1. Из множества $\{1, 2, \ldots, 37\}$ выбрали 10 чисел. Докажите, что среди выбранных найдутся 4 такие, что сумма двух из них равна сумме двух других.
2. В первом ряду кинотеатра расположены 330 сидений. Некоторые 25 из них заняты зрителями. Докажите, что среди попарных расстояний между зрителями найдутся одинаковые.
3. Prove that there is no set of 2021 different positive integers such that the sum of any 2019 of them is divisible by the sum of the rest two numbers.
4. The sum of several (not necessary different) positive integers not exceeding 10 is equal to S. Find all possible values of S such that these numbers can always be partitioned into two groups with the sum of the numbers in each group not exceeding 70.
5. Let n be a positive integer. What is the smallest value of m, $m > n$, such that the set $M = \{n, n+1, \ldots, m\}$ can be partitioned into subsets so that in each subset there is a number which equals to the sum of all other numbers of this subset?
6. Let S be a set of positive integers with 100 digits. A number from S is called *bad* if it is not divisible by the sum of any two (not necessarily distinct) numbers from S. Find the maximal possible number of elements of S if it is known that S contains at most 10 bad numbers.
7. Let us call a set of positive integers *nice*, if its number of elements is equal to the average of all its elements. Call a number n *amazing*, if one can partition the set $\{1, 2, \ldots, n\}$ into nice subsets. **a)** Prove that any perfect square is amazing. **b)** Prove that there exist infinitely many positive integers which are not amazing.
8. Is it possible to arrange 100 positive integers (not all equal to 1) along the circle so that for any three consecutive numbers one of them is the product of two others?
9. **a)** Let (a_n) be an increasing sequence of positive integers. The number a_k is said to be *funny* if it can be represented as the sum of other (not necessarily distinct) terms of the sequence. Prove that only finite number of terms of the sequence can be not funny. **b)** Is the same statement true for the sequence of positive rationals?
10. For a set M of positive integers with n elements, where n is odd, a nonempty subset T of M is called *good*, if the product of the elements of T is divisible by the sum of the elements of M, but not divisible by its square. If M is good, find the maximum possible number of the good subsets of M.
11. Let n be a positive integer and S be the set of $2n$ positive integers. A pairing of elements of S is called *squarefree* if none of the products of numbers in pairs is a perfect square. Suppose S has a squarefree pairing. Prove that it has at least $n!$ squarefree pairings.
12. Find the minimal possible number of colors which is required to color all vertices, edges and diagonals of a convex n-gon so that: 1) if two segments share an endpoint they has different color, 2) the color of every vertex differs from the color of any adjacent segment.
13. Find all positive integers $n \geqslant 3$ such that it is possible to mark the vertices of a regular n-gon with the numbers from 1 to n so that for any three vertices A, B and C with $AB = AC$ the number in A is greater or smaller than both numbers in B and C.
14. For a positive integer k denote by $f(k)$ the number of positive integers m such that the remainder of km modulo 2019^3 is greater than m. Find the amount of different numbers among $f(1)$, $f(2)$, $\ldots$, $f(2019^3)$.
15. Let N be a positive integer. Consider a sequence a_1, a_2, $\ldots$, a_N of positive integers, none of which is a multiple of 2^{N+1}. For $n \geqslant N+1$ the number a_n is defined as follows: Choose k to be the number among 1, 2, $\ldots$, $n-1$ for which the remainder obtained when a_k is divided by 2^n is the smallest, and define $a_n = 2a_k$ (if there are more than one such k, choose the largest such k). Prove that there exist M for which $a_n = a_M$ holds for $n \geqslant M$.