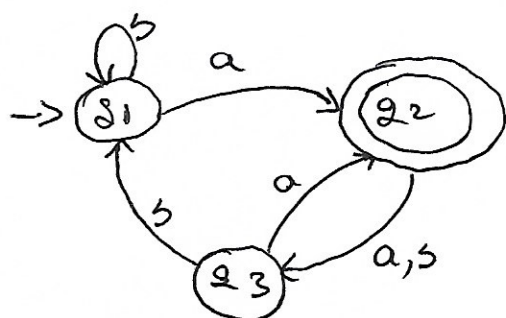
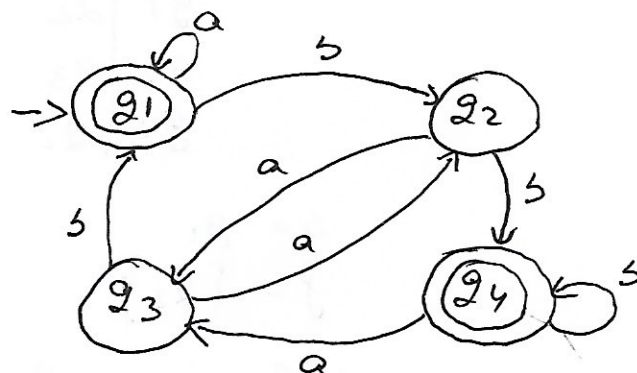


Exercise 1.1 page 83

The following are the state diagrams of two DFAs, M_1 and M_2 . Answer the following questions about each of these machines.



M_1



M_2

a. What is the start state?

M_1 the start state is q_1

M_2 the start state is q_1

b. What is the set of accept states?

For M_1 the set of accept state is $\{q_2\}$

For M_2 the set of accept state is $\{q_1, q_4\}$

c. What sequence of states does the machine go through on input $aabbb$?

For $M_1 \Rightarrow q_1, q_2, q_3, q_1, q_1,$

For $M_2 \Rightarrow q_1, q_1, q_1, q_2, q_4$

d. Does the machine accept the string $aabbb$?

M_1 Doesn't accept the string $aabbb$.

M_2 accept the string $aabbb$.

e. Does the machine accept the ϵ ?

M_1 Doesn't accept the ϵ .

M_2 accept the ϵ .

Exercise 1.2 page 83

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Give the formal description of the machines M_1 and M_2 pictured in Exercise 1.1.

For M_1

$$Q = \{q_1, q_2, q_3\}$$
$$\Sigma = \{a, b\}$$

$$\delta =$$

	a	b
q_1	q_2	q_1
q_2	q_3	q_3
q_3	q_2	q_1

$$s = q_1$$
$$f = \{q_2\}$$

$$\Rightarrow M_1 = (\{q_1, q_2, q_3\}, \{a, b\}, \delta, q_1, \{q_2\})$$

For M_2

$$Q = \{q_1, q_2, q_3, q_4\}$$
$$\Sigma = \{a, b\}$$

$$\delta$$

	a	b
q_1	q_1	q_2
q_2	q_3	q_4
q_3	q_2	q_1
q_4	q_3	q_4

$$s = q_1$$
$$f = \{q_1, q_4\}$$

$$\text{For } M_2 = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_1, q_4\})$$

Exercise 1.3. page 83

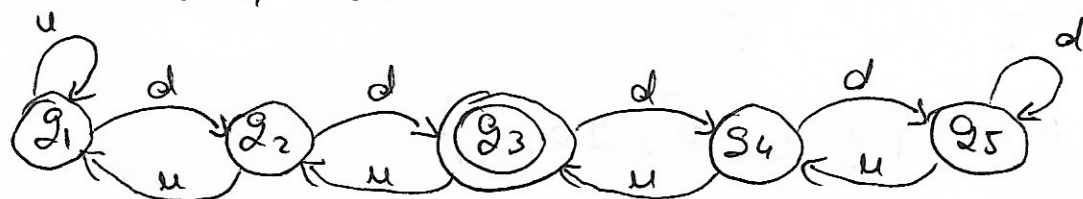
PS 3

The formal description of a DFA is

$M(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_1, \{q_3\})$,

where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

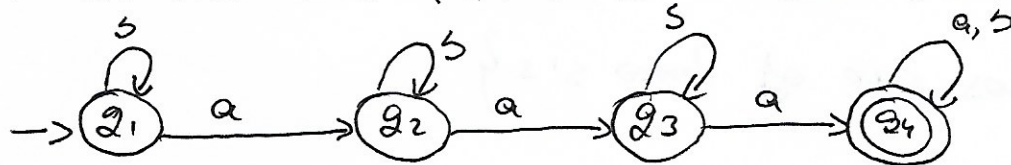


Exercise 1.4 (a, c, f, g, only; use the product construction) page 83

Each of the following language is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3, to give the state diagram of the DFA for the language given. In all parts $\Sigma = \{a, b\}$.

(a) $\{w, w \text{ has at least three } a\text{'s and at least two } b\text{'s}\}$

M_1 - at least 3 a $\{w \mid w \text{ has at least 3 a's}\}$

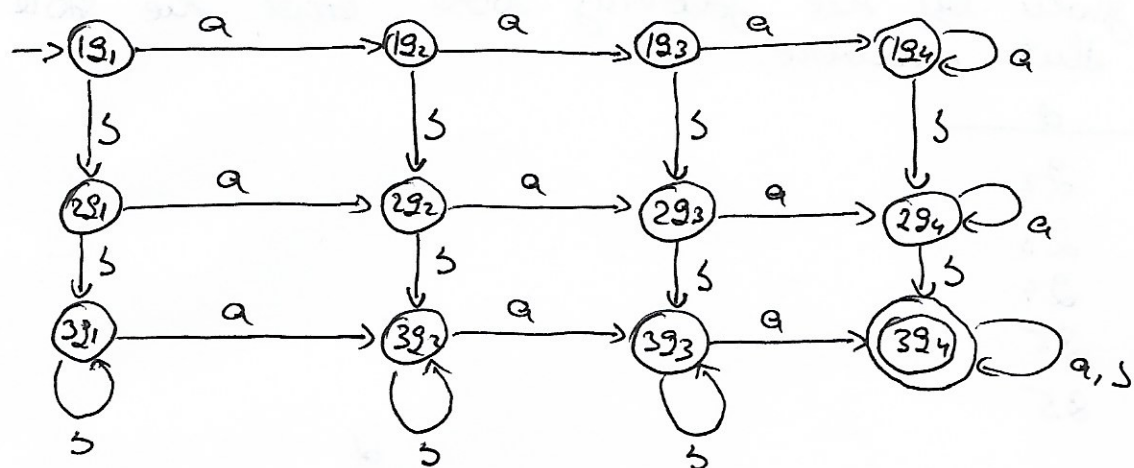


M_2 - $\{w \mid w \text{ has at least 2 b's}\}$



$M_3 \rightarrow \{w \mid w \text{ has at least 3 a's and at least 2 b's}\}$

(P84)

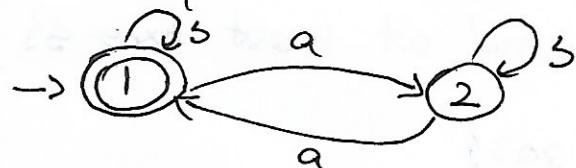


Description of $M_3 = (Q_3, \Sigma, \delta, q_0, F_3)$

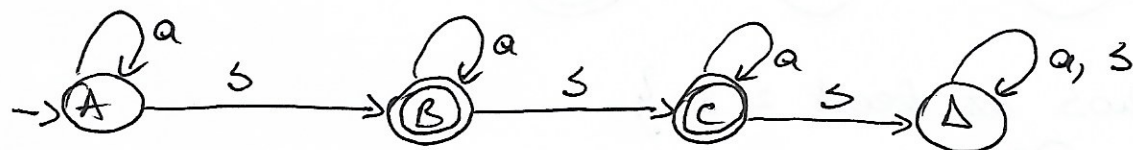
- ① $Q_3 = Q_1 \times Q_2$
- ② Σ is the same as M_1 and M_2
- ③ $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$
- ④ q_0 is the pair $(1, 2_1)$
- ⑤ $F_3 = F_1 \times F_2$

© $\{w \mid w \text{ has an even number of a's and one or two b's}\}$

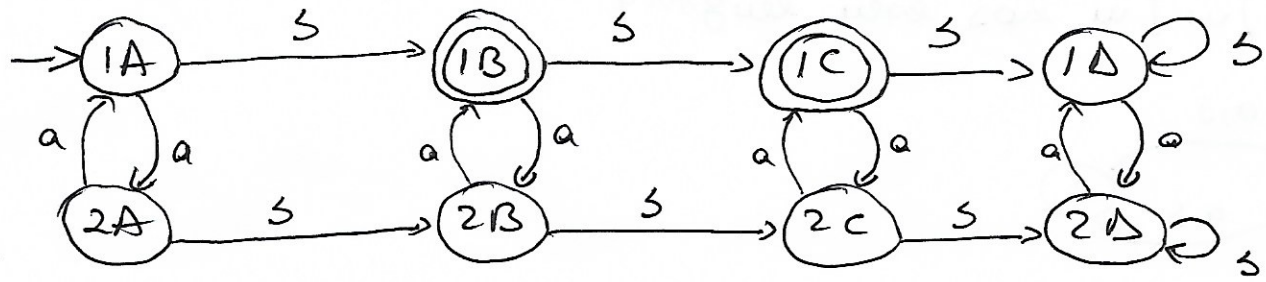
$M_1 \rightarrow \{w \mid w \text{ has an even number of a's}\}$



$M_2 \rightarrow \{w \mid w \text{ has one of two b's}\}$

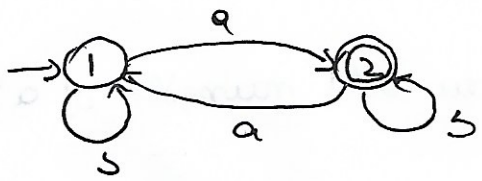


$M_3 \rightarrow \{w \mid w \text{ has an even number of } a \text{ and } 1 \text{ or } 2 \text{ } b\text{'s}\}$

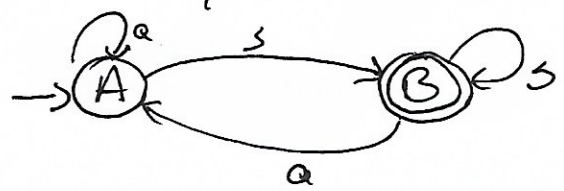


⑧ $\{w \mid w \text{ has an odd number of } a\text{'s and ends with a } b\}$

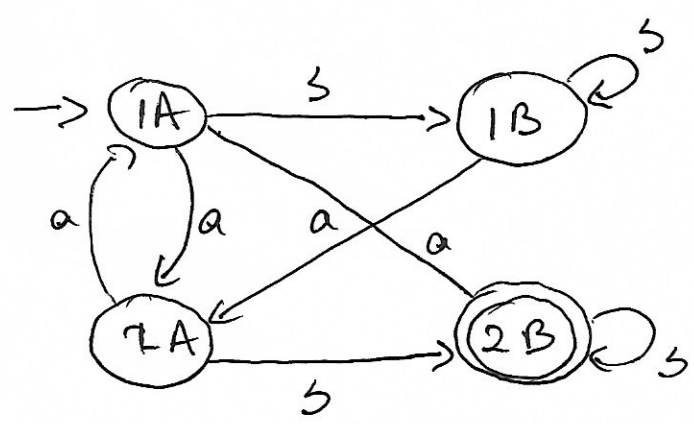
$M_1 \rightarrow \{w \mid w \text{ has an odd number of } a\text{'s}\}$



$M_2 \rightarrow \{w \mid w \text{ ends with a } b\}$

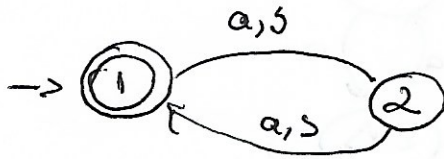


$M_3 \rightarrow \{w \mid w \text{ has an odd number of } a\text{'s and ends in } b\}$

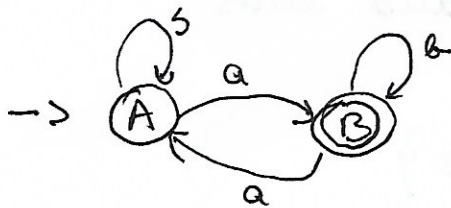


⑨ $\{w \mid w \text{ has even length and an odd number of } a\text{'s}\}$

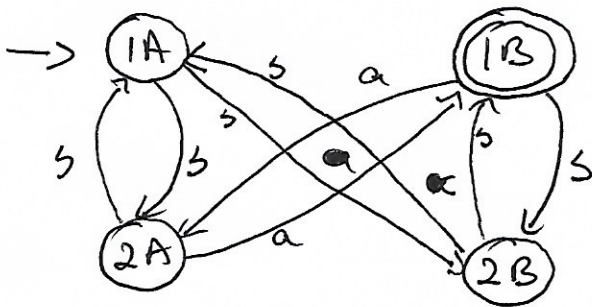
$M_1 \rightarrow \{w \mid w \text{ has even length}\}$



$M_2 \rightarrow \{w \mid w \text{ has an odd number of } a\text{'s}\}$



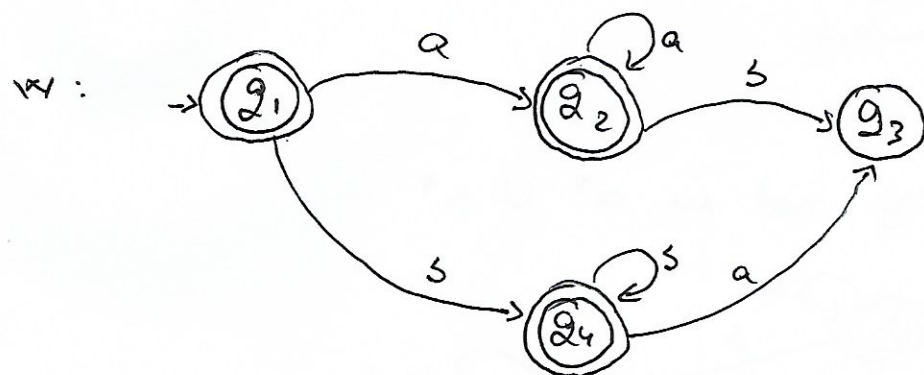
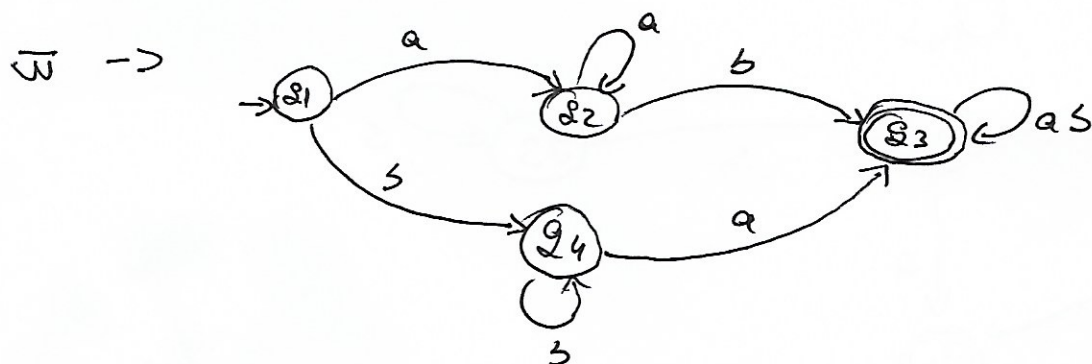
$M_3 \rightarrow \{w, w \text{ has even length and an odd number of } a\text{'s}\}$



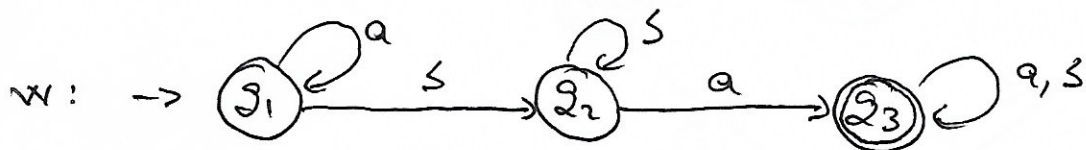
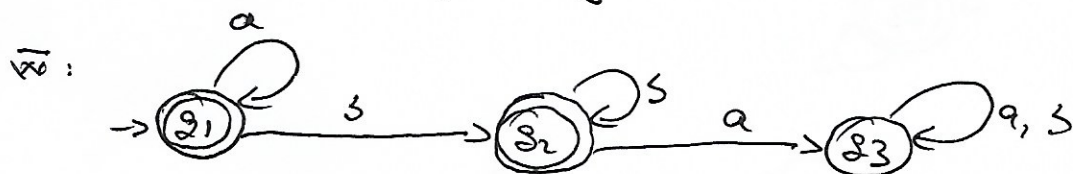
Exercise 1.5 (c, d, e, f, g, h) pg 84.

Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

(c) $\{w, w \mid \text{contains neither the substring } ab \text{ nor } ba\}$

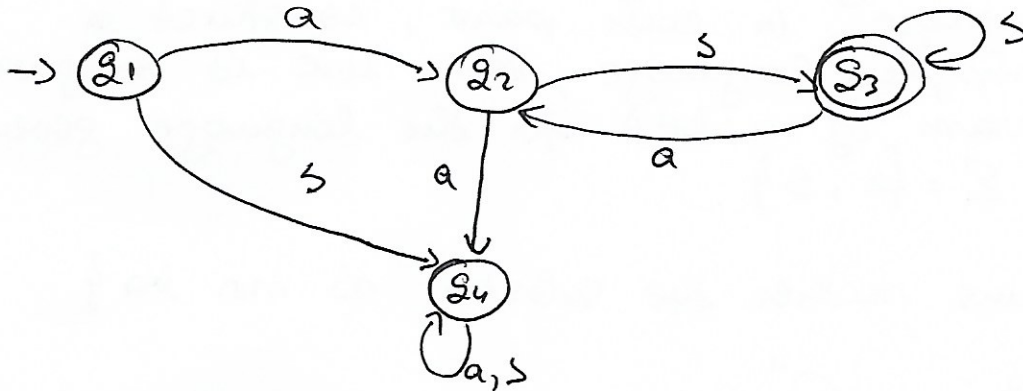


(d) $\{w, w \mid \text{is any string not in } a^*b^*\}$

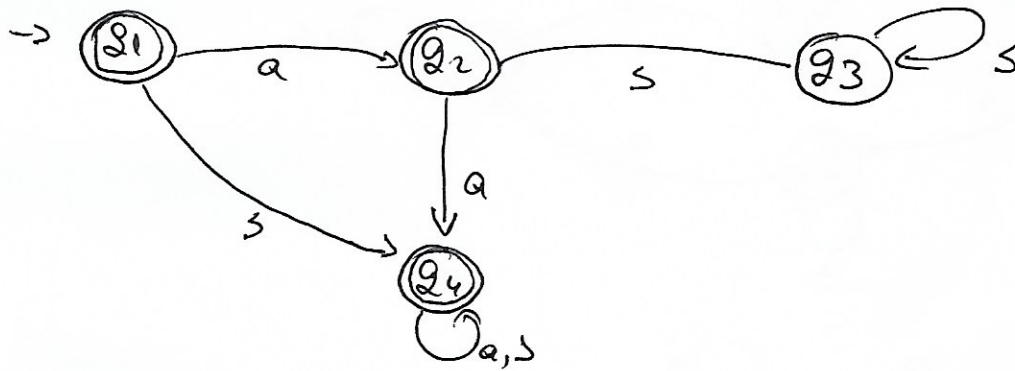


② $\{w \mid w \text{ is any string not in } (as^+)^*\}$

\bar{w} :

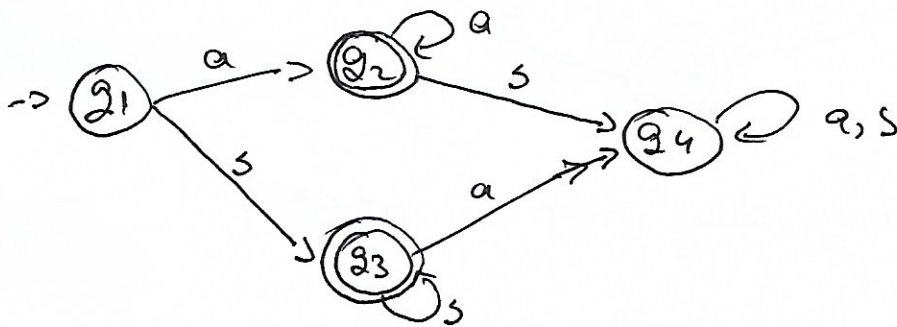


w :

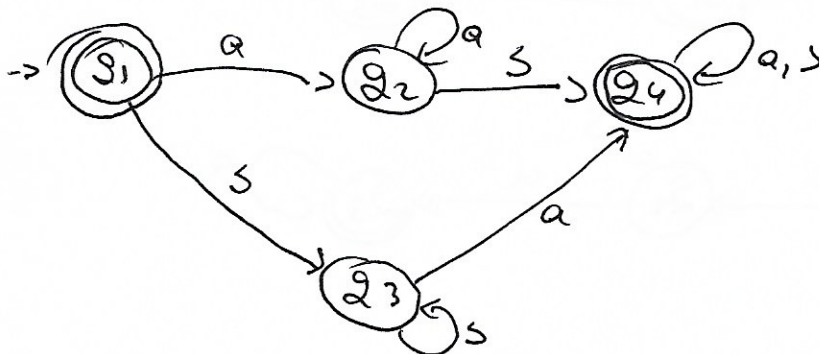


③ $\{w \mid w \text{ is any string not in } a^* \cup s^*\}$

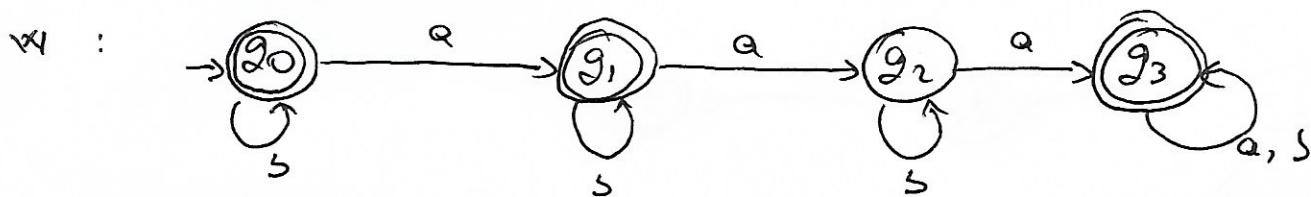
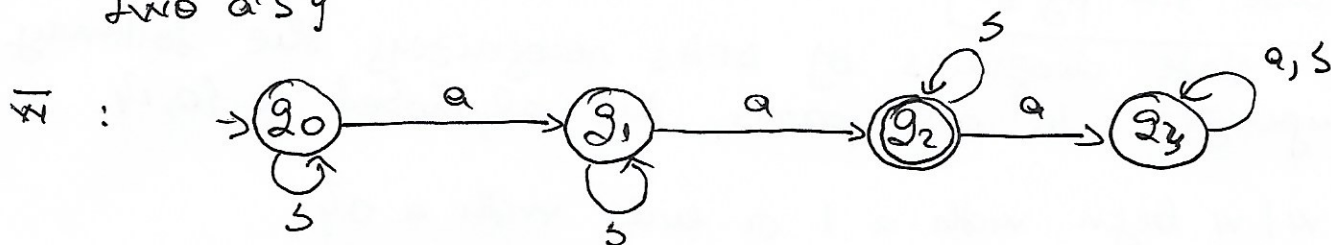
\bar{w} :



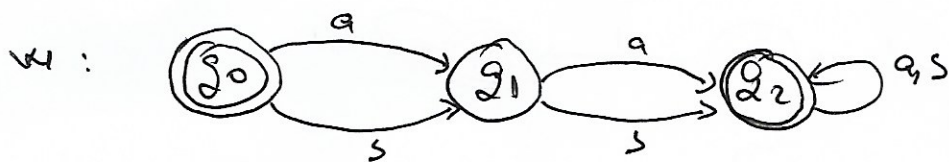
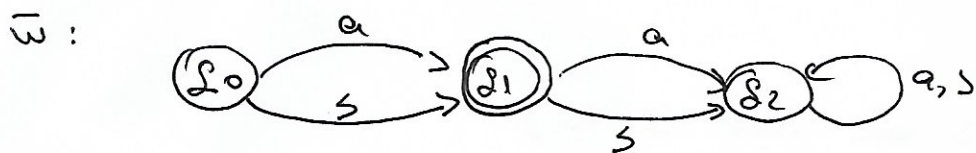
w :



g) $\{w \mid w \text{ is any string that doesn't contain exactly two a's}\}$



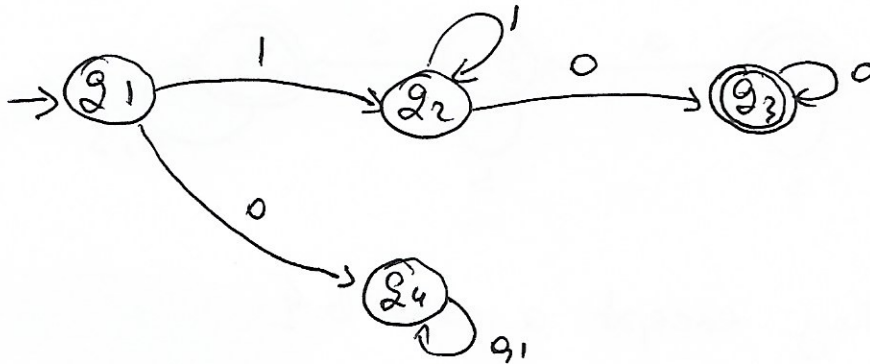
h) $\{w \mid w \text{ is any string except a and b}\}$



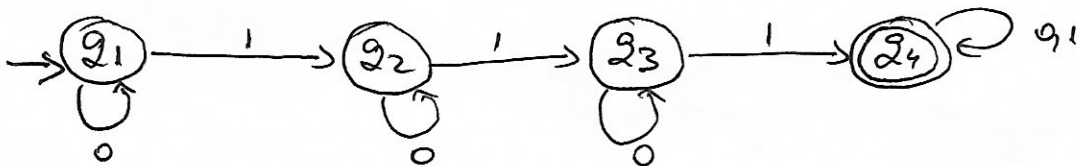
Exercise 1.6 pg 84

Give state diagrams of DFA's recognizing the following languages. In all parts, the alphabet is $\{0,1\}$.

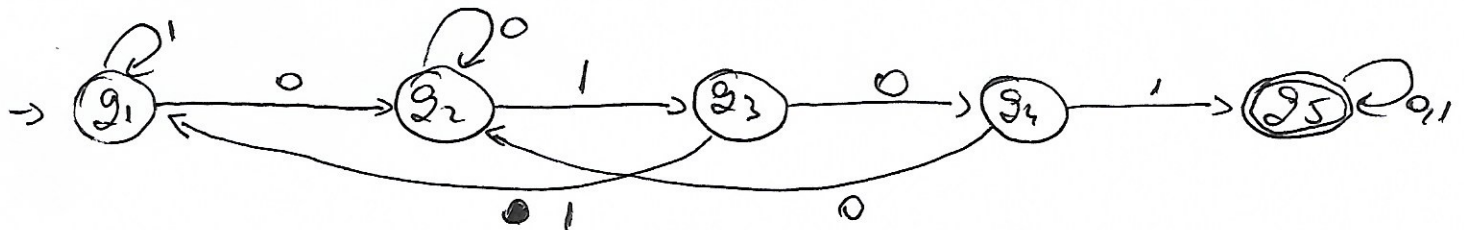
(a) $\{w \mid w \text{ begins with a 1 or ends with a 0}\}$



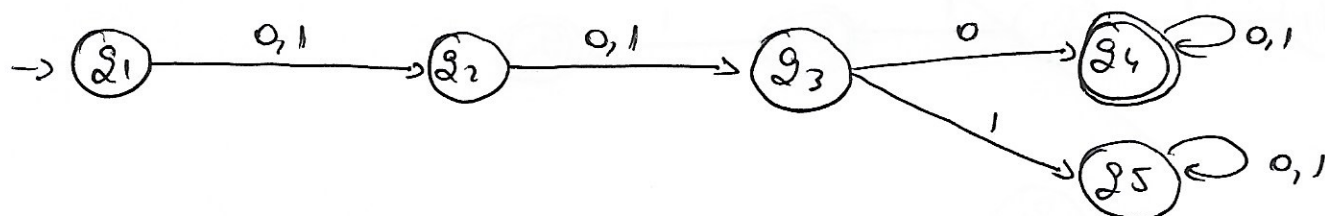
(b) $\{w \mid w \text{ contains at least 3 1's}\}$



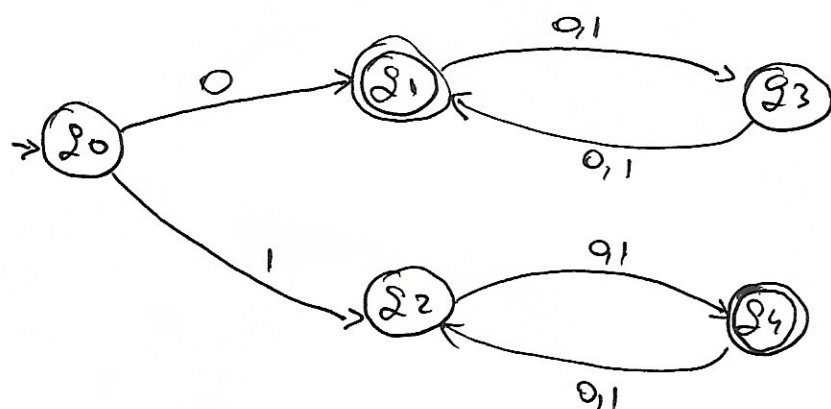
(c) $\{w \mid w \text{ contains the substring 0101}\}$



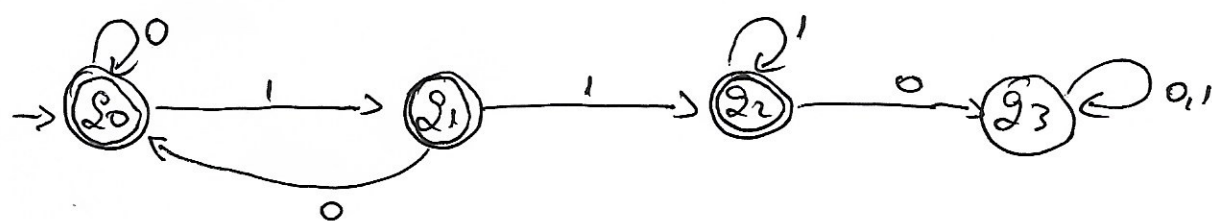
(d) $\{w \mid w \text{ has length at least 3 and its third symbol is a 0}\}$



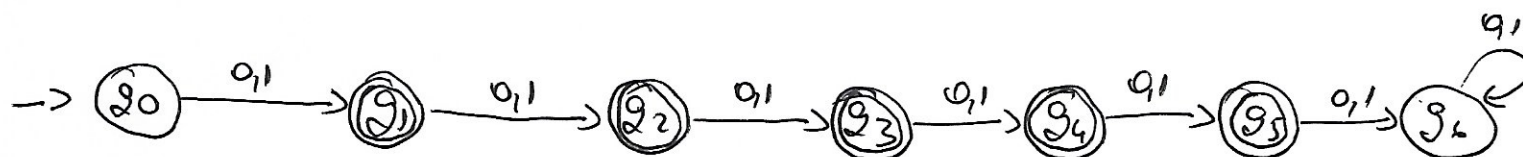
(e) $\{w \mid w \text{ starts with a 0 and has odd length, or starts with a 1 and has even length}\}$



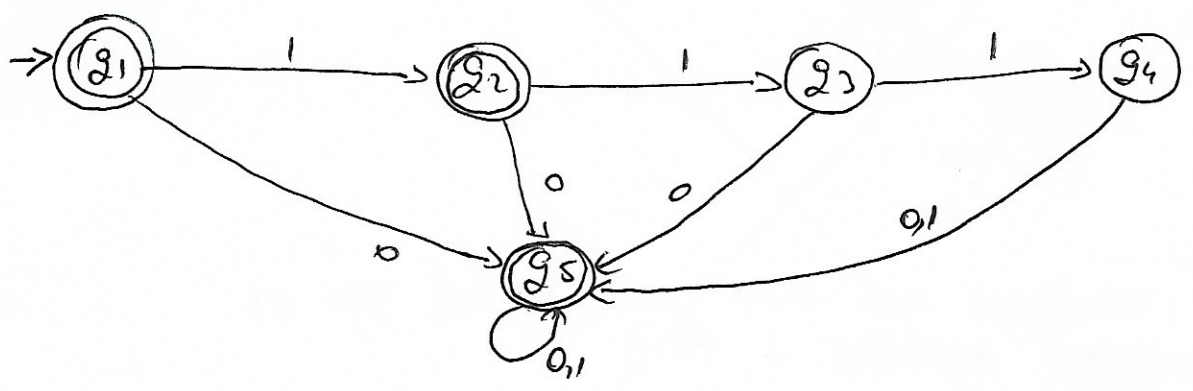
(f) $\{w \mid w \text{ doesn't contain the substring } 110\}$



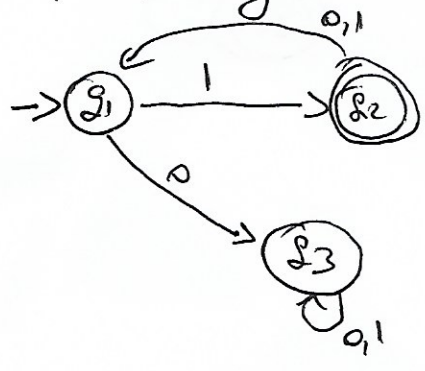
(g) $\{w \mid \text{the length of } w \text{ is at most 5}\}$



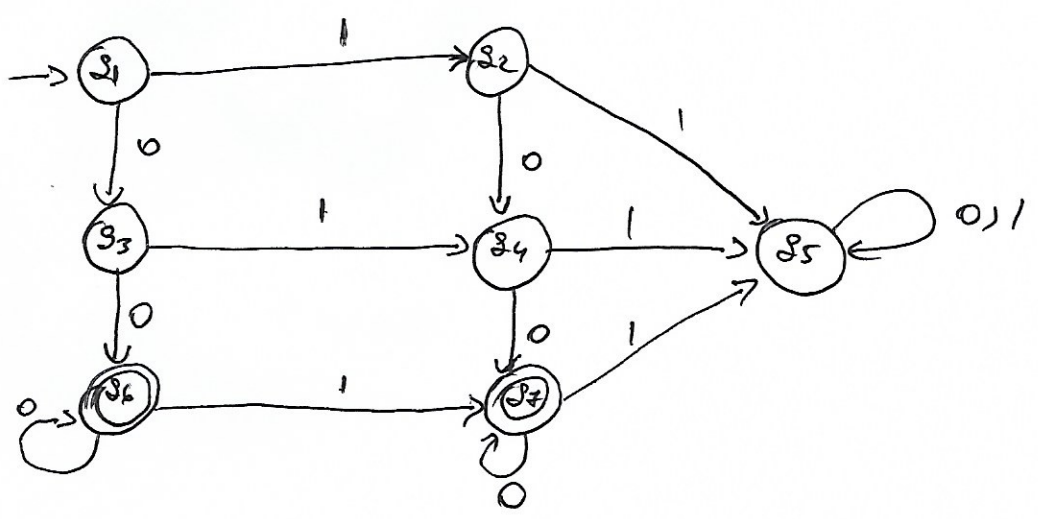
h) $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$



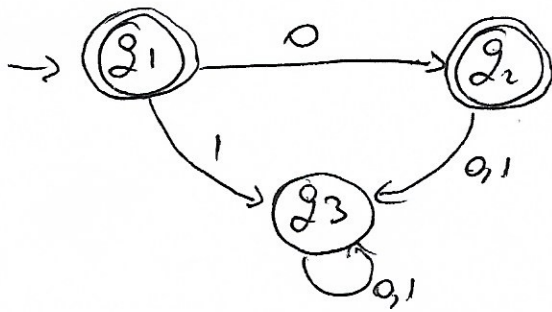
i) $\{w \mid \text{every odd position of } w \text{ is a } 1\}$



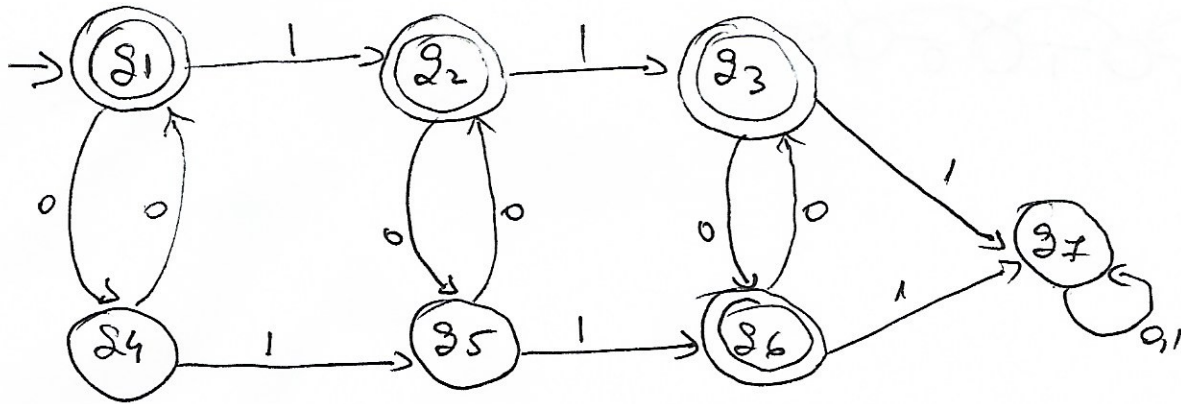
g) $\{w \mid w \text{ contains at least 2 0's and at most 1 1's.}\}$



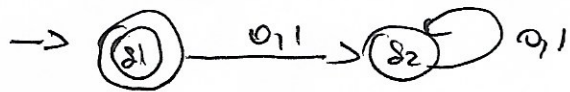
(k) $d \in \{0, 1\}$



(l) $\{w \mid w \text{ contains an even number of 0's, or contains exactly 2 1's}\}$



(m) the empty set



(n) All strings except the empty set

