CMPS 102 — Quarter Spring 2017 – Homework 1

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I have read and agree to the collaboration policy.Vladoi Marian
I want to choose homework heavy option.
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Solution to Problem 3: Induction

1. Uniform shuffling

Claim 1. *The program takes as input an array of n elements (called A) and generates a Uniform Random Shuffle of A. Running time of this Algorithm is* $O(n)$ *.*

Proof. A Permutation is a one-to-one and onto function π : $\{$1,2 ...n$\}$ $\rightarrow$ $\{$1,2...n$\}$. A permutation defines a reordering of the Array A. Because the size of the arry is n, we have n! permuations . A uniform shuffle of A , is a random permutation of its elements. Any permutation from the set of permutations S of size n! is equally probable.

(a) The the algorithm produce any permutation of $A[1...n]$. Suppose we choose one of the permutation of A from the set of permutations S, called A'. We sort in ascending order A' with a sorting algorithm. Then $A'[1] < A'[2] < ...A'[n]$. We can proof that this algorithm can produce the ascending order permutation. If the algorith will randomly choos j as the smallest element of the A, we get A' sorted in ascending order.

(b) We have to show that A' is uniformly shuffled, meaning that an element from A is placed on index i in A' with probability $\frac{1}{n}$ were n is the size of A.
Show that an element from A is placed in A'[i] with a probability:

$$\prod_{i=1}^{n-1} \frac{n-i}{n-i+1}$$

We prove it by induction on array A' index (called i)
Base case: i = 1.
The first element is placed on A' with probability = $\prod_{i=1}^{n-1} \frac{n-i}{n-i+1} = \frac{n-1}{n} * \frac{n-2}{n-1} * ... \frac{1}{n-i} = \frac{1}{n}$
Assume for $1 < i \leq (k-1)$, the probability of k-1 element of A to be placed on A' = $\prod_{i=k-1}^{n-1} \frac{n-i}{n-i+1} = \frac{1}{n}$
We have to show that for $i = k$, $\prod_{i=k}^{n-1} \frac{n-i}{n-i+1} = \frac{1}{n}$
$\prod_{i=k}^{n-1} \frac{n-i}{n-i+1}$ = $\prod_{i=k-1}^{n-1} \frac{n-i}{n-i+1} * \frac{n-k}{n-k+1}$ = $\frac{n-(k-1)}{n-(k-1)+1} * ... \frac{1}{n-k} * \frac{n-(k)}{n-k+1} = \frac{1}{n}$

The algorith running time is O(n), because if we assume that A[0],...,A[$n-1$] are already shuffled, we randomly select each element A[j] from A[i],...,A[$n-1$]and exchange it with A[i].

$\square$

2. Point out the error of the Induction

Let B be a set of $b+1$ buses .

$B = \{b_1, b_2, b_3, ...b_{n+1}\}$

Then when we remove a bus for the first time we create a new set

$B_1 = \{b_2, b_3, ...b_{n+1}\}$ = $B - \{b_1\}$. which has b buses

When we remove the second time a bus we create a new set

$B_2 = \{b_1, b_3, ...b_{n+1}\}$ = $B - \{b_2\}$. which has b buses

The proposition being proved is false because the inductive step was not quantified properly. It should have been proved that $\forall b \geq 1 P(n) \rightarrow P(n+1)$. Instead it is proved that $\forall b > 1 P(n) \rightarrow P(n+1)$. It is false that $P(1) \rightarrow P(2)$. It is proved only that $P(2) \rightarrow P(3), P(3) \rightarrow P(4)$....