CMPS 102 — Quarter Spring 2017 – Homework 1

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April 28, 2017

I have read and agree to the collaboration policy.Vladoi Marian
I want to choose homework heavy option.
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Solution to Problem 2: Time Complexity

a) The following functions are ranked by increasing order of growth:
gacrepqmobhdfsjltikn
$\sum_{i=1}^{n} \frac{1}{2}^{i} < \ln(\ln n) < [log(n^2), 14\log_3 n] < \log^2(n) < [\sqrt[2]{n}, 2^{\log_4 n}] < [n, 2^{\log n}] <$
$< [n\log n, \log(n!)] < [n^2, 4^{\log_2 n}, \sum_{x=5}^{n} \frac{x+1}{2}] < n^{\log 7} < 2^{log^2(n)} < \left(\frac{5}{4}\right)^n < [3^n, \sum_{i=1}^{n} 3^i]$
$< n!$

g) $\sum_{i=1}^{n}(\frac{1}{2})^i = \int_1^n (\frac{1}{2})^n dn = \frac{\frac{1}{2}-2^{-n}}{\ln 2} \approx \frac{1}{2^n} \to \sum_{i=1}^{n}(\frac{1}{2})^i < \ln\ln n$ because $\lim_{x\to\infty} \frac{\frac{\frac{1}{2}-2^{-n}}{\ln 2}}{\ln\ln n} = 0$
a) $\ln\ln n \to \lim_{x\to\infty} \frac{\ln\ln n}{14\log_3 n} = 0, \ln\ln n < 14\log_3 n$
c) $14\log_3 n \to \lim_{x\to\infty} \frac{14\log_3 n}{\log_2(n^2)} = \frac{7\ln 2}{\ln 3}, 14\log_3 n \approx \log_2(n^2)$
r) $\log_2(n^2) \to \lim_{x\to\infty} \frac{\log_2(n^2)}{(\log_2 n)^2} = 0, \log_2(n^2) < (\log_2 n)^2$
e) $(\log_2 n)^2 \to \lim_{x\to\infty} \frac{(\log_2 n)^2}{2^{\log_4 n}} = 0, (\log_2 n)^2 < 2^{\log_4 n}$
p) $2^{\log_4 n} = n^{\log_4 2} = n^{\frac{1}{2}}$
q) $\sqrt[2]{n} = n^{\frac{1}{2}}$
m) $2^{\log_2 n} = n^{\log_2 2} = n$
o) n
b) $nlog(n)$
h) $log(n!) \approx nlog(n) stirling\ formula$
d) n^2
f) $4^{log_2 n} = n^{log_2 4} = n^2$
s) $\sum_{x=5}^{n} \frac{x+1}{2} = \frac{1}{4}(n^2+3n-28) \to lim_{x\to\infty} \frac{\sum_{x=5}^{n}\frac{x+1}{2}}{n^2} = constant$
j) $n^{log(7)},\ log(7) > 2$
l) $2^{log^2(n)}\ \ exponential > polinominal$
t) $\left(\frac{5}{4}\right)^n \to lim_{x\to\infty} \frac{2^{log^2(n)}}{\left(\frac{5}{4}\right)^n} = 0$
i) $3^n,\ because\ 3 > \frac{5}{4}$
k) $\sum_{i=1}^{n} 3^i = \frac{3}{2}(3^n - 1) \to \lim_{x\to\infty} \frac{\sum_{i=1}^{n} 3^i}{3^n} =\ constant$
n) $n!\ factorial > exponential$

b)

Claim 1. $f(n) + g(n) = \Omega(max(f(n), g(n)))$

This statement is ALWAYS TRUE

Proof. Let f (n) and g(n) be asymptotically non-negative functions.
Using the definition of Ω
$\exists$ a positive constant n_0 such that f (n) ≥ 0 and g(n)≥ 0 $\forall n \geq n_0$
For such n we have :
$0 \leq max(f(n), g(n)) \leq min(f(n), g(n)) + max(f(n), g(n))$
But $f(n) + g(n) = min(f(n), g(n)) + max(f(n), g(n))$, so $\forall n \geq 0$ we have :
$0 \leq 1 * max(f(n), g(n)) \leq f(n) + g(n)$
Thus $f(n) + g(n) = \Omega(max(f(n), g(n)))$, as required . □

Claim 2. $f(n) = \omega(g(n))$ *and* $f(n) = \mathcal{O}(g(n))$

This statement is ALWAYS FALSE

Proof. I will proof by contradiction.
If our statement is true , it means that $\omega((g(n)) \cap \mathcal{O}(g(n)) \neq \emptyset$.
Let g(n) be an asymptotically non negative function.
Assume that f(n) $\in \omega g(n) \cap \mathcal{O} g(n)$.
(1)Since f(n) = $\mathcal{O} g(n)$: $\exists\, c_1 > 0,\ \exists n_1 > 0$, $\forall n \geq n_1$: $0 \leq f(n) \leq c_1 g(n)$.
(2)Also since f(n) = $\omega g(n)$: $\exists\, c_2 > 0,\ \exists n_2 > 0$, $\forall n \geq n_2$: $0 \leq c_2 g(n) < f(n)$.
Let $c_2 = c_1$ and m = $max(n_1, n_2)$. Then by (1) and (2) we have :
$0 \leq c_1 g(m) < f(m) \leq c_1 g(m)$ which implies that $c_1 g(m) \leq c_1 g(m)$, which is a contradiction.
Our assumption was false and $\omega((g(n)) \cap \mathcal{O}(g(n)) = \emptyset$.
$f(n) = \omega(g(n))$ *and* $f(n) = \mathcal{O}(g(n))$ is false .

□

Claim 3. *Either* $f(n) = \mathcal{O}(n)$ *or* $f(n) = \Omega(g(n))$ *or both.*

This statement is ALWAYS TRUE

Proof. Let f (n) and g(n) be asymptotically non-negative functions.
Either is the case by definition of $\mathcal{O}$
$\exists$ positive constants n_1, c_1 such that f (n) ≥ 0 and g(n)≥ 0 $\forall n \geq n_1$
$0 \leq f(n) \leq c_1 g(n)$
Either is the case by definition of Ω
$\exists$ positive constants n_2, c_2 such that f (n) ≥ 0 and g(n)≥ 0 $\forall n \geq n_2$
$0 \leq c_2 g(n) \leq f(n)$
Either both by the definition of Θ
$\exists$ positive constants $n_3 = max(n_1, n_2), c_1, c_2$ such that f (n) ≥ 0 and g(n)≥ 0 $\forall n \geq n_3$
$0 \leq c_2 g(n) \leq f(n) \leq c_1 g(n)$

□