CMPS 130

Computational Models
Homework Assignment I
VLADOI MARIAN

Spring 2017

1.29. (a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Consider the set $S = \{0^i \mid i \geq 1\}$
The set S is an infinite set
Any 2 members $\in S$ have the form $a = 0^c$
and $b = 0^d$ where $c \neq d$
the string $x = 1^c 2^c$ distinguishes them
$ax = 0^c 1^c 2^c \in A_1$
$bx = 0^d 1^c 2^c \notin A_1$
$0^c \cancel{R_{A_1}} 0^d$ if $c \neq d$
Therefore A_1 is not regular by Myhill-Nerode Theorem

(c) $A_3 = \{a^{2^n} \mid n \geq 0\}$
Consider the set $X = \{0^{2^i} \mid i \geq 1\}$
The set X is an infinite set
Any 2 members $\in X$ have the form $a = 0^{2^c}$
and $b = 0^{2^d}$ where $c \neq d$
string $z = 0^{2^c}$ distinguishes them
We have 2 cases:

① $c < d$
$2^c < 2^d$
$2^c < 2^d < 2^d + 2^c < 2^d + 2^d = 2^{d+1}$
We show that $0^d < 0^{(2^d + 2^c)} < 0^{2^{d+1}}$
$az = 0^{2^c} 0^{2^c} = 0^{2^{c+1}} \in A_3$
$bz = 0^{2^d} 0^{2^c} = 0^{2^c + 2^d} \notin A_3$ because
$0^{2^d} 0^{2^c}$ is between 0^{2^d} and $0^{2^{d+1}}$

② $c > d$
$2^c > 2^d$
$2^d < 2^c < 2^d + 2^c < 2^c + 2^c < 2^{c+1}$

$az = 0^{2^d} 0^{2^d} = 0^{2^{d+1}} \in A_3$

$sz = 0^{2^c} 0^{2^d} = 0^{2^c + 2^d} \notin A_3$ because

$0^{2^d} 0^{2^c}$ is between 0^{2^d} and $0^{2^{d+1}}$

A_3 is not regular by the Myhill Nerode theorem

$\boxed{\text{Problem 1.46}}$ page 50

(b) $A_2 = \{0^m 1^n \mid m \neq n\}$

Consider the set $X = \{0^i \mid i \geq 1\}$

the set X is an infinite set

Any 2 members in X have the form

$a = 0^c$

$s = 0^d$

In which $c \neq d$

string $z = 1^c$ distinguishes them

$az = 0^c 1^c \notin A_2$

$sz = 0^d 1^c \in A_2$

A_2 is not regular by the Myhill Nerode theorem

(c) $A_3 = \{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$

Consider the set $X = \{0^i 1 \mid i \geq 1\}$

the set X is an infinite set

Any 2 members $\in$ X have the form

$a = 0^c 1$

$s = 0^d 1$

In which $c \neq d$

string $z = 10^c$ distinguishes them

$az = 0^c 110^c \notin A_3$ this is a palindrome

$sz = 0^d 110^c \in A_3$ this is not a palindrome

A_3 is not regular by the Myhill Nerode theorem

II Prove a language is regular
$L = \{0^n \mid n \% 2 = 0\}$

Let A_1 and A_2 be two languages such that
$A_1, A_2 \subseteq \Sigma^*$
Let $A_1 = L$
Let $A_2 = \{0^n \mid n \% 2 \neq 0\}$

(1) We say that all the members of A_1 are equivalent to each other because:
Let $a = 0^i$
$b = 0^j$
a and b are two members of A_1
i and j are both divisible by 2
$i \neq j$
Then it is the case that $0^i R_{A_1} 0^j$
since for any $z \in \Sigma^*$ say $z = 0^k$
both $\left.\begin{array}{l} az = 0^i 0^k \\ bz = 0^j 0^k \end{array}\right\}$ are in A_1 if $k \% 2 = 0$

neither $\left.\begin{array}{l} az = 0^i 0^k \\ bz = 0^j 0^k \end{array}\right\}$ are in A_1 if $k \% 2 \neq 0$

(2) We say that all the members of A_2 are equivalent to each other because:
let $a = 0^i$
$b = 0^j$
a and b are two members of A_2
a and b are not divisible by 2
$i \neq j$
then it is the case that $0^i R_{A_2} 0^j$
since for any $z \in \Sigma^*$ say $z = 0^k$

both $\left.\begin{array}{l} az = 0^i 0^k \\ bz = 0^j 0^k \end{array}\right\}$ are in A_2 if $k \% 2 \neq 0$

neither $\left.\begin{array}{l} az = 0^i 0^k \\ bz = 0^j 0^k \end{array}\right\}$ are ~~not~~ in A_2 if $k \% 2 = 0$

We showed that any member of Σ^* is either in A_1 or A_2.
$A_1 \cup A_2 = \Sigma^*$.
the language L is a regular language.

$\boxed{\text{Exercise 2.3}}$ page 155.

1. the set of variables is $\{R, S, X, T\}$ and the
2. set of terminals is $\{a, b\}$. 3. R is the start symbol
4. (ab, ba, aab)
5. (a, b, aa)
6. $T \Rightarrow aba$ False
7. $T \Rightarrow^* aba$ True
8. $T \Rightarrow T$ False
9. $T \Rightarrow^* T$ True
10. $XXX \Rightarrow^* aba$ True
11. $X \Rightarrow^* aba$ False
12. $T \Rightarrow^* XX$ True
13. $T \Rightarrow^* XXX$ True
14. $S \Rightarrow^* \varepsilon$ False
15. Every word in $L(G)$ has the form $w_1 a w b w_2$ or $w_1 b w a w_2$ for some $w, w_1, w_2 \in \{a, b\}^*$ such that $|w_1| = |w_2|$

$\boxed{\text{Exercise 2.4}}$ page 155

① $L = \{ w \mid w$ contains at least 3 1's $\}$:

$$X \to Y1Y1Y1Y$$
$$Y \to 0Y \mid 1Y \mid \varepsilon$$

② $L = \{ w \mid w$ starts and ends with the same symbol $\}$

$$X \to 0Y0 \mid 1Y1$$
$$Y \to 0Y \mid 1Y \mid \varepsilon$$

③ $L = \{ w \mid$ the length of w is odd

$$X \to 0Y \mid 1Y$$
$$Y \to 0X \mid 1X \mid \varepsilon$$

④ $L = \{ w \mid$ the length of w is odd and its middle is $0 \}$

$$X \to 0X0 \mid 0X1 \mid 1X0 \mid 1X1 \mid 0$$

⑤ $L = \{ w \mid w = w^R$ that is w is a palindrome $\}$

$$X \to 0X0 \mid 1X1 \mid 0 \mid 1 \mid \varepsilon$$

⑥ the empty set

$$X \to X$$

PS6

III. Give a CFG for the language $\{x \in \{a,b\}^* \mid x \neq ww \text{ for some } w \in \{a,b\}^*\}$.

$S \to A \mid B$

$S \to AB \mid BA$

$A \to LAL \mid a$

$B \to LBL \mid b$

$L \to a \mid b$