\left(g_{rr'}p^{r'}\right)\left(g_{rr'}p^{r'}\right) - \frac{L^2}{r^2} - m_\chi^2 \tag{B.6}$$

$$= \frac{E_\chi^2}{B(r)} - m_\chi^2 A(r)\left(\frac{dr}{d\tau}\right)^2 - \frac{L^2}{r^2} - m_\chi^2 \tag{B.7}$$

To find $dt/d\tau$, we use

$$p^t = m_\chi \frac{dt}{d\tau} = g^{tt} p_t = \frac{E_\chi}{B(r)} \tag{B.8}$$

$$\implies \frac{dt}{d\tau} = \frac{1}{B(r)} \frac{E_\chi}{m_\chi} \tag{B.9}$$

This gives

$$\left(\frac{dr}{dt}\right)^2 = \frac{B}{\tilde{E}_\chi^2 A} \left[\tilde{E}_\chi^2 - B(r) \left(1 + \frac{\tilde{L}^2}{r^2}\right)\right] \tag{B.10}$$

For simplicity, consider orbits that are a straight line ($\tilde{L} = 0$), which has a radial extent R. This is related to $\tilde{E}_\chi$ through

$$\tilde{E}_\chi^2 = B(R) \tag{B.11}$$

$$\implies R = \frac{2GM_\star}{1 - \tilde{E}_\chi^2}, \quad R > R_\star \tag{B.12}$$

using $B(r > R_\star) = 1 - 2GM_\star/r$.

It is important to note that E_χ so far has been the *conserved* energy along the orbit, which for the initial approach is $E_\chi = m_\chi + \frac{1}{2} m_\chi u^2 \sim m_\chi$. We now call this energy E_χ^{orbit}, which is related to the DM energy as seen by a distant observer, E_χ^{int}, and is the energy used in calculating the interaction rates, through

$$E_\chi^{\text{orbit}} = \sqrt{g_{tt}} E_\chi^{\text{int}} = \sqrt{B(r)} E_\chi^{\text{int}} \tag{B.13}$$

and as $E_\chi^{\text{orbit}} < m_\chi$ for all subsequent scatters after capture, eq. B.12 is always positive.

These "orbits" are straight lines that pass through the star's centre and extend an amount $R - R_\star$ on either side. Due to the symmetry of the motion, the period of the orbit is then

$$T_{\text{orbit}} = 4 \int_0^R \frac{1}{dr/dt} dr \tag{B.14}$$

More relevant to this application is the time spent inside and outside the star, which is given by

$$T_{\text{inside}} = 4 \int_0^{R_\star} \frac{1}{dr/dt} dr \tag{B.15}$$

$$T_{\text{inside}} = 4 \int_{R_\star}^R \frac{1}{dr/dt} dr \tag{B.16}$$

B.2 Checking Newtonian/Non-Relativistic Limit

In the Newtonian limit, we take

$$B - 1 \approx 2\phi \ll 1, \tag{B.17}$$
$$A - 1 \approx -2GM(r)/r \equiv -2V(r) \ll 1, \tag{B.18}$$
$$\tilde{L}^2/r^2 \ll 1, \tag{B.19}$$
$$\tilde{E} - 1 = \varepsilon \ll 1, \tag{B.20}$$

with ε the non-relativistic energy per unit mass. Then expanding Eq. B.10 we get

$$\left(\frac{dr}{dt}\right)^2 = (1+2\phi)(1+2V) - (1+2\phi)^2(1+2V)(1-2\varepsilon)\left(1+\frac{\tilde{L}^2}{r^2}\right) \tag{B.21}$$
$$= 1 + 2\phi + 2V - \left(1 + 4\phi + 2V + \frac{\tilde{L}^2}{r^2} - 2\varepsilon\right) \tag{B.22}$$
$$= -2\phi - \frac{\tilde{L}^2}{r^2} + 2\varepsilon \tag{B.23}$$
$$\implies \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{\tilde{L}^2}{2r^2} + \phi = \varepsilon \tag{B.24}$$

which is the standard result for a Newtonian orbit.

B.3 Procedure for calculating kinetic heating time

- Select a point in the star for the DM to scatter off, $r_{\text{scatter},0}$.
- DM comes in from infinity with initial energy $E_\chi \approx m_\chi$
- Boost DM to local energy of $m_\chi/\sqrt{B(r_{\text{scatter}})}$
- Scatter the DM and calculate initial ΔE_χ
- Set local DM energy to $E_\chi \equiv p^t = m_\chi/\sqrt{B(r_{\text{scatter}})} - \Delta E_\chi$
- Calculate the new conserved energy per unit mass along the orbit as

$$\tilde{E}_\chi^{\text{orbit}} = \sqrt{B(r_{\text{scatter}})}E_\chi/m_\chi = \frac{\sqrt{B(r_{\text{scatter}})}}{m_\chi}\left(m_\chi/\sqrt{B(r_{\text{scatter},0})} - \Delta E_\chi\right) \tag{B.25}$$

- Use Equation B.11 to solve for the maximum radius of the orbit, R_{orbit}.
- Use equations B.15 and B.16 to calculate $T_{\text{in}}/(T_{\text{in}} + T_{\text{out}})$
- Adjust the time interval between scatter by $dt \to dt(T_{\text{in}}/(T_{\text{in}} + T_{\text{out}}))^{-1}$
- Iterate until $R_{\text{orbit}} < R_{\star}$

Definition of Symbols and Abbreviations

$\boldsymbol{C}_{\mathrm{geo}}$ Geometric Capture Rate

DM Dark Matter
$\boldsymbol{K}_\chi$ Dark Matter Kinetic Energy
$\boldsymbol{\rho}_\chi$ DM halo density
$\boldsymbol{m}_\chi$ Dark Matter Mass

EFT Effective Field Theory
EoS Equation of State

$\boldsymbol{f}_{\mathrm{FD}}$ Fermi-Dirac Distribution
$\boldsymbol{\varepsilon}_{F,i}$ Fermi kinetic energy of target species

$|\overline{\boldsymbol{\mathcal{M}}}|^{\mathbf{2}}$ Spin-averaged squared matrix element
$\boldsymbol{\mu}$ DM-Target mass ratio, m_χ/m_i

NS Neutron Star

PB Pauli Blocking

QMC Quark-Meson-Coupling EoS

$\boldsymbol{\sigma}_{\mathbf{th}}$ Threshold Cross Section

$\boldsymbol{T}_{\mathrm{eq}}$ Equilibrium Temperature
$\boldsymbol{t}_{\mathrm{eq}}$ Capture-Annihilation equilibrium time
$\boldsymbol{T}_\star$ Temperature of the star
$\boldsymbol{t}_{\mathrm{therm}}$ Thermalisation time

$\boldsymbol{v}_d$ DM halo dispersion velocity
$\boldsymbol{v}_\star$ Star velocity

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