- Matthew Virgin, COS 675
  Hw2 q2 full solution
  3/3 5pm

$$\min_{w,b} \sum_{i=1}^{n} C_i \left(y_i - w^T x_i - b\right)^2$$

$$= \sum_{i=1}^{n} C_i \left(y_i - \underbrace{(w^T x_i + b)}_{\;= \phi^T x_i}\right)^2$$

→ like in q1, eliminate b by incorporating into w and X, replace w with ϕ

$$\min_{\phi} \sum_{i=1}^{n} C_i \left(y_i - \phi^T x_i\right)^2$$

- derivative w/ respect to ϕ:

$$-2 \sum_{i=1}^{n} C_i \left(y_i - \underbrace{\phi^T x_i}_{= x_i^T \phi}\right) x_i = 0$$

$$\sum_{i=1}^{n} \left(C_i y_i x_i - C_i x_i^T \phi x_i\right) = 0$$

$$\sum_{i=1}^{n} C_i y_i x_i - \phi \sum_{i=1}^{n} C_i x_i^T x_i = 0$$

matrix form:

$$\overset{\text{(T)}}{X^T} C y - \phi X^T C X = 0$$

to make dimensions work: dxn . nxn . nx1

$$\phi X^T C X = X^T C y$$

$$\phi = \left(X^T C X\right)^{-1} X^T C y$$

when $C = I$, you'll have $\phi = \left(X^T X\right)^{-1} X^T y$.