CMPS 102 — Quarter Spring 2017 – Homework 1

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April 28, 2017

I have read and agree to the collaboration policy.Vladoi Marian
I want to choose homework heavy option.
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Solution to Problem 4: Divide and Conquer

One of the algorith that Jack and Anthony used to find the Median book of the joint book collection could be the folowing:

1. We have 2 sets of sorted books , J and A, each set has the size n. We can find the median of this two sets in constant time.
2. Find median of J set , caled j , j= $J[\lceil \frac{n}{2} \rceil]$
3. Find median of A set , caled a, a = $A[\lceil \frac{n}{2} \rceil]$
4. If $j == a$ return j (or a) , we found the median book
5. If $j > a$, then look for the median in the following subarrays : $J[\lceil \frac{n}{2} \rceil \ to \ (n-1)]$, $A[0 \ to \ \lceil \frac{n}{2} \rceil]$
6. If $j < a$, then look for the median in the following subarrays : $J[0 \ to \ \lceil \frac{n}{2} \rceil]$, $A[\lceil \frac{n}{2} \rceil \ to \ (n-1)]$
7. Repeat the algorithm (divide and conquer) until size of both sets becomes 2
8. If size of the two sets is 2, we have 4 books or 3 books . If $J[1] \leq A[0] \, return \, J[1], \, else \, return \, A[0]$.

Our subproblem size reduces by a factor of half and we spend only constant time to compare the medians of J and A.The recurrence relation is : T(n) = T(n/2) + $\Theta(c)$
Initial condition: the time to find the median book in a Set of size 2 is constant
T(1) = Θ(1) = 1
Then we apply telescoping: reducing the recurrence relation to n/2, n/4, , 2

T(n) = T($\frac{n}{2}$) + 1

T($\frac{n}{2}$) = T($\frac{n}{4}$) + 1

T($\frac{n}{4}$) = T($\frac{n}{8}$) + 1

....

T(4) = T(2) + 1

Next we sum up the terms on left and right side of the ecuation:

T(n) + T($\frac{n}{2}$) + T($\frac{n}{4}$) + T($\frac{n}{8}$) ... + T(2) = T($\frac{n}{2}$) + T($\frac{n}{4}$) + T($\frac{n}{8}$) ... + T(2) + (1 + 1 + 1+ 1.. + 1)

There are (log n) number of 1's.

After canceling the terms on both side of the ecuation, we have

T(n) = T(2) + (log n)

T(n) = c + log n

Therefore the runing time of the algorithm is T(n) = Θ (log n)

Proving by induction that the algorithm works.

Claim 1. *Let P(n) be the assertion that algorithm works correctly for Sets of size n.Prove that P(n) is true for all n, then the algoritm works on all possible input Sets.*

Proof. □

Base case: In the case where the size of the Sets is 2 , the algorith works.

Inductive Step: We assume that the algorithm works for Sets of size $\leq k$;

We prove that the algorithm works for Sets of size k + 1;

When size of the sets is k+1 , we have 3 cases:

1. median of J == median of A(the algorithm works)

2. median of J$>$ median of A (in this case we look for the median on $J[\lceil \frac{n}{2} \rceil\ to\ (n-1)]$, $A[0\ to\ \lceil \frac{n}{2} \rceil]$) So assuming that the recursive calls works correctly, this call works too.

3.median of J$<$ median of A (in this case we look for the median on $J[0\ to\ \lceil \frac{n}{2} \rceil]$, $A[\lceil \frac{n}{2} \rceil\ to\ (n-1)]$) So assuming that the recursive calls works correctly, this call works too.

The inductive step works correctly in all cases, we can conclude that the algorithm works correctly.