CMPS 130

Computational Models
Homework Assignment 1
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PS1

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- Exercises from pages 25, 26 and 27 of book: 0.1 – 0.9

0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

a. $\{1, 3, 5, 7, \ldots\}$ => this is the set of Odd Natural Numbers

b. $\{\ldots, -4, -2, 0, 2, 4 \ldots\}$ => This is the set of even Integers Numbers

c. $\{n \mid n = 2m$ for some m in $N\}$

=> this is the Set of Even Natural Numbers

d. $\{n \mid n = 2m$ for some m in N, and $n = 3K$ for some K in $N\}$

=> This is the set of natural numbers divisible by both 3 and 2.

e. $\{w \mid w$ is a string of 0s and 1s and w equals the reverse of $w\}$.

=> this is the set of strings formed by 0s and 1s that can be defined as palindrome. Can be read the same from beginning to end or from end to beginning

Ex 01110, 0110,

f. $\{n \mid n$ is an integer and $n = n+1\}$

=> this is the set of all Integers, $\mathbb{Z}$.

0.2 Write formal descriptions of the following sets.

a. The set containing the numbers 1, 10, 100

$\boxed{\{n \mid n = 10^m \text{ for } m \in \{0,1,2\}\}}$

b. the set containing all integers that are greater than 5

$\boxed{\{n \mid n > 5 \text{ for } n \in \mathbb{Z}\}}$

c. the set containing all natural numbers that are less than 5

$\boxed{\{n \mid n < 5 \text{ for } n \in \mathbb{N}}$

d. the set containing the string aba

$\boxed{\{ab\}}$

e. the set containing the empty string

$\boxed{\{\} \text{ or } \varepsilon}$

f. the set containing nothing at all

$\boxed{\emptyset}$

Q.3 Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

a. Is A a subset of B?

$\boxed{\text{No, } A \text{ is Not a subset of } B.}$

b. Is B a subset of A?

$\boxed{\text{Yes, } B \text{ is a subset of } A}$

c. What is $A \cup B$?

$\boxed{A \cup B = \{x, y, z\}}$

d. What is $A \cap B$?

$\boxed{A \cap B = \{x, y\}}$

e. What is $A \times B$?

$\boxed{A \times B = \{(x,x), (x,y), (y,x), (y,y), (z,x), (z,y)\}}$

f. What is the power set of B?

$\boxed{P(B) = \{\emptyset, \{x\}, \{y\}, \{x,y\}\}}$

PS4

Q.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

For each element in A there will be b ordered pairs. A has a elements $\Rightarrow$ $A \times B = a \times b = |A| \times |B|$.

Q.5 If C is a set with c elements, how many elements are in the power set of C. Explain your answer?

The number of elements in a power set S is $|P(S)| = 2^n$. (S is the set, n is the number of elements in the set)

To extend to our example $|P(C)| = 2^c$

0.6 Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f: X \to Y$ and the binary function $g: X \times Y \to Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. what is the value of $f(2)$?

$\boxed{f(2) = 7}$

b. what are the range and domain of f?

the Range $= \{6, 7\}$

the Domain $= \{1, 2, 3, 4, 5\}$

c. what is the value of $g(2, 10)$?

$g(2, 10) = 6$

d. what are the range and domain of g?

the Range $= \{6, 7, 8, 9, 10\}$

the Domain $= \{(1,6), (1,7), (1,8), (1,9), (1,10),$
$(2,6), (2,7), (2,8), (2,9), (2,10),$
$(3,6), (3,7), (3,8), (3,9), (3,10),$
$(4,6), (4,7), (4,8), (4,9), (4,10),$
$(5,6), (5,7), (5,8), (5,9), (5,10)\}$

PS6

e. what is the value of $f(4, f(4))$?

$f(4) = 7$

$f(4,7) = 8$

0.7. For each part, give a relation that satisfies the condition:

a. Reflexive and symmetric, but not transitive

$S = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$

b. Reflexive and Transitive, but not symmetric

$S = \{1, 2, 3\}$

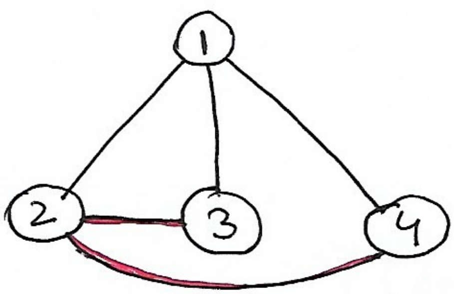
$R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$

c. Symmetric and Transitive, but not Reflexive

$S = \{1, 2, 3\}$

$R = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$

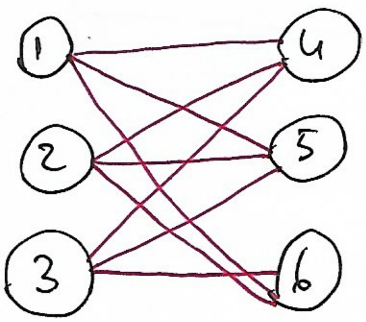
Q.8 Consider the undirected graph $G = (V, E)$ where V, the set of nodes, is $\{1, 2, 3, 4\}$ and E, the set of edges, is $\{\{1,2\}, \{2,3\}, \{1,3\}, \{2,4\}, \{1,4\}\}$. Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.



Degree of Node
- 1 = 3
- 2 = 3
- 3 = 2
- 4 = 2

Path from 3 to 4 is 3 – 2 – 4.
(shortest Path) (or 3 – 1 – 4

Q.9 Write a formal description of the following graph.



$(\{1,2,3,4,5,6\},$
$\{(1,4),(1,5),(1,6),$
$(2,4),(2,5),(2,6),$
$(3,4),(3,5),(3,6)\})$

0.16 Find the error in the following proof that $2=1$. Consider the equation $a=b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$ to get $a+b=b$. Finally, let a and b equal 1, which shows that $2=1$.

$a = b$ $(\times a)$

$a^2 = ab$ $(-b^2)$

$a^2 - b^2 = ab - b^2$ (factor)

$(a+b)(a-b) = (a-b)b$ (divide by $(a-b)$)

$a+b = b$ (replace $b=1$, $a=1$)

$2 = 1$

$\boxed{\text{Since } a=1 \text{ and } b=1 \text{, when we divide by } (a-b) \text{, we divide by } 0. \text{ Division by } 0 \text{ is not allowed. Therefore our proof is false}}$

PSS

0.12 Find the error in the following proof that all horses are the same color.

CLAIM : In any set of h horses, all horses are the same color

PROOF : By induction on h

BASIS : For $h = 1$. In any set containing just one horse, all horses, clearly are the same color.

Induction step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k+1$.

Take any set H of $k+1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore, all the horses in H must be the same color, and the proof is complete.

We can not assume that H_1 and H_2 are the same color. When we remove a horse and replace it with another horse, we can not prove that the two horses have the same color.

PS10

0.13 Show that every graph with two or more nodes contains two nodes that have equal degree

Proof by Contradiction
Assume that every Graph with two or more nodes have unique degrees.
We know that the set of possible degrees for a graph with n vertices is $= \{0, 1, \dots n-1\}$

$\Rightarrow$ we have n unique degrees to assign to our n vertices

$\Rightarrow$ We must assign the degree 0 to one of our node this ~~vertices~~ vertex is not conected to other vertex.

$\Rightarrow$ then we assign the degree $n-1$. this vertex is conected to every other vertices in the graph.

this is a contradiction, because it is impossible to have a node conected to all nodes and other node that is not connected to ~~other~~ any nodes.

therefore, our assumption is false, and there are at least 2 vertices, (nodes) with the same degree in any graph with at least 2 vertices.

Prove from the definitions of set union, intersection, complement and equality, that
$(\overline{A \cap B}) = (\bar{A} \cup \bar{B})$

This theorem states that 2 sets $\overline{A \cap B}$ and $\bar{A} \cup \bar{B}$ are equal.
We prove by showing that every element of one, also is an element of the other, and vice versa.

a) $x \in \overline{A \cap B}$
$x \notin A \cap B$
$x \notin A$ or $x \notin B$
$x \in \bar{A}$ or $x \in \bar{B}$
$\boxed{x \in (\bar{A} \cup \bar{B})}$

b) $x \in \bar{A} \cup \bar{B}$
$x \in \bar{A}$ or $x \in \bar{B}$
$x \notin A$ and $x \notin B$
$x \notin A \cap B$
$\boxed{x \in \overline{A \cap B}}$

$\Rightarrow \boxed{\overline{A \cap B} = \bar{A} \cup \bar{B}}$ from a and b

Show that the set of odd numbers is countable.

We need to find a bijection from the natural numbers to the set of odd numbers.

Define $f : N \rightarrow \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$

by $f(n) = \begin{cases} n & , \text{ if } n \text{ is odd} \\ 1-n & , \text{ if } n \text{ is even} \end{cases}$

for all $n \in N$.

- Prove by induction on n

For all $n \in \mathbb{N}$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Proof by induction

Base case $n=1$, we have

$$\sum_{i=1}^{1} i^2 = \frac{n(n+1)(2n+1)}{6} \Rightarrow 1^2 = \frac{1(2)(3)}{6} \Rightarrow 1 = 1$$
$\Rightarrow$ True

Assume that for $n = K$, $K \geq 1$:

$$\sum_{i=1}^{K} i^2 = \frac{K(K+1)(2K+1)}{6}$$ Induction Hypothesis:

We must show that $\displaystyle\sum_{i=1}^{K+1} i^2 = \boxed{\frac{(K+1)(K+2)(2K+1)}{6}}$ RHS

$$\sum_{i=1}^{K+1} i^2 = \sum_{i=1}^{K} i^2 + (K+1)^2$$

$$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$$ By inductive Hypothesis

$$= \frac{1}{6} K(K+1)(2K+1) + (K+1)^2$$

$$= (K+1)\left(\frac{1}{6} K(2K+1) + (K+1)\right)$$

$$= \frac{1}{6}(K+1)(2K^2 + 7K + 6)$$

$$= \boxed{\frac{1}{6}(K+1)(K+2)(2K+1)}$$

Since the statement is true for $n=1$, and $n=K$ implied that the statement is true for $n=K+1$.

By mathematical induction the statement is true.