CMPS 102 — Quarter Spring 2017 – Homework 3

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I have read and agree to the collaboration policy.Vladoi Marian
I want to choose homework heavy option.
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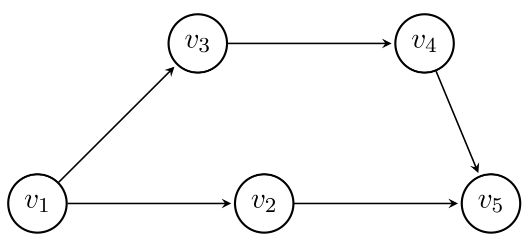
Solution to Problem 3: Dynamic Programming

A) The algorithm from part A , does not correctly solve this example:



The algorithm will chose the path $\{v_1, v_2, v_5\}$ when the correct algorithm would be $\{v_1, v_3, v_4, v_5\}$.

B)This is my dynamic programming algorithm for finding the longest path in an ordered graph:

The Optimal Substructure :

Lets assume we have an ordered path. $v1$ should be the first node on the path for all $v_i, i > 1$. Then we have to notice that there might not be a path from v_1 to all nodes v_i. Another observation is that: the longest path, in an ordered path, through v_1 is one edge longer that the longest path through any node to which v_1 is connected.

Notation : OPT(i) = value of optimal solution to the problem consisting of v_i nodes i = 1, 2, ..., n.

OPT(i) = 0 if i = n (we start with the last node in order to use memoisation)
OPT(i) = $-\infty$ (in case when v_i has no children nodes)
OPT(i) = $max_{c \in childredn\ i}\, OPT(1 + OPT(c))$ (otherwise) ;

The dynamic algorithm for this problem:

1. Create an OPT table of size n (n = number of nodes).

2. Create a T stack (that will store the path nodes, which nodes won the OPT max expression)
3. Push v_n to T
4. Set OPT[n] = 0;
3. For i = n - 1 to 1
4. Check the children nodes of i
5. Set OPT[i]= $max_{c \in childredn\ i}\ OPT(1 + OPT(c))$
6. Push the node v_i that wom the max expression to T

The algorithm would fill the OPT table in decreasing order OPT[n], OPT[n-1] ... OPT [1]. We want to make sure that each application of the reccurence will only use precomputed values. When the for loop terminates OPT[1] holds the longest path from v_1 to v_n.
The stackT contains the path fron v_1 to v_n.
To reconstruct the path we just pop elements from the stack T . This would have O(n) running time .

Proof : The value OPT[i] is the longest path that we can achieve for the subset of nodes v_1 to v_i. At each step of the for loop iteration we add the node v_k to the longest path from $v_1\ to\ v_{k-1}$. Assume that at step k, the algorithm choose a solution that is not optimal. Then it would have choosen to add the node v_k to a path that is not the longest path from $v_1\ to\ v_{k-1}$. We know that this can not happen because we checked which choice result in a longest path from $v_1\ to\ v_{k-1}$. in our max OPT expression. Then it is garanteed that the solution choosen by the algorithm is optimal at each node v_i for $0 > i > n$.

Running Time of this algorithm:
n = number of nodes
m = number of edges
Running time is O(n + m)
Space : O(n) .