CMPS 130

Computational Models
Homework Assignment 6
VLADOI MARIAN

Spring 2017

Exercise 1.51 page 80
Show $\equiv_L$ is an equivalence relation
We have to show that $\equiv_L$ is
a) Reflexive
b) Symmetric
c) Transitive

a) Reflexive: $x \equiv_L x$ is true
For all strings z, xz is in L iff xz is in L
therefore $x \equiv_L x$ is true. Hence $\equiv_L$ is reflexive

b) Symmetric: $x \equiv_L y$ implies $y \equiv_L x$
If $x \equiv_L y$ is true then $\forall z$, xz is in L iff yz in L
this is also equivalent to this statement:
$\forall z$, yz is in L iff xz is in L
Therefore $y \equiv_L x$ is also true
Hence $\equiv_L$ is symmetric

c) Transitive: If $a \equiv_L b$ and $b \equiv_L c$ then $a \equiv_L c$
This can be written as $\forall z$ $az \in L$ iff $bz \in L$ and
$\forall z$, $bz \in L$ iff $cz \in L$
therefore $\forall z$, $az \in L$ iff $cz \in L$
$a \equiv_L c$ is true. Hence $\equiv_L$ is transitive
Since we proved $\equiv_L$ is reflexive, symmetric and
transitive we can say $\equiv_L$ is an equivalence relation

PJ2

2.A

	a	b
1	6	3
2	5	6
3	4	5
4	3	2
5	2	1
6	1	4

	1	2	3	4	5	6
1						
2	2					
3	1	1				
4	1	1	0			
5	2	0	1	1		
6	0	2	1	1	2	

$(1,2) \xrightarrow{a} (6,5)$
$ \xrightarrow{b} (3,6)$

$(1,5) \xrightarrow{a} (2,6)$
$ \xrightarrow{b} (1,3)$

$(2,5) \xrightarrow{a} (2,5)$
$ \xrightarrow{b} (1,6)$

$(3,5) \xrightarrow{a} (3,4)$
$ \xrightarrow{b} (1,5)$

$(1,6) \xrightarrow{a} (1,6)$
$(1,6) \xrightarrow{b} (3,4)$

$(2,6) \xrightarrow{a} (1,5)$
$ \xrightarrow{b} (4,6)$

$(6,5) \xrightarrow{a} (2,1)$
$ \xrightarrow{b} (1,4)$

the states that are unmarked and we can combine are (2,5), (1,6), (3,4)

	a	b
(1,6)	(1,6)	(3,4)
(2,5)	(2,5)	(1,6)
(3,4)	(3,4)	(2,5)

2B.

	a	b
1	2	3
2	5	6
3	1	4
4	6	3
5	2	1
6	5	4

1	X	X	X	X	X
2	2	X	X	X	X
3	1	1	X	X	X
4	1	1	0	X	X
5	2	0	1	1	X
6	0	2	1	1	2
	1	2	3	4	5

$(1,2) \xrightarrow{a} (2,5)$
$ \xrightarrow{b} (3,6)$

$(3,4) \xrightarrow{a} (1,6)$
$ \xrightarrow{b} (4,3)$

$(1,5) \xrightarrow{a} (2,2)$
$ \xrightarrow{b} (1,3)$

$(2,5) \xrightarrow{a} (2,5)$
$ \xrightarrow{b} (1,6)$

$(1,6) \xrightarrow{a} (2,5)$ (
$ \xrightarrow{b} (3,4)$

$(2,6) \xrightarrow{a} (5,5)$
$ \xrightarrow{b} (1,6)$

$(5,6) \xrightarrow{a} (2,5)$
$ \xrightarrow{b} (1,4)$

We can reduce the points $(3,4)$, $(2,5)$ and $(1,6)$

	a	b
(1,6)	(2,5)	(3,4)
(2,5)	(2,5)	(6,1)
(3,4)	(1,6)	(4,3)

2 C.

	a	s
0	3	2
1	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

	0	1	2	3	4	5
0						
1	0					
2	1	1				
3	1	1	2			
4	1	1	3	2		
5	1	1	0	0	3	
6	1	1	2	0	2	2

	a	s
0, 1	2, 3, 5, 6	2, 3, 5, 6
4	2, 3, 5, 6	4
2, 3, 5, 6	2, 3, 5, 6	2, 3, 5, 6

2^{nd} poss:

$(0,1) \xrightarrow{a} (3,3)$
$\xrightarrow{s} (2,5$

$(2,6) \xrightarrow{a} (2,5)$
$\xrightarrow{s} (0,6)$

$(2,5) \xrightarrow{s} (3,6)$

3^{rd} poss:

$(0,1) \xrightarrow{s} (2,5)$

Equivalent class $(0,1)\ (2,5)\ (3,6)$
$(3,5)$

P55

2.D

	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4	0	2
5	1	6
6	2	6

0						
1	0					
2	2	2				
3	2	2	0			
4	1	1	1	1		
5	1	1	1	1	0	
6	2	2	0	0	1	1

$(0,1) \xrightarrow{a} (3,2)$
$\xrightarrow{b} (5,4)$

$(2,3) \xrightarrow{a} (6,6)$
$\xrightarrow{b} (3,6)$

$(2,6) \xrightarrow{a} (6,2)$
$\xrightarrow{b} (3,6)$

$(3,6) \xrightarrow{a} (6,2)$
$\xrightarrow{b} (6,6)$

$(4,5) \xrightarrow{a} (0,1)$
$\xrightarrow{b} (2,6)$

$(0,2) \xrightarrow{a} (3,6)$
$\xrightarrow{b} (5,3)$

$(0,3) \xrightarrow{a} (3,6)$
$\xrightarrow{b} (5,6)$

$(0,6) \xrightarrow{a} (3,2)$
$\xrightarrow{b} (5,6)$

$(1,2) \xrightarrow{a} (2,6)$
$\xrightarrow{b} (4,3)$

$(1,3) \xrightarrow{a} (2,6)$
$\xrightarrow{b} (4,6)$

$(1,6) \xrightarrow{a} (2,2)$
$\xrightarrow{b} (4,6)$

Equivalent classes (0,1), (2,6), (3,2), (3,6), (4,5)

	a	b
(0,1)	(3,2)	(5,4)
(4,5)	(0,1)	(2,3,6)
(2,3,6)	(2,3,6)	(2,3,6)