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CMPS 130
Homework #5

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29 a) $A_1 = \{0^n1^n2^n \mid n \geq 0\}$
We first assume that A_1 is regular. We will let p be the pumping length given by the pumping lemma. We also will let s be the string $0^p1^p2^p$. ($s = 0^p1^p2^p$)

Since s is the member of A_1 and s is longer than p, the pumping lemma guarantees s is able to be split into 3 parts, s = xyz for any $i \geq 0$ the string xy^iz is in A_1.

We can consider 2 possibilities:

1) String y consists only of 0s, only of 1s, only of 2s. xy^2z won't have equal numbers of 0s, 1s, and 2s. Hence xy^2z isn't a member of A_1
2) String y consists of more than one kind of symbol. xy^2z have 0s, 1s, or 2s in a different order. Hence xy^2z isn't a member of A_1

Looking at both these cases we arrive and a contradiction. Therefore A_1 isn't regular.

b) $A_2 = \{www \mid w \varepsilon \{a,b\}^*\}$
We first assume that A_2 is regular. We will let p be the pumping length given by the pumping lemma. We will also let s be the string $a^pba^pba^pb$

$s \varepsilon A_2$ since $s=(a^pb)^3$ and $|s| = 3(p+1) \geq p$ so the pumping lemma holds. We can further split the string into 3 pieces, s = xyz satisfying the conditions $xy^iz \varepsilon A_2$ for each $i \geq 0$, $|y| > 0$, $|xy| \leq p$.

The first p symbols of s are all a's, the condition implies x and y consists only of a's. |y| > 0 y has at least one a. Thus we may say: $x = a^j$ for $j \geq 0$

$y = a^k$ for $k \geq 1$

$z = a^mba^pba^pb$ for $m \geq 0$

s = xyz we take the individual value of x, y, and z multiply them out to get $a^{j+k+m}ba^pba^pb$ thus j+k+m = p. We implied $xy^2z = a^{p+k}ba^pba^pb$ since j+k+m = p. Thus $xy^2z \notin A_2$ because $k \geq 1$ we get a contradiction, and hence A_2 is a nonregular language.

c) $A_3 = \{a^{2^n} \mid n \geq 0\}$
First we will assume A_3 is regular. We will let p be the pumping length given by the pumping lemma. We will then choose s to be the string a^{2^p}. Since s is a member of A_3 and s is longer than p, we can say the pumping lemma guarantees s can be split into 3 parts s = xyz thus we satisfy the 3 conditions of the pumping lemma.

The 3rd condition tells us that $|xy| \leq p$. Further, $p < 2^p$ and so $|y| < 2^p$. Therefore $|xy^2z| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$. The second condition requires $|y| > 1$ so $2^p < |xy^2z| < 2^{p+1}$. The length of xy^2z cannot be a power of 2. Hence xy^2z is not a member of A_3, a contradiction. Therefore, A_3 is not regular.

30) We first will set p to be the pumping length of 0*1* and string s to be 0^p1^p. In other words $s = 0^p1^p$. We can further split s into 3 parts xyz. We can take an example and set xy^iz is equal to $0^{p+(j-1)|y|}1^p$ this is a flaw since xy^iz is still contained within in 0*1*.
Therefore this proof is flawed and 0*1* is regular.

42) Let $M_A = (Q_a, \Sigma, \delta a, q_a, F_a)$ be a DFA recognizing A and $M_B = (Q_B, \Sigma, \delta b, q_B, F_B)$ be a DFA recognizing B. The NFA N for the shuffle of A & B, simulate both M_A & M_B.

We can formally mention the definition of N as:

1) $Q = (Q_A \times Q_B) \cup \{q_0\}$ where $Q_A \times Q_B$ are all possible current states of M_A and $M_{\mathbf{B}}$, and q_0 is the state when nothing is read.
2) $q = q_0$
3) $F = (F_A \times F_B) \cup \{q_0\}$ in which N accepts the string if both $M_A \times M_B$ are in accept states, or N accepts the empty string.
4) δ is defined as follows:

i) $\delta(q_0, \varepsilon) = (q_A, q_B)$ which states q_0 is the start state. N can make M_A in q_A and M_B in q_B without reading anything

ii) $(\delta A(x,a), y) \in \delta((x,y),a)$ which states if the current state of M_A is x, the current state of M_B is y, when a is read, we change the state of A to $\delta A(x,a)$ while state B isn't changed

iii) $(x, \delta B(y,a)) \in \delta((x,y),a)$

46a) $\{0^n1^m0^n \mid m, n \geq 0\}$
To prove that L isn't a regular language, we should use a proof by contradiction. We first assume L is regular, and by the pumping lemma we can say there exists a p(pumping length) for L such that for any string $s \in L$ where $|s| \geq p$, $s = xyz$ contains these following conditions:

(1) $|y| > 0$
(2) $|xy| \leq p$
(3) $\forall i > 0, xy^iz \in L$.

We then choose $s = 0^p10^p$. We can clearly see, $|s| \geq p$ and $s \in L$.
By condition (2) we can say that x and y are composed solely of zeros.
By condition(1) we can say that $y = 0^k$ for some value $k > 0$
By condition (3) we can take i to be 0, and the string will still be contained within L.
Thus xy^0z should be in L. $xy^0z = xz = 0^{(p-k)}10^p$. but it clearly isn't contained within L

This is a contradiction, therefore the assumption we made in the beginning is incorrect, and L isn't a regular language.

b) $\{0^m1^n \mid m \neq n\}$

We shall use a proof by contradiction to show L isn't a regular language. We first assume L is a regular language. Using the pumping lemma for regular languages, we see there exists a p(pumping length) for L such that for any string $s \in L$ where $|s| \geq p$, $s = xyz$ contains these following 3 conditions:

(1) $|y| > 0$

(2) $|xy| \leq p$, and

(3) $\forall i > 0, xy^iz \in L$.

We shall choose $s = 0^p110^p1$. We can clearly see that $s \in L$ with $w = 0^p1$ and $t = 1$, and $|s| \geq p$.

By condition (2), it is pretty noticeable that xy is composed solely of 0s

by (1) and (2), if shows that $y = 0^k$ for some $k > 0$.

By condition (3), we can take any i and xy^iz will still be contained within in L.

We can set i to be 2, then $xy^2z \in L$. $xy^2z = xyyz = 0^{(p+k)}110^p1$.

There is no possible way to split the string into wtw as required to be in L, thus $xy^2z \notin L$

This contradicts condition (3) of the pumping lemma.

Thus our assumption in the beginning is false and L isn't a regular language.

c) L = {w| w ε {0,1}* isn't a palindrome}

We will prove L by contradiction. We first assume L is regular, then we set L to its complement L` which says {w| w $\varepsilon$ {0,1}* is a palindrome}. In this case we can say L` is also regular.

If L` is regular by the pumping lemma, there exists an integer $p > 0$ which is the pumping length, such that any word $w \in$ L`|w| $|w| \geq p$ can further be split into 3 parts such as w = xyz(with $|xy| \leq p$ and $|y| > 0$) in such a way that, for any $i \geq 0$, the word xy^iz also belongs to L`.

Now we should consider the word $w = 0^p10^p$, We can see $|w| = 2p + 1$ so it satisfies the condition $|w| \geq p$ of the theorem. Any partition of w into parts x, y, z must be such that $y = 0^k$ for some integer $k > 0$, and then $x = 0^{p-k}$ and $z = 10^p$. If the word is formed as w` = $xy^0z$ then we can further say w` = $0^{p-k} \in 10^p = 0^{p-k}10^p$ which isn't a palindrome since $p-k < p$. Thus w` $\notin$ L` which contradicts the result of the pumping lemma.

d) L = {wtw | w, t $\in$ {0, 1} +}

We first want to prove that L is not a regular language, so we use a proof of contradiction. First we assume that L is a regular language. By the pumping lemma for Regular Languages there's a p(pumping length) for L such that for any string $s \in L$ where $|s| \geq p$, $s = xyz$ has the 3 following conditions: (1) $|y| > 0$

(2) $|xy| \leq p$, and

(3) $\forall i > 0, xy^iz \in L$.

We shall choose $s = 0^p110^p1$. We clearly can see that $s \in L$ with $w = 0^p1$ and $t = 1$, and $|s| \geq p$. By condition (2), it is obvious that xy is composed only of zeros, and further, by (1) and (2), it follows that $y = 0^k$ for some $k > 0$. By condition (3), we can take any value i and $xy^i z$ will still be in L.

By taking i to be 2, then $xy^2z \in L$. $xy^2z = xyyz = 0^{(p+k)}110^p1$. We can't find a way to split this string into wtw which is required to be in L thus $xy^2z \notin L$. This is a contradiction with condition (3) of the pumping lemma. Thus L isn't a regular language.

47)
Let $\Sigma = \{1, \#\}$ and let

$A = \{w \mid w = x_1\#x_2\#\cdots\#x_k,\ k \geq 0,\ \text{each } x_i \in 1^* \text{ and } (i \neq j) \Rightarrow (x_i \neq x_j)\}$

We can let p be a pumping lemma constant for A.
We can let $u = 1^p\#1^{(p+1)}\# \cdots \#1^{(2p)}$.
Another way we can write u is as $u = u_0\#u_1\# \cdots \#u_k$, where $k = p$ and $u_i = 1^{(p+i)}$.
Let $xyz = u$ such that $|y| > 0$ and $|xy| \leq p$.
Let $v = xy^2z$. Note that we can write $v = v_o\#v_1\# \cdots \#v_k$, where $k = p$, $v_0 = 1^{p+|y|}$ for $1 \leq i \leq p$, $v_i = 1^{(p+1)}$. Because $1 \leq |y| \leq p$, we conclude $p + 1 \leq (p + |y|) \leq 2p$ and $v_0 = v_{p+|y|}$. Thus, $v \notin A$
We can see that A does not satisfy the conditions of the pumping lemma. Therefore, A is not regular.

55) e) (01)* Let s be the string in the language. S could be ε but it can't be pumped so the length isn't 0. Next s could be 01, if we divide into xyz since x is an empty string ε, y is 01, and z is everything after, then it satisfies the 3 conditions of the pumping lemma

Therefore the minimum pumping length is 1.

f) ε Let s be a string in the language then we can say s is in ε and according to the pumping lemma it can't be pumped

Therefore the minimum pumping length is 0

i) 1011: Let s be a string in the language. If we divide s into xy^iz we get x as 10, y as 1, and z is an empty string.

Therefore the minimum pumping length is 3

j) Σ^*
We can say s to be a string in the language. According to the pumping lemma if we divide s in xyz then x can be the empty string, y is (ε | 0| 1) and z is the empty string. ε can't be pumped.

Therefore the minimum pumping length is 1