CMPS 130

Computational Models
Homework Assignment 8
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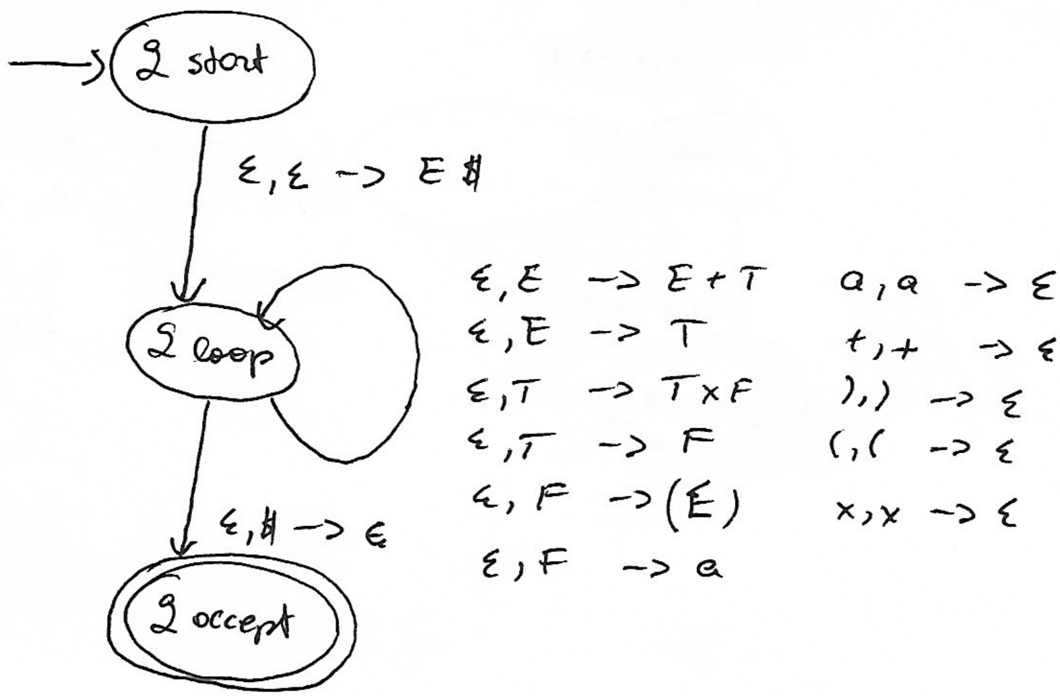
PS1

2.11 The CFG G_4 is:

$E \rightarrow E + T \mid T$
$T \rightarrow T \times F \mid F$
$F \rightarrow (E) \mid a$

Assuming that we can write the entire string to the stack in one PDA step, the PDA is:



PS2

$\boxed{2.12}$

$R \to XRX \mid S$

$S \to aTb \mid bTa$

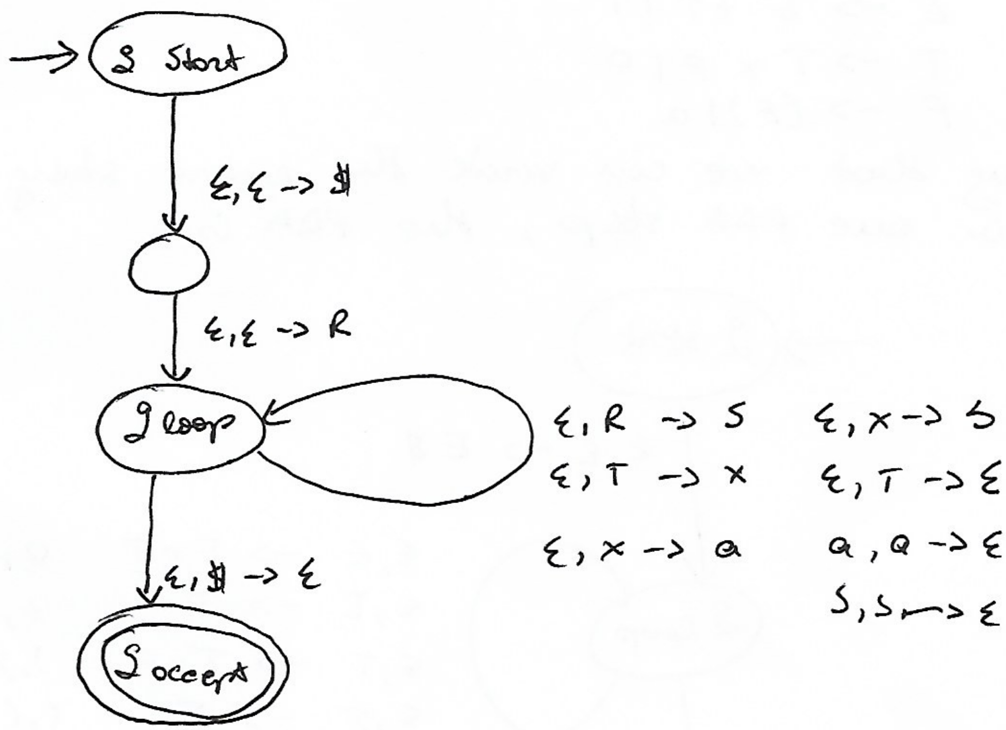
$T \to XTX \mid X \mid \varepsilon$

$X \to a \mid b$

$\varepsilon, R \to XRX$ $\quad$ $\varepsilon, T \to XTX$

$\varepsilon, S \to aTb$

$\varepsilon, S \to bTa$



2.

PS3

$\boxed{2.14}$

$A \to BAB \mid B \mid \varepsilon$
$B \to 00 \mid \varepsilon$

$A \to BAB \mid B \mid BB \mid BA \mid AB$
$B \to 00$

$A \to BAB \mid 00 \mid BB \mid BA \mid AB$
$B \to CC$
$C \to 0$

$A \to BAB \mid CC \mid BB \mid BA \mid AB$
$B \to CC$
$C \to 0$

$A \to BF \mid CC \mid BB \mid BA \mid AB$
$B \to CC$
$C \to 0$
$F \to AB$

2.30 a) $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Assume L is context free
Let p be the pumping length by pumping lemma
Let $s = 0^p 1^p 0^p 1^p$
If $s = uv^i w x^i y$, this string can not be pumped
consider the case when v is all 0's and
x is all 1's
We see that increasing the number of 0's will cause the resulting string not to be in the language

PS4

Then, suppose both v and x contains only 0's
In this case, when increase i, will cause the resulting string not to be in the language.
By pumping lemma, we showed that this language is not context free.

5) $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$
Assume L is context free
Let p be the pumping length
Let $s = 0^p \# 0^{2p} \# 0^{3p}$.
Now divide s into $u x^i w y^i z$
such that $|xyz| \leq p$ and $|xy| \geq 1$

In the case when x is all 0's and
y is all 0's
we increase i and we can see that n and 2n are not matching anymore.

Next x or y can have #,
In this case, increasing i makes the ratio of n, 2n, 3n not maintained.

By pumping lemma, we showed that the language $0^n \# 0^{2n} \# 0^{3n}$ is not a context free language.

PS5

c) $\{ w \# t \mid w$ is a substring of t, where $w, t \in \{a, b\}^* \}$
Assume L is context free
Let p be the pumping length by pumping lemma.
Let $s = a^p b^p \# a^p b^p$
We show that $z = u v^i x w^i y$ can not be pumped

Let v and w be both a's
Then increasing the i will cause having more a's in one side.
This case also apply for both v and w be b's
Now consider the case where v or w contains #
Increasing the i we can not maintain the ratio and the resulting string would not be in the language
Then, consider the case when $v = a$ and $w = b$
or $v = b$ and $w = a$.
Increasing the i will cause the string not to be in the language
By pumping lemma, we showed that the language L is not a context free.

d) $L = \{ t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2,$ each $t_i \in \{a, b\}^*$ and $t_i = t_j$ for $i \neq j \}$

Assume L is context free
Let p be the pumping length by pumping lemma.
Let $z = a^p b^p \# a^p b^p$
We show that the string $s = u v^i w x^i y$ can not be pumped.
Consider the case that either v or x contains #
Increasing i will cause the string not to be in L
Then, consider the case that both v and x are a's
In this case, increasing the i will make z not to be in the language
Then, consider v ~~x~~ is all a's and x is all b's, or v is all b's and x is all a's
Increasing the i will change the number of a's and b's.
We showed that L is not a context free language.

PS7

2.31 L = all palindromes over $\{0,1\}$

Assume L is context free

Let p be the pumping length

Let $s = 0^p 1^{2p} 0^p \in L$

By the pumping lemma, we may choose u, v, x, y, z

such that $s = uvxyz$, $|v| > 0$ or $|y| > 0$,

$|vxy| \leq p$, and $uxz \in L$

Consider the case when v and y ~~so~~ are all 1's.

then $uxz = 0^p 1^{2p-|vy|} 0^p$ does not have the same number of 0's and 1's.

Otherwise, v or y contains k symbols of m from one of the sets of 0's and some amout n from the set of 1's $(k > 0)$, but not from the other set of 0's, because $|vxy| \leq p$.

We get 4 cases:

the set of 0's intersecting vy can be 0 or 1

and the substring containing 0's and 1's can be v or y.

then uxz can be ① $0^{p-m} 1^{2p-n-|y|} 0^p$

② $0^{p-m-|v|} 1^{2p-n} 0^p$

③ $0^p 1^{2p-n-|v|} 0^{p-m}$

④ $0^p 1^{2p-n} 0^{p-m-|y|}$

None of this is a palindrome since $m > 0$, the $uxz \notin L$.

PS 8

2.32 Assume L is context free language
Let's say L has pumping length p
Let $s = 1^p 3^p 2^p 4^p \in L$ $|s| > p$
Therefore, there exists $uvxyz$ such that

a) $uv^i x y^i z \in L$ for all $i \geq 0$
b) $xy > 0$
c) $vxy \leq p$

Case I: If vxy contains a 1
then $uv^2xy^2z \notin L$, because we do not have the same number of 1's and 2's
Hence, by condition ©, vxy can not contain any 2's.

Case II: vxy contains a 2
Then $uv^2xy^2z \notin L$, because we do not have the same number of 1's and 2's.
Hence, by condition © vxy can not contain any 1's.

Case III: vxy contains a 3
Then $uv^2xy^2z \notin L$, because we do not have the same number of 3's and 4's.
Hence by condition ©, vxy can not contain any 4's.

Case IV: vxy contains a 4
then $uv^2xy^2z \notin L$, because we do not have the same number of 3's and 4's.
Hence, by condition ©, vxy can not contain any 3's.

From condition ⓑ we contradict ⓐ
then L is not context free.

2.35

Since G is a CFG in Chomsky normal form, Every derivation can generate at most 2 non terminals.

In any parse tree using G, an internal node can have at most 2 children.

If the parse tree has height K, then the tree has at most $2^K - 1$ internal nodes.

If G generates a string in 2^b steps, the parse tree of this string will have 2^b internal nodes. The height of this parse tree is at least b+1 implies that there is a path from root to leaf containing b+1 nodes. (variables)

By pigeonhole principle, there is one variable occuring at least twice.

We can use the technique described in the proof of pumping lemma, to construct infinitely many strings which are all in L(G).