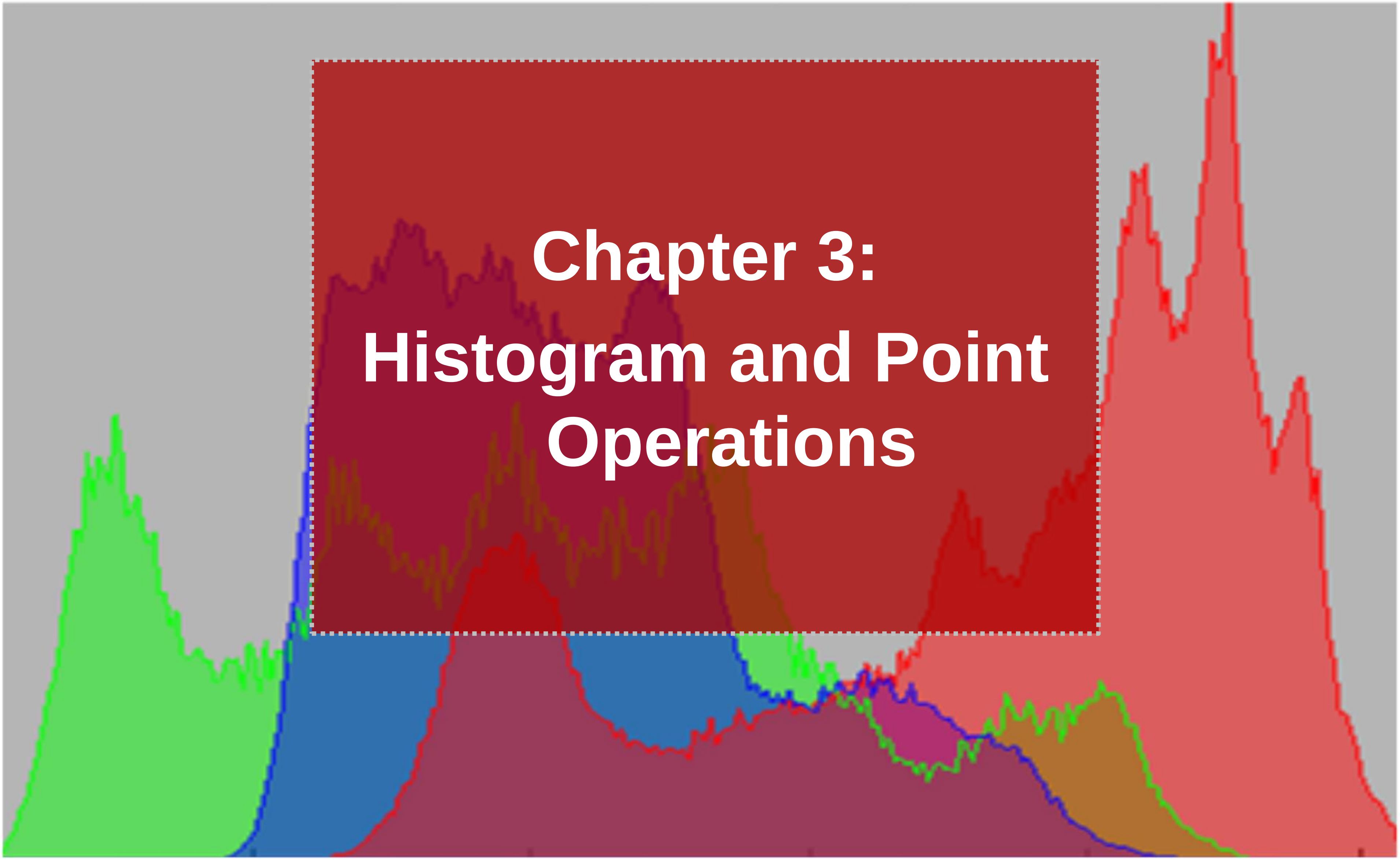
Solution Exercise 2: Thresholding



Chapter 3: Histogram and Point Operations

Histogram calculation

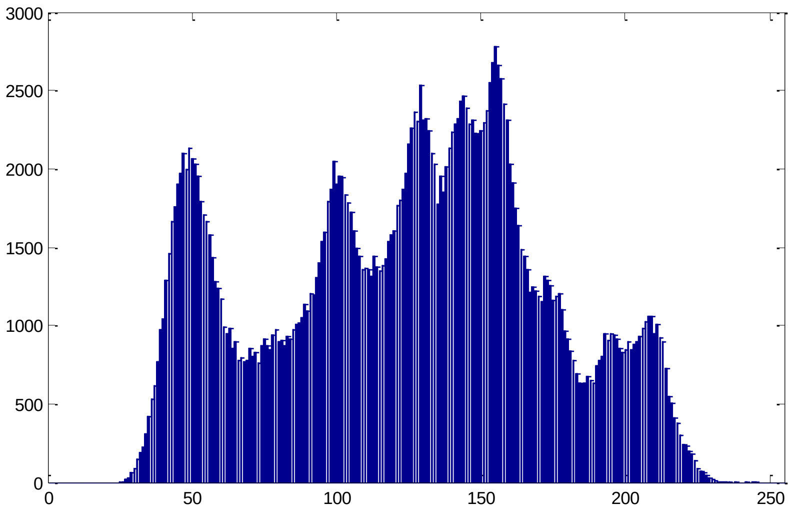
- Each pixel represents a quantized measurement of the intensity
- The Histogram is a vector, which contains one element of each quantization level
  - N-bit resolution → 2^N quantization levels → $K=2^N$ histogram elements
  - E.g. 8-bit → $K=2^8=256$ entries
- Each element contains the number of pixels whose value corresponds to the index of the element
- Example: calculation of a histogram in MATLAB:

```
% read image
I = imread('image.bmp');
[rows cols] = size(I);

% initialize histogram with zeros
h = zeros(256,1);

% iterate over all pixels
for n = 1:rows
    for m = 1:cols
        % increment histogram value at index I(n,m)
        h(I(n,m)+1) = h(I(n,m)+1) + 1;
    end
end

% show histogram
bar(h);
```



Histogram distribution of Lena

Repetition Histogram Operations

Mean value (expectation value)

- Provides information about the brightness of the image (mean value μ)

Calculation from the histogram: $\mu = \sum_{i_{\min}}^{i_{\max}} i \cdot h_n(i)$

Calculation from the image: $\mu = \frac{1}{M \cdot N} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(n,m)$

Variance

- Provides information about the dispersion of the values

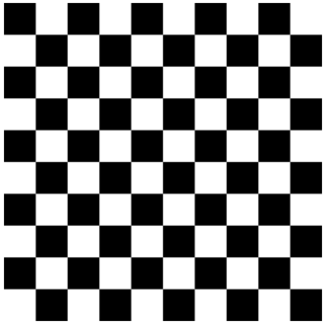
Calculation from the histogram: $\sigma^2 = \sum_{i_{\min}}^{i_{\max}} (i-\mu)^2 \cdot h_n(i)$

Calculation from the image: $\sigma^2 = \frac{1}{M \cdot N} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I^2(n,m) - \mu^2$

Examples:

$\mu = 127$

$\sigma^2 = 0$

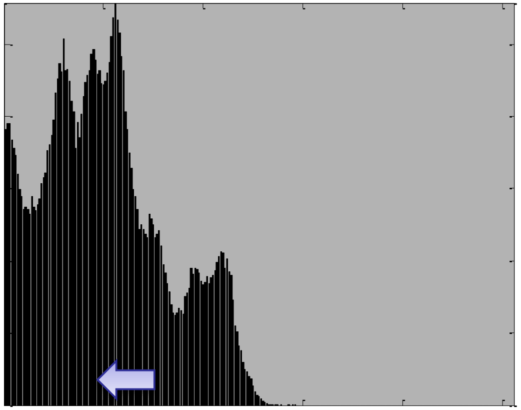


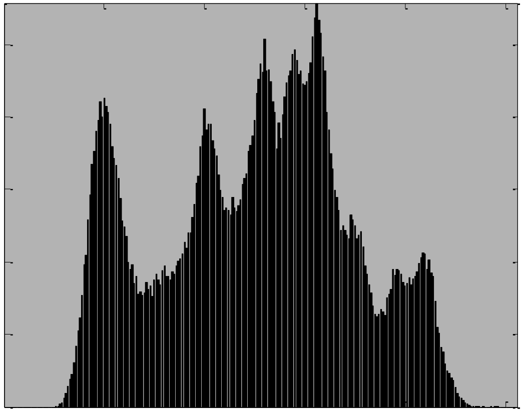
$\mu = 127,5$

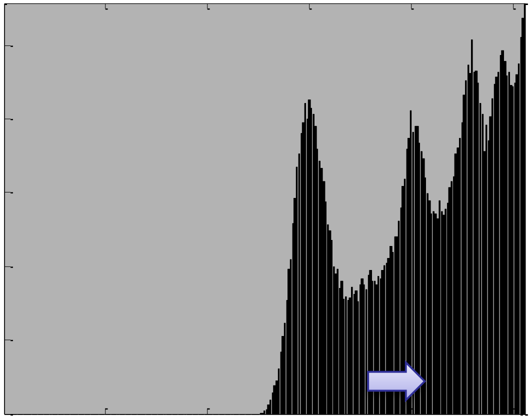
$\sigma^2 = 127,5^2 = 16256,25$

Examples

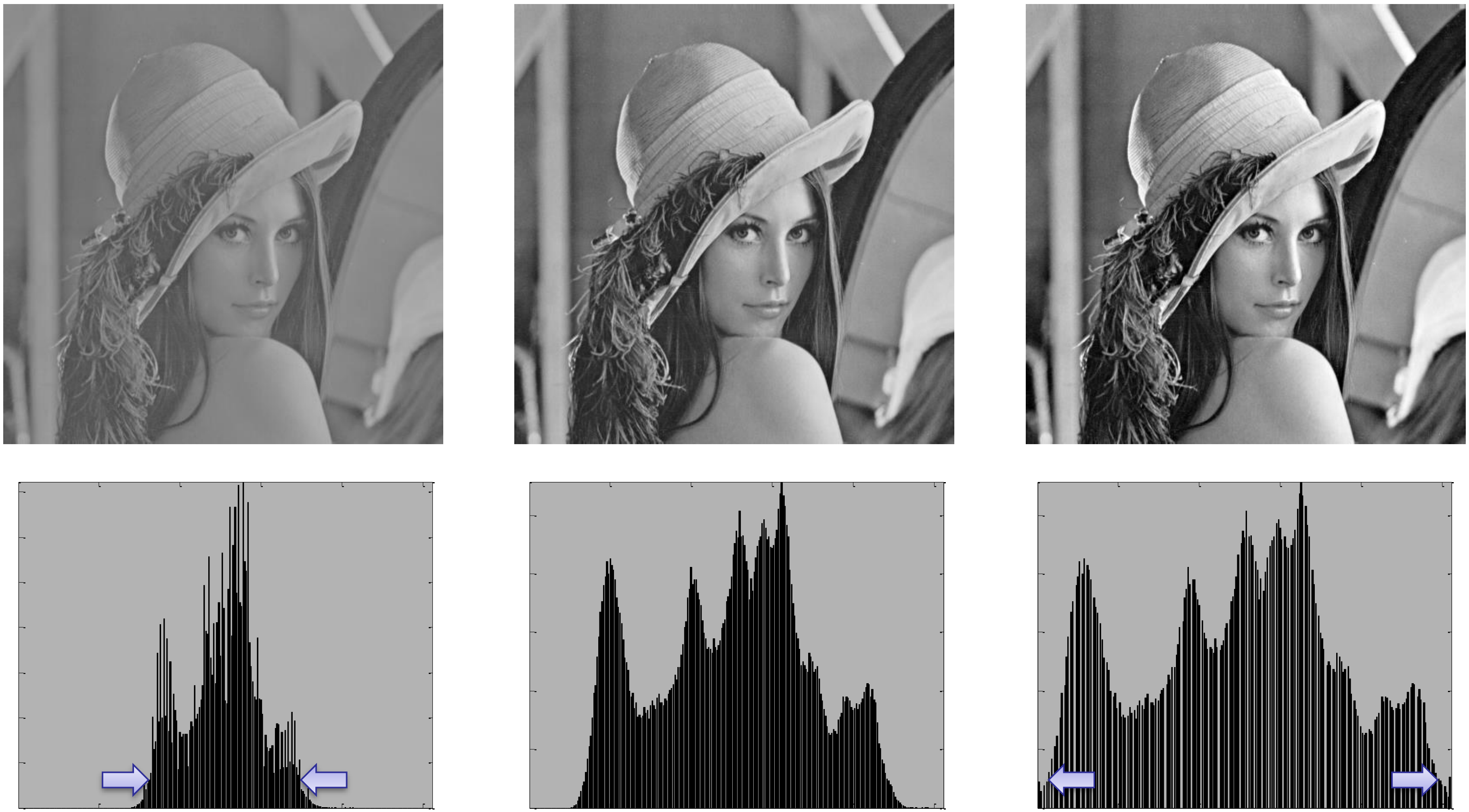
Brightness





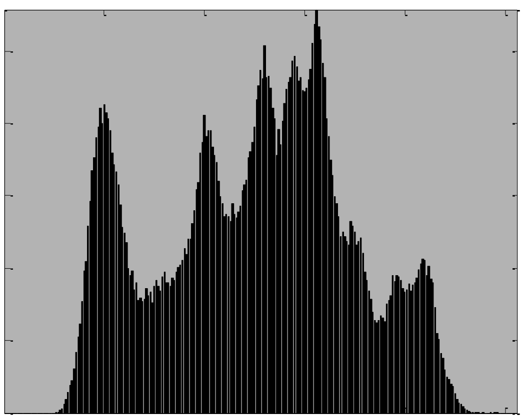
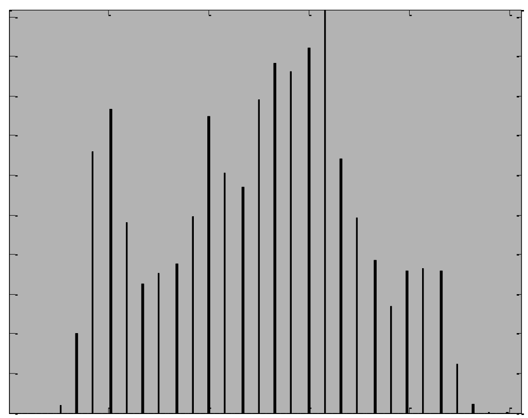
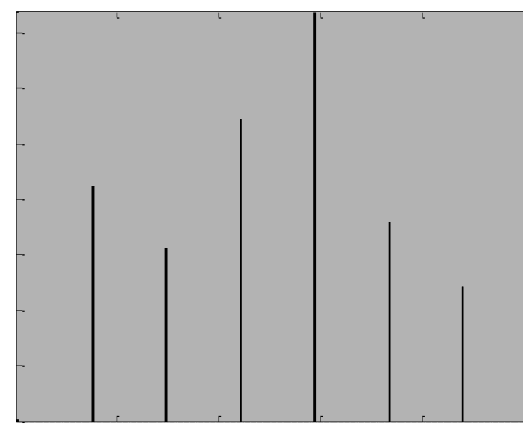


Contrast



Quantization noise



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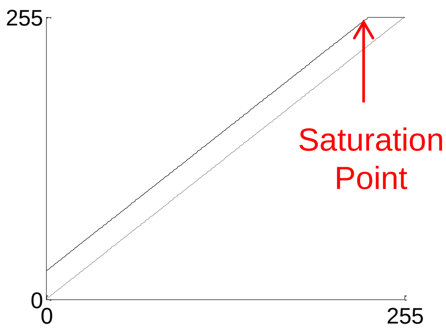
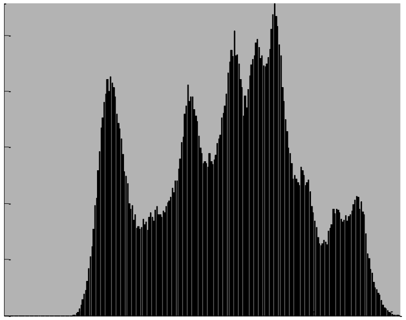
Brightness Adjustment

$$f(m,n) = I(m,n) + a \qquad a \in [0,255]$$

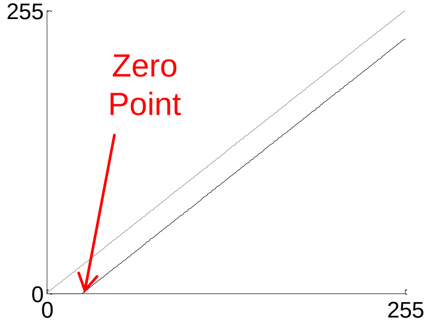
$$J(m,n) = \begin{cases} f(m,n), & \text{if } 0 \le f(m,n) \le 255 \\ 0, & \text{if } f(m,n) < 0 \\ 255, & \text{if } f(m,n) > 255 \end{cases}$$

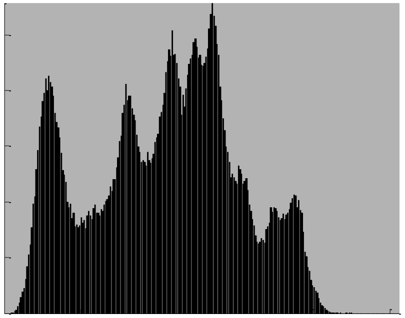
$a > 0:$





$a < 0:$





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Contrast Adjustment

$\alpha \rightarrow$ Contrast factor.

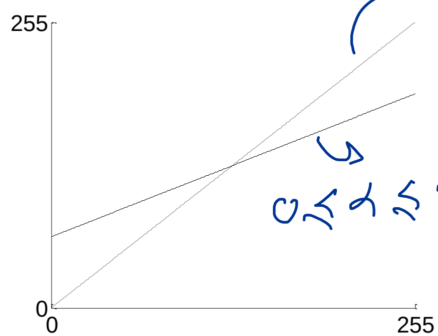
$$f(m,n) = a(I(m,n) - s) + s \qquad a, s \in [0,255]$$

$$J(m,n) = \begin{cases} f(m,n), & if\ 0 \le f(m,n) \le 255 \\ 0, & if\ f(m,n) < 0 \\ 255, & if\ f(m,n) > 255 \end{cases}$$

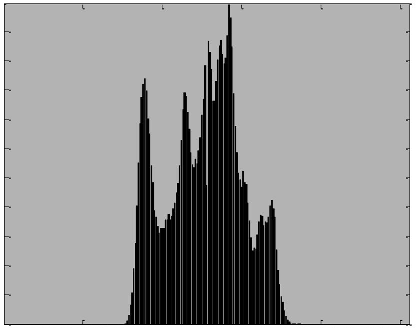
s is the center of the contrast function, e.g $s = 127$

$\alpha = 1$

$0 \le a < 1$:

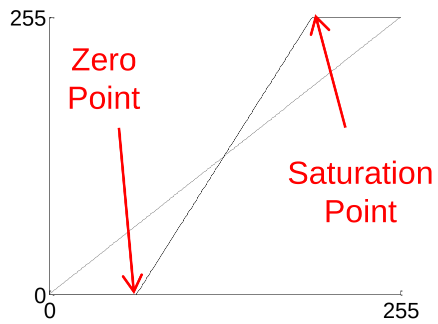


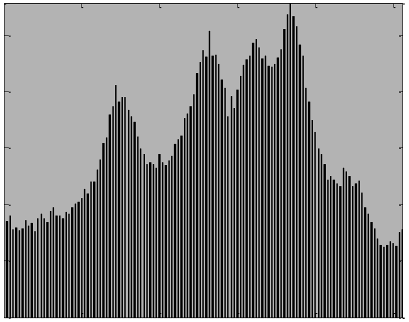
$0 \le \alpha \le 1$



$s = 127$

$a > 1$:

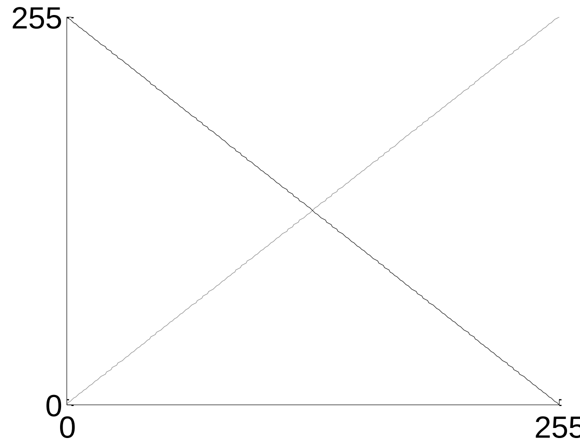


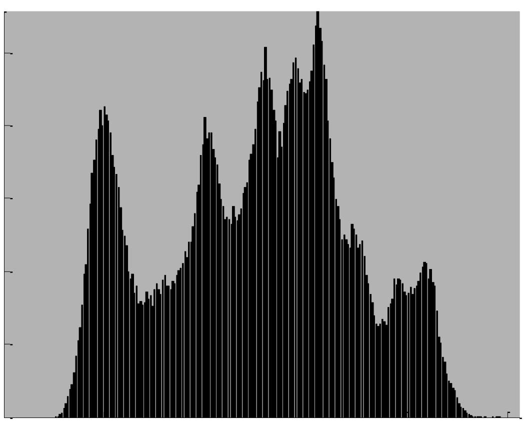


Repetition Histogram Operations

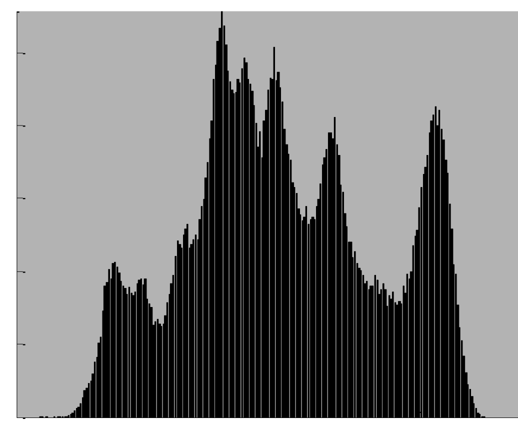
Inversion

$$J(m,n) = 255 - I(m,n)$$





Histogram flipped



Binning (quantization)

- Images with a high bit depth would generate huge histograms
  - Example: 14-bit → $K=2^{14}=16384$ histogram entries
- To reduce the size of the histogram, binning can be done
- The intensity values are counted in B intervals (bins) $[i_j,i_j+1)$
- For equally sized bins, the interval size is $k_B=K/B$
- The bin index can be calculated with $j = \left\lfloor I(m,n)\dfrac{B}{K} \right\rfloor$

Example: 14-bit image, histogram with 256 bins

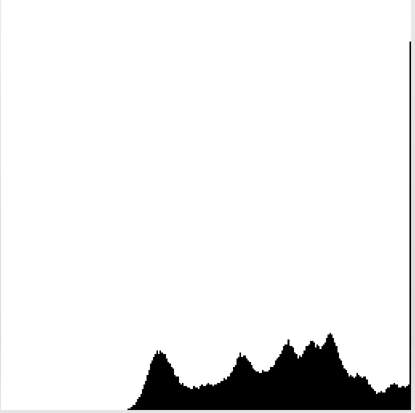
$$
\begin{array}{cccc}
h(0) & \leftarrow\ 0 & \le I(n,m) < & 64 \\
h(1) & \leftarrow\ 64 & \le I(n,m) < & 128 \\
\vdots & \leftarrow\ \vdots & \le I(n,m) < & \vdots \\
h(j) & \leftarrow\ i_j & \le I(n,m) < & i_{j+1} \\
\vdots & \leftarrow\ \vdots & \le I(n,m) < & \vdots \\
h(255) & \leftarrow 16320 & \le I(n,m) < & 16384
\end{array}
$$

Third exercise

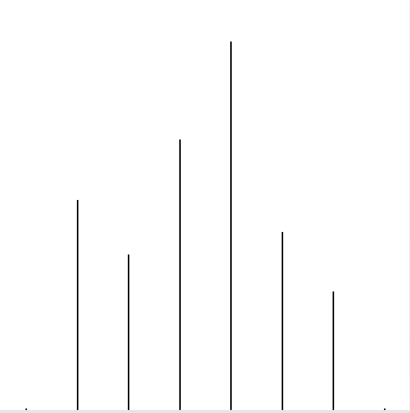
- Implement some histogram operations

Expected Output (3 of 6)

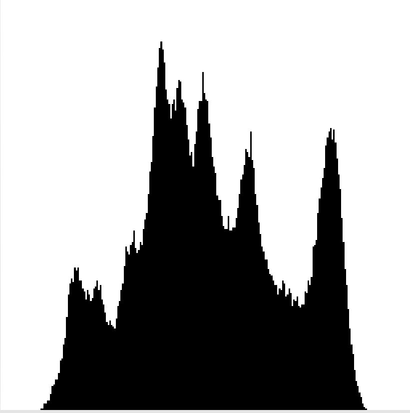
Adjusted Brightness



Quantized



Inverted



That is all for today.