Q Multiply two numbers without * but you can recursion

$n \geq 0$

a, n

$a = 5$
$n = 7$

$a * \boxed{n} = a + a * (n-1)$

Rec. $\boxed{f(a, n) = a + f(a, n-1)}$

5×7
$= 35$

$5 + 5 \times 6$
$= 5 + 30$
$= 35$

Base case $\boxed{f(a, 0) = 0}$

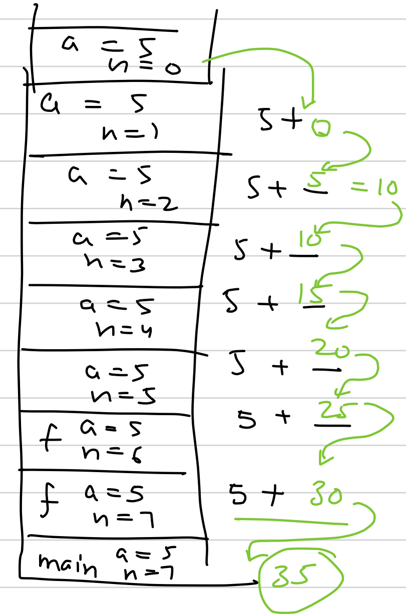
```
static int multiply(int a,int n){
  → if(n==0){        a==0   || n==0
        return 0;
    }
  → return a + "multiply(a,n-1)";
}
```

$$f(a,n) = a + f(a,n-1);$$

$\boxed{5\times 7}$

$5 \times 7 = 5 + \underbrace{5+5+5+5+5+5}_{\text{multiply}(5,6)}$

add 5 seven times



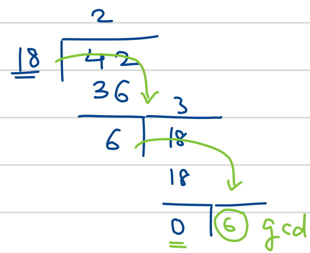
MATHS

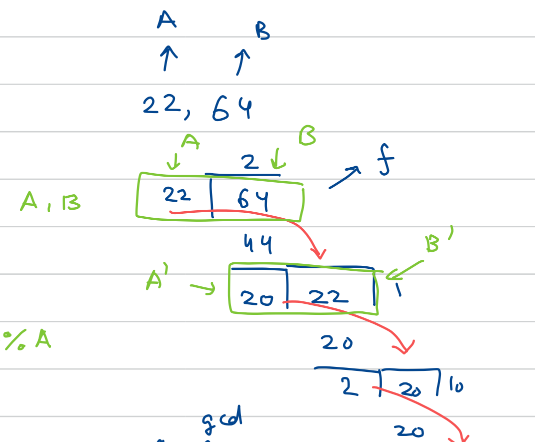
- **GCD** – greatest common divisors

$A = 18$

$B = 42$ $\qquad gcd(42, 18) = 6$

Pen & paper





$$f(A, B) = f(A', B')$$
$$= f(\underline{B \% A}, A)$$

$A' = B \% A$

$B' = A$

$gcd(22, 64) = gcd(20, 22) = gcd(2, 20) = gcd(\underline{0, 2})$

gcd

spl case

Rec Case

gcd (A, B) = gcd (B % A , A)

Base Case   when A == 0 return B

Euclid's Algorithm

Loop →

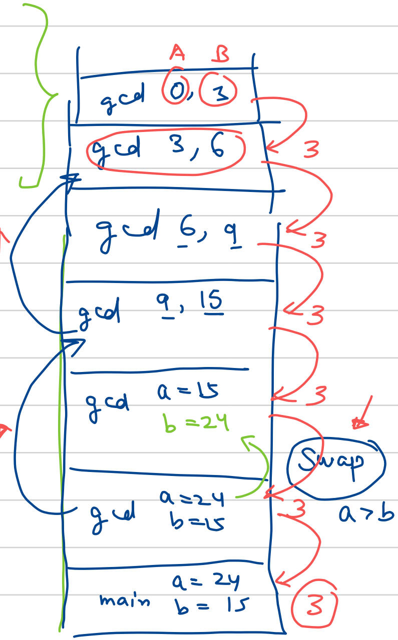
```java
static int gcd(int a,int b){
    if(a==0){
        return b;
    }
    return gcd(b%a,a);
}
```

A' = B % A
B' = A

a = 24
b = 15

15 % 24

Max Calls



N
24, 15
15, 24
9, 5
6, 9
3, 6
0, 3
$\boxed{\text{Log N}}$

a → a/2 → a/4 ...

N steps

No of Steps = No of fn calls

$A = 24$ $\quad$ $A' = B \% A = 15 \% 24 = 15$

$B = 15$ $\quad$ $B' = A = 24$

$\log_2 1000 = 10$

$\log_2 (1000)^2 = 20$

gcd $\quad$ a $\quad$ b

1 $\quad$ ✓ $\quad$ ✓

2 $\quad$ ✓ $\quad$ ✓

⋮ $\quad$ ✓ $\quad$ ✓

⋮

min(A, B)

N ↑

~~to~~

$A = 10^6$

Stack overflow

A $\quad$ Numbers / Calls

$\log_2 A$ $\quad$ Calls

$\approx$ 20 calls

Logarithm

↳ mathematical fn

$\log_A N = val$ $\qquad\qquad$ $\boxed{A^{val} = N}$

$\log_{10} 1000 = ?$ $\qquad\qquad$ $10^{?} = 1000$

$10^3 = 1000$

$\boxed{\log_{2} 128} = 7$ $\qquad\qquad$ $2^7 = 128$

$\Rightarrow \log_2 N^3 = 3 \log_2 N$

$\log_2 1024 = 10$

$\log_2 1000 = 9.xx \simeq 10$

$\log_2 10^{18} = \log_2 (1000)^6$

$= 6 \log_2 1000$

$= 60$

10^6

$\log_2 1000000$

$= \log_2 (1000)^2$

$= 2 \log_2 1000$

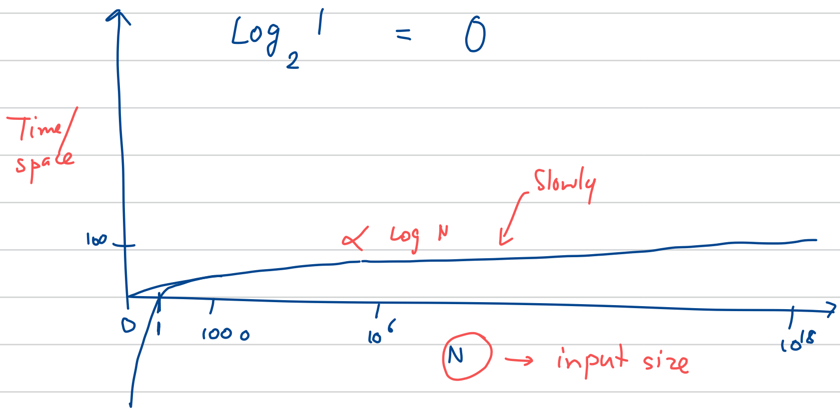
$\simeq 20$

Steps

$1 \rightarrow 0$

$10^{18} \rightarrow 60$

$\log_2 1 = 0$



$N \rightarrow$ input size

Time Funda

Online Judge $\quad 10^8$ instructions in 1 s

$1s \longrightarrow 10^8$ steps

$N = 10^9$, $\boxed{N = 10^{18}}$

for (i=1 to i = N)
gcd = ____ if a & b are div

$\log_2 N$ steps

Euclids $\quad a \rightarrow N$

$a/2 \rightarrow a/4 \rightarrow a/8 \rightarrow \cdots \rightarrow 1$

$10^8 \longrightarrow 1s$

$1 \text{ step} \rightarrow \frac{1}{10^8} s$

$10^{18} \text{ steps} \rightarrow \frac{10^{18}}{10^8} = 10^{10}$

10^8 steps $\rightarrow$ 1s

10^9 steps $\rightarrow$ 10 second

10^{18} steps $\rightarrow$ $\boxed{10^{10} \text{ seconds}}$

Several years

60 steps $\rightarrow \frac{60}{10^8}$

$= \boxed{6 \times 10^{-7} \text{ seconds}}$

Speed up $\Rightarrow \frac{10^{10}}{6 \times 10^{-7}} = \boxed{10^{16} \text{ approx}}$

N

1028

512

256

128

64

32

16

8 → 4 → 2 → 1 → 0

$\log_2 1028$ ~~+ 1~~

$= \log_2 N$ steps

Euclid's

gcd

100

50

25

12

6

3

2

1

0

10^8

$\log_2 N$

7-8 steps

long

$$\log_2 10^{18} = \log_2 (1000)^6$$

$$= 6 \boxed{\log_2 1000}$$

$$= 60 \text{ steps}$$

1 steps $= 10^{-8}$ s

60 steps $= 6 \times 10^{-7}$ seconds

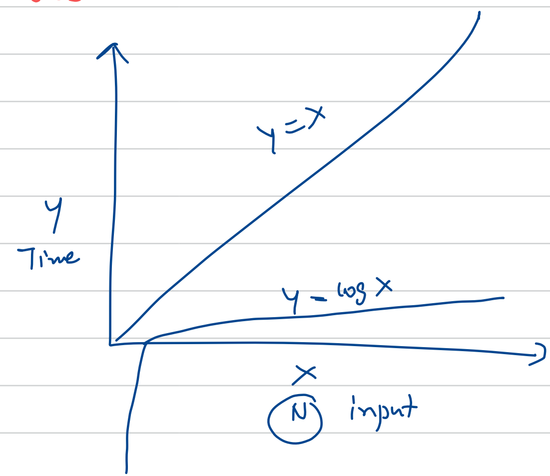
Other:

10^{18}

$A = 20$, $B = 48$ gcd

$i = 1$

$i = \min(20, 48)$



$\log_2 N$ = what power of base(2) should we keep s.t

$$2^{\boxed{\text{power}}} = N$$

Fact

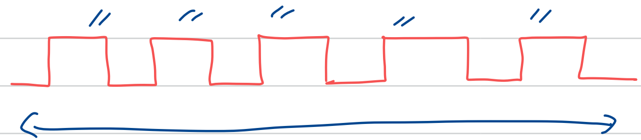
$1s \rightarrow 10^8$ steps

1 step $\rightarrow 10^{-8}$ seconds

$\Rightarrow$ Acc of most processors

GHz

clock signal



10^{10}

$c = a + b$

10^8 steps in 1s

2 -3

100

int c = a + b - d ;

~~3-4 steps~~ >10 steps

$1\ \text{GHz} \rightarrow 10^9$

10^8 steps $\rightarrow$ 1 s

$10^8 \times 10 \rightarrow 10$ s

1 step $\rightarrow \frac{1}{10^8}$

10^9 step $\rightarrow \frac{10^9}{10^8} \leftarrow 10$ s

2.4 GHz

5 GHz

In $\boxed{10^9}$

$\downarrow$

$\boxed{10^8 \text{ ins}}$

each ins $\rightarrow$ 10 steps

Load A $\longrightarrow$ Load B $\longrightarrow$ Addition $\longrightarrow$ C

$\boxed{c = a + b} \Leftarrow$

Processor

Load A

Prime No → $\sqrt{N}$ optimisation

GCD → Linear Approach $O(N)$

Euclid's Alg $O(\log N)$

GCD of N inputs

HW

$\boxed{40, 50}$, ~~35~~, 48, 72

10

2

2

for(________)

→ = gcd(prevgcd, next no)

LCM Least Common Multiple

A , B

10, 12

Basic Approach

24 No

36 No ~~A % B →~~

48 No multiple % A

60 Yes $\boxed{60 \% 10}$ ↓ 0

First

5 , 7
14
21
28
$\boxed{35}$

many steps

5, 7
co-prime

2, 9

Formula
Second

$$a * b = gcd * LCM$$

1

$$LCM = a * b$$

$$LCM = \left(\frac{a * b}{gcd}\right) = \frac{\cancel{10}^{5} \times 12}{\cancel{2}} = 60$$

optimised

Log N

10, 12

10, 12, 5, 8.

60

60

120

$$\frac{60 \times \cancel{8}^{2}}{\cancel{4}}$$

$\boxed{\text{Series}}$ A P , G P , Binomial Coefficients

Airthmetic Progression

Series → diff b/w 2 consecutive terms is constant.

$$\overset{3}{\leftrightarrow}\ \overset{3}{\leftrightarrow}\ \overset{3}{\leftrightarrow}\ \overset{3}{\leftrightarrow}$$

3 ← d

5 , 8 , 11 , 14 , 17 ....

↑ a ↑ N^{th}

$$\boxed{T_N = a + (n-1)d}$$

$N = 6$

$= 5 + (6-1)3$

$= 5 + 15$

$= 20$

+3 +3 +3 +3 +3

5, 8, 11, 14, 17, 20

1 2 3 4

$\frac{6}{2}(10 + 5 \cdot 3)$

$= 25 \times 3$

$= 75$

$5 + (5)3$

$5 + (6-1) \times 3$

$$\boxed{T_n = a + (n-1)d}$$

$$S_n = \boxed{\frac{n}{2}(2\underline{a} + (n-1)d)} = \frac{n}{2}(a + \underline{a + (n-1)d})$$

$$= \boxed{\frac{n}{2}(a + \underline{a_n})}$$

$\frac{\cancel{6}^{3}}{\cancel{2}}(5 + 20)$

$= 75$

6 Terms

$\xrightarrow{3}$

8, 11, ...

a

$$S_6 = \frac{6}{2}(2\cdot 8 + 5\cdot 3)$$

$= 3 \; (16+15)$

$= 93$

6 Terms

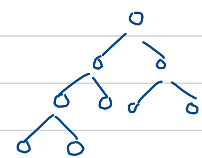
8, ... , 40

a_n

$\frac{n}{2}(48)$

$= \frac{48 \times \cancel{6}\,3}{\cancel{2}} = 144$

Geometric Progression



Every terms bears a constant ratio (r) to prev term

$p=0$

$a, \; ar^1, \; ar^2, \; ar^3 \; .... \; ar^{n-1}$

1 2 3

$= 3, 6, 12, 24, 48 \; .....$

Nth Terms

$\boxed{r=2}$

$\boxed{a=3}$

Finite GP (n is finite)

$\boxed{r = 1}$ $\quad a, a, a, a \ldots\ldots a$ $\quad$ n times

$$S_n = an$$

$\boxed{r \neq 1}$

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

1, 2, 4, 8, 16, 32, 64 $\quad \boxed{n = 7}$

$$= \frac{1(2^7 - 1)}{2 - 1} = \boxed{127}$$

infinite GP

$r = \frac{1}{2}$

$|r| < 1$ $\quad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots\ldots \infty$

Converge

$$= S_\infty = \frac{a}{1 - r} = \frac{1}{1 - 1/2} = 2$$

$r \geq 1 \Rightarrow \infty$

[Divergent]

P&C

Binomial Coefficient $^{N}C_{R}$

Combination

$\boxed{^{N}C_{R}}$ denotes the no of ways of choosing R objects out of N objects

B1, B2, B3, ..., B10,

$\boxed{2 \text{ Boys}}$ $^{10}C_{2}$

$$^{N}C_{R} = \frac{N!}{(N-R)!\,R!}$$

$$1 = \frac{10!}{8!\,2!} = \frac{\cancel{8!}\ 9 \times \cancel{10}\ 5}{\cancel{8!}\ 1 \times \cancel{2}} = 45 \text{ ways}$$

Numerator

$$^{\boxed{10}}C_{\boxed{2}} = \frac{\overset{2}{\cancel{10}} \times 9}{1 \times \cancel{2}} = 45$$

$$^{\boxed{6}}C_{\boxed{3}} = \frac{\cancel{6} \times 5 \times 4}{1 \cancel{\times 2 \times 3}} = \boxed{20}$$

$$^{N}C_{R} = {}^{N}C_{\boxed{N-R}}$$

Proof

$$\frac{N!}{(N-R)!\,R!} \qquad \frac{N!}{(N-(N-R))!\,(N-R)!}$$

Same

$$= \frac{N!}{R!\,(N-R)!}$$

Advantage

$$^{10}C_{8} = {}^{10}C_{10-8} \qquad r > \frac{N}{2}$$

$$= {}^{10}C_{2} = \frac{10 \times 9}{1 \times 2} = 45$$

$$^6C_5 = {}^6C_1 = \frac{6}{1} = 6$$

^NC_R in Combinations

↳ choose R objects out of N objects

→ 2 ppl for the post of Captain & Vice-Captain

Permutation : Choose + Arrange.

$^5C_2$

$= \frac{5 \times 4}{2} = 10$ ways

A, B, C, D, E

B A
C. VC

AB, AC, AD, AE

BC, BD, BE

CD, CE

DE

$4 + 3 + 2 + 1 = 10$ ways

$= 10 \times 2!$

$= 20$

Arrange.

$$^NP_R = {}^NC_R * R!$$

$$= \frac{N!}{(N-R)!\,\cancel{R!}} \times \cancel{R!} = \frac{N!}{(N-R)!} \rightarrow {}^NP_R$$

$3!$ ways = 6 ways

1, 2, 3

1, 3, 2

2, 1, 3

2, 3, 1

3, 1, 2

3, 2, 1

G1 | G2 G3

G1 → 2 ways

G2 → 2 way

G3 → 2 ways

3×2

$4 \times \underbrace{\boxed{\quad}}_{3!}$

$= 4!$

$r \Rightarrow r!$

$$^NP_R = {}^NC_R \cdot R! = \frac{N!}{(N-R)!}$$

Q) 5 Boys   3 girls

→ Choose 2 Boys and 1 girl

Product Rule

$$\frac{5 \times 4}{2} \times 3 = 30$$

$$^5C_2 \times {}^3C_1$$

A, B	G1
A, C	G1
A, B	G2
A, C	G2

G3 G1 G2

G1 G2 G3

$\boxed{B, B, G}$

$\boxed{G, B, B}$

→ Sum Rule : X set 1 Y set 2 OR

$^5C_2 + {^3C_1}$

$= 10 + 3 =$ 13 ways

X + Y ways to choose an element that belongs to set or set B

Pascal's Triangle

Pattern

3 ways

i=1	C0	C1	C2	C3	C4	
0	1					
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4 +	6	4	1	
5	1	5	10	10	5	1

many mul
1 mul
1 addition

store prev row

2D Array

Time

$$^NC_R = \frac{N!}{(N-R)!\,R!}$$

$= \frac{1 \times 2 \times 3 \times 4 \times 5}{\text{— — — —}}$

= ans

$^5C_2 = \frac{5 \times 4}{2} = 10$

^NC_R

$^4C_3 = 4$

(2)

^NC_R

Y1 Y2

$X \to {^NC_R}$

$$^NC_R = {^{N-1}C_R} + {^{N-1}C_{R-1}}$$

	R−1	R
N−1	4	6
N		10

$$^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

$^{2}C_{2} \quad {}^{2}C_{1} \quad {}^{2}C_{0}$

1

1 1

⇒ 1 2 1

1 3 3 1

$^{5}C_{r} = k\,{}^{5}C_{r-1}$

$^{5}C_{0} \xrightarrow{k} {}^{5}C_{1} \xrightarrow{k} {}^{5}C_{2} \xrightarrow{k} {}^{5}C_{3} \xrightarrow{k} {}^{5}C_{4} \xrightarrow{k} {}^{5}C_{5}$

1 5 10 10 5 1

formula

$^{N}C_{R}$ → $\boxed{k} \cdot {}^{N}C_{R-1}$

next term

$$^{N}C_{R} = f\left({}^{N}C_{R-1}\right)$$

$$\frac{N!}{(N-R)!\,R!} = k \cdot \frac{N!}{(N-R+1)!\,(R-1)!} = \frac{k}{(N-R+1)}$$

$$\Rightarrow \frac{1}{(N-R)!\,(R-1)!\,R} = \frac{k}{(N-R+1)\times(N-R)!\,(R-1)!}$$

$$\boxed{k = \frac{N-R+1}{R}}$$

Row 5

1 5 10

0 1

$\frac{5-1+1}{1}$ $\frac{5-2+1}{2}$

i=0
i=2
⋮
N

r →

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

for (i=0, i <= N ; i++) { term = 1
for (r = 1 ; r <= i, r++) {
term = k * term ;
print (term) ;

3

3

$\rightarrow \left(k = \frac{i - r + 1}{r}\right)$

Row col

$term = k = \left(\frac{5 - 2 + 1}{2}\right)$ 4

$= 3 \times \cancel{4} 2 = 6$

KOOL :)

0
1
2
3
4
5
6

1	4	6	- - -
0	1	2	

r=1

row NO

$k = \left(\frac{4 - 1 + 1}{1}\right)$ 1

$k = \frac{4 - 2 + 1}{2}$

$= \frac{3}{2} (4) 2$