(pg 1)

CMPS 130

Computational Models
Homework Assignment 5
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Exercise 1.29 page 88

Use the pumping lemma to show that the following languages are not regular.

a. $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Proof by contradiction.

Suppose A_1 is regular.

Let p be the pumping length.

Choose $s = 0^p 1^p 2^p$

By the lemma $|xy| \leq p$ and $|y| > 0$ therefore $p \geq 0$

so $s \in A_1$

$|s| \geq p$ thus $s = xyz$ for some $x y z$

Since $|xy| \leq p \Rightarrow xy = 0^k$, where $1 \leq k \leq p$

Let us write $x = 0^a$, $y = 0^b$, $z = 0^c 1^p 2^p$

The number of 0, 1, 2 in s are given by $a+b+c=p$

Let $i=0 \Rightarrow s' = xy^i z = xz$

The number of 1's in $s' = p$

The number of 0's in $s' = a+c$

$s' \in A$, the number of 0 must equal the number of 1

$\Rightarrow a+c = p$

$\Rightarrow a+c = a+b+c$ when $b=0$

$|y| > 0$ and $|y| = b$, $b > 0$, thus $s' \notin A$, a contradiction. Therefore A_1 is not regular.

PS2

b. $A_2 = \{www \mid w \in \{a,b\}^*\}$

Proof by contradiction:
Suppose A_2 is regular
Let p be the pumping length
choose $s = www$ where $w = a^p b$
Clearly $s \in A_2$ and $|s| \geq p$, thus $s = xyz$
Since $|xy| \leq p$, xy cannot extend the first p symbol of $s \Rightarrow xy = a^k \quad 1 \leq k \leq p$
$x = a^a$, $y = a^b$, $z = a^c b a^p b a^p b$
the number of a in w is given $a + b + c = p$
Let $i = 0$ and $s' = xy^iz = xz$
$\Rightarrow xz = w_1 w_2 w_3 \quad (w_1 = a^a a^c b, \quad w_2 = w_3 = a^p b$
the number of a's in $w_2, w_3 = p$
The number of a's in $w_1 = a + c$
For $s' \in A \quad a + c = p$
Substituting for $p \Rightarrow a + c = a + b + c \Rightarrow b = 0$
Because $|y| > 0$ and $|y| = b$, $b > 0$ thus $s' \notin A$, a contradiction. therefore A_2 is not regular.

c. $A_3 = \{a^{2^n} \mid n \geq 0\}$ n is an integer
Proof by contradiction:
Suppose A_3 is regular
Let p be the pumping length
choose $s = a^{2^n}$, by lemma $|xy| \leq p$ and $|y| > 0$
therefore $p \geq 0$ and $s \in A_3$
$|s| = 2^p \geq p$, thus $s = xyz$ for some xyz.
Let us write $x = a^a$, $y = a^b$, $z = a^c$
the number of a's in $s = a + b + c = 2^p$
Let $i = 2$ and $s' = xy^iz = xyyz$
the number of a's in $s' = a + 2b + c = 2^p + b$
Since $|y| > 0$ and $|y| = b$, $b > 0$

Substituting for s $\Rightarrow$ $a + 2s + c = 2^p + s$

$\Rightarrow$ $a + 2s + c = 2^p + 2^p - a - c$

Since $|xy| \le p$, $c = |xyz| - |xy| \ge 2^p - p > 0$

$\Rightarrow$ $a + 2s + c < 2^{p+1}$

Thus Number of a's in $s' < 2^{p+1}$

$\Rightarrow 2^p <$ the Number of a's in $s' < 2^{p+1}$

$\Rightarrow$ the Number of a's in s' is not even power of 2

$\Rightarrow$ $s' \notin A_3$, a contradiction

therefore A_3 is non-regular

Exercise 1.30 page 88.

Describe an error in the following proof that 0^*1^* is not a regular language.

The example 1.73 (pg 80) gives a proof that the language $B = \{0^n 1^n \mid n \ge 0\}$ is not regular

It means that $0^p 1^p$ can not be pumped with respect to the language B, not with respect to the language $0^* 1^*$. Strings from $0^p 1^p$ belong to language $0^* 1^*$.

Exercise 1.42 page 88

For languages A and B, let the shuffle of A and B be the language

$\{w \mid w = a_1 b_1 \dots a_k b_k$, where $a_1 \dots a_k \in A$ and $b_1 \dots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$

Show that this kind of regular language is closed under shuffle.

Let $A = (Q_A, \Sigma, \delta_A, s_A, F_A)$

$B = (Q_B, \Sigma, \delta_B, s_B, F_B)$

be 2 DFA's that recognize A and B

PS4

Proof by construction:

a) $Q = (Q_A \times Q_B) \cup \{q_0\}$

b) $q = q_0$

c) $F = (F_A \times F_B) \cup \{q_0\}$

d) δ is :

① $\delta(q_0, \varepsilon) = (q_A, q_B)$
② $(\delta_A(x, a), y) \in \delta((x, y), a)$
③ $(x, \delta_B(y, a)) \in \delta((x, y, s))$

e) the Σ_A is the same with Σ_B

Problem 1.46 page 80

Prove that the following languages are non regular

a. $\{0^n 1^m 0^n \mid m, n \geq 0\}$

Proof by contradiction: Assume the Language is regular
Let p (the pumping length) > 0

$q \geq 0$ and $w = 0^p 1^q 0^p$
then $|w| = 2p + q > p$. using the fact that
$|xy| \leq p$, it follows that $xy, (y)$ have only 0
Choose $i = 2$, then $xyyz \in L$ (More 0 in the left side of the string)
thus we have a contradiction

this conditions apply to s

a) $|y| > 0$

b) $|xy| \leq p$, and

c) $\forall i \geq 0,\ xy^iz \in L$

Choose $s = 0^p 1 1 0^p 1$

$s \in L$ with $w = 0^p 1$, and $t = 1$, and $|s| \geq p$

By condition b) xy is composed only of 0

By a), b) $y = 0^k$ for $k > 0$

Taking $i = 2$, then $xy^2z \in L$

$xyyz = 0^{(p+k)} 1 1 0^p 1$. This string can not be divided into wtw $\Rightarrow$ $xy^2z \notin L$

This is a contradiction. Then L is not a regular language.

Problem 1.47 page 80

Let $\Sigma = \{1, \#\}$ and let

$Y = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k$ for $k \geq 0$

each $x_i \in 1^*$, and $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$.

Prove that Y is not regular.

Let p be the pumping length

Let $s = 1^p \# 1^{p+1} \# \ldots \# 1^{2p}$

We can write $s = s_0 \# s_1 \# \ldots \# s_k$ where $k = p$

and $s_i = 1^{p+i}$

Let $s = xyz$ such that $|y| > 0$ and $|xy| \leq p$

Let $v = xy^2z$ for $(i = 2)$

We can write $v = v_0 \# v_1 \# \ldots \# v_k$

PSF

b) $\{0^m 1^n \mid m \neq n\}$

Consider $\bar{L} \cap 0^*1^* = \{0^k 1^k \mid k \geq 0\}$
We already proved that $0^k 1^k$ is not a regular language then L must be a non regular language also.

c) $L = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$

Proof by contradiction : Assume $\bar{L} = \{w \mid w \in \{0,1\}^*$ is a palindrome$\}$.
Let p be the pumping length
Let $s \in \bar{L}$ be $0^p 1^{p+1} 0^p$ which we can divide into xy^iz for $i \geq 0$.
We know that $|xy| \leq p$
Let $x = 0^g \quad 0 \leq g \leq p$
$y = 0^{g'} \quad 0 \leq g' \leq p$
$z = 0^{p-(g+g')} 1^{p+1} 0^p$

Let $i = 1 \Rightarrow xy^iz = 0^g 0^{g'} 0^{p-(g+g')} 1^{p+1} 1^p$ and

For $i = 2 \Rightarrow xyyz = 0^{g'} 0^p 1^{p+1} 0^p$

$\Rightarrow \bar{L}$ is not a regular language $\Rightarrow L$ is not a regular language.

d) $\{wtw \mid w, t \in \{0,1\}^+\}$

Proof by contradiction Assume L is regular language then $\exists p$ the pumping length for L such that any string $s \in L$ where ~~$s \in L$~~ $|s| \geq p$,
Let $s = xyz$

where $k = p$

$v_0 = 1^{p+|y|}$

and for $1 \le i \le p$, $v_i = 1^{p+i}$

Because $1 \le |y| \le p$ we conclude

$\Rightarrow p+1 \le (p+|y|) < 2p$ and

$v_0 = v_p + |y|$, thus, $v \notin A$

the language does not satisfy the condition of pumping Lemma. therefore the language is not regular.

Problems 1.55 (e, f, i, and j) page 81

e) $p = 2$. the minimum string is 01 which has length of 2. If we have xy^iz where $x = \varepsilon$, $y = 01$, z is the rest we can repeat 01 and pump 01 many times.

f) $p = 1$ since 1 is the minimum value that p can have $p \neq 0$

i) $p = 5$ since 5 is the minimum number of transition states for language 1011 in a DFA

j) $p = 1$ since 1 is the minimum number of states that are required for a language with minimum of 1 of its alphabet.