

**Proof reading for “Matrix identities involving multiplication  
and transposition” by Auinger et al**

Location	Type	In the proofs	In the original	Should be
Throughout the text	Editor’s intervention	non-finitely based	nonfinitely based	As in the proofs (we accept the change)
P.1, Background and Motivation, line +9	Editor’s intervention	...much attention as well: see, for instance ...	...much attention as well, see, for instance ...	As in the proofs (we accept the change)
P.1, footnote, line +4	Update	21000	21000	21101
P.1, footnote, line +5	Update	Faculty of Mathematics and Mechanics, Ural State University	Faculty of Mathematics and Mechanics, Ural State University	Institute of Mathematics and Computer Science, Ural Federal University
P.1, footnote, line +6	Update	620083	620083	620000
P.2, line +21	Typo (our fault)	...may be <b>a</b> summarized ...	...may be <b>a</b> summarized ...	...may be summarized ...
P.2, Theorem, line +1	Editor’s intervention	<b>None of the</b> following sets of matrix identities admits <b>a</b> finite identity basis:	Each of following sets of matrix identities admits no finite identity basis:	As in the proofs (we accept the change)
P.2, Theorem, lines +2, +4, +6, +9 (4 times)	Editor’s intervention	the identities <b>for</b> ...	the identities of ...	As in the proofs (we accept the change)
P.3, line +5	Editor’s intervention	$\langle$ displayed formula $\rangle$	$\langle$ inline formula $\rangle$	As in the proofs (we accept the change)
P.3, line +18	Editor’s intervention	... then so is $u^*$ .	... then so is $(u)^*$ .	As in the original (we <b>do not</b> accept the change)
P.3, line +20	Editor’s intervention	$u \mapsto u^*$ .	$u \mapsto (u)^*$ .	As in the original (we <b>do not</b> accept the change)
P.3, line –3	Typo (our fault)	A variety is <b>is</b> said to be ...	A variety is <b>is</b> said to be ...	A variety is said to be ...
P.4, lines 1–2	Editor’s intervention	... forming direct products <b>and</b> taking unary subsemi-groups ...	... forming direct products, taking unary subsemi-groups ...	As in the proofs (we accept the change)

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Location	Type	In the proofs	In the original	Should be
P.4, line −2	Editor's intervention	if $p_{jk} = 0$ ,	if $p_{jk} = 0$ ;	As in the proofs (we accept the change)
P.5, line +4	Editor's intervention	If the group $\mathcal{G}$ involved	If the involved group $\mathcal{G}$	As in the proofs (we accept the change)
P.5, display (1.1)	Editor's intervention	otherwise,	otherwise;	As in the proofs (we accept the change)
P.5, line +1 after display (1.1)	Editor's intervention	...semigroup that will be quite useful is ...	...semigroup that will be quite useful in the sequel is ...	As in the proofs (we accept the change)
P.5, line −5	Editor's intervention	...has dimension $n - 1$ , whence ...	...has dimension $n - 1$ whence ...	As in the proofs (we accept the change)
P.6, line −16	Editor's intervention	The following easy observation will be useful as it helps ...	The following easy observation will be useful in the sequel as it helps ...	As in the proofs (we accept the change)
P.6, line −8	Editor's intervention	$H(\mathcal{T}) \in \text{var } H(\mathcal{S})$ , and so $H(\text{var } \mathcal{S}) \subseteq \text{var } H(\mathcal{S})$ .	$H(\mathcal{T}) \in \text{var } H(\mathcal{S})$ . Since this holds for an arbitrary $\mathcal{T} \in \text{var } \mathcal{S}$ , we conclude that $H(\text{var } \mathcal{S}) \subseteq \text{var } H(\mathcal{S})$ .	As in the proofs (we accept the change)
P.7, line +1	Editor's intervention	...there exists a group $\mathcal{G} \in \mathbf{V} \setminus H(\mathbf{V})$	...there exists a group $\mathcal{G} \in \mathbf{V}$ for which $\mathcal{G} \notin H(\mathbf{V})$ .	As in the proofs (we accept the change)
P.7, line −10	Editor's intervention	denotes the $n \times n$ -matrix	denotes the $n \times n$ -matrix of the form	As in the proofs (we accept the change)
P.7, matrix $M_n(g)$ , entry (4,4)	Editor's intervention	$\vdots$ (produced by <code>\vdots</code> )	$\ddots$ (produced by <code>\ddots</code> )	As in the original (we <b>do not</b> accept the change)
P.7, line −8	Editor's intervention	(This construction is in a sense a combination of those of [3] and [53].)	(This construction is in a sense a combination of those of the first and the third authors' papers [3] and [53].)	As in the proofs (we accept the change)
P.8, line +6	Overfull	The row of dots is too long		
P.8, line −9	Editor's intervention	As $2k < n$ according to ...	Using that $2k < n$ according to ...	As in the proofs (we accept the change)
P.9, line +3	Editor's intervention	For each $i$ with ...	For each $i$ such that ...	As in the proofs (we accept the change)
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Location	Type	In the proofs	In the original	Should be
P.10, line +11	Editor's intervention	such that $\mathcal{G} \in \mathbf{V} \setminus P_d(\mathbf{V})$ ...	such that $\mathcal{G} \in \mathbf{V}$ but $\mathcal{G} \notin P_d(\mathbf{V})$ ...	As in the proofs (we accept the change)
P.10, line +18	Editor's intervention	These words have already been used ...	These words already have been used ...	As in the proofs (we accept the change)
P.10, line -14	Editor's intervention	Let $x_1, x_2, \dots$ be a sequence of letters.	Let $x_1, x_2, \dots, x_n, \dots$ be a sequence of letters.	As in the proofs (we accept the change)
P.10, line -8	Editor's intervention	<b>Aiming at a</b> contradiction, suppose ...	Arguing by contradiction, suppose ...	As in the proofs (we accept the change)
P.11, line +2	Editor's intervention	... in <b>Fig. 1 (left)</b> .	... shown in the left hand part of Fig. 1	As in the proofs (we accept the change)
P.11, lines 2-3	Editor's intervention	All odd- <b>numbered</b> columns ...	All odd columns ...	As in the proofs (we accept the change)
P.11, line +4	Editor's intervention	All <b>even-numbered</b> columns	All even columns	As in the proofs (we accept the change)
P.11, line +5	Editor's intervention	... to <b><math>(1, 2, \dots, r, \dots, 1, 2, \dots, r)^t</math></b> where the block $1, 2, \dots, r$ occurs $r$ times.	... to the transpose of the row $(1, 2, \dots, r, \dots, 1, 2, \dots, r)$ in which the block $1, 2, \dots, r$ occurs $r$ times.	We <b>do not</b> accept the change in the proposed form. The notation $(\dots)^t$ for the transpose is inconsistent with the notation elsewhere in the paper. We suggest: ... to the transpose of $(1, 2, \dots, r, \dots, 1, 2, \dots, r)$ where the block $1, 2, \dots, r$ occurs $r$ times.
P.11, line +8	Editor's intervention	(shown in <b>Fig. 1, right</b> )	(shown in the right hand part of Fig. 1)	As in the proofs (we accept the change)
P.11, line +11	Editor's intervention	Let $v_t$ be the word in the $t^{\text{th}}$ row of $M_A$ .	Let $v_t$ be the word in the $t^{\text{th}}$ row of the matrix $M_A$ .	As in the proofs (we accept the change)
P.12, line +13	Editor's intervention	$\varphi(q)$ is not 0; say $\varphi(p) \neq 0$ .	$\varphi(q)$ is not equal to 0; (without loss of generality) assume that $\varphi(p) \neq 0$ .	As in the proofs (we accept the change)
P.12, line -15	Editor's intervention	<b>which</b> may	that may	As in the proofs (we accept the change)
P.13, footnote, line +1	Editor's intervention	the expression <b>that follows</b> is not ...	the following expression is not ...	As in the proofs (we accept the change)
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Location	Type	In the proofs	In the original	Should be
P.14, line +14	Editor's intervention	... the <b>six</b> matrices	... the 6 matrices	As in the proofs (we accept the change)
P.15, line +10	Editor's intervention	... have recently been obtained	... have been recently obtained	As in the proofs (we accept the change)
P.15, line -15	Editor's intervention	... for each $a \in S$ .	... for each element $a \in S$ .	As in the proofs (we accept the change)
P.16, line +3	Editor's intervention	This implies that such <b>a</b> $\mathcal{T}$ ...	This implies that such $\mathcal{T}$ ...	As in the proofs (we accept the change)
P.16, line -8	Editor's intervention	For every $g \in \mathcal{F}$ ...	For every element $g \in \mathcal{F}$ ...	As in the proofs (we accept the change)
P.17, line +4	Editor's intervention	<b>hence</b> belongs to $\text{var } \mathcal{G}$ and so is locally finite.	whence this group belongs to $\text{var } \mathcal{G}$ and so is locally finite.	As in the proofs (we accept the change)
P.17, line -2	Editor's intervention	Then for $a \in \mathcal{S}$ ...	Then for an arbitrary $a \in \mathcal{S}$ ...	As in the proofs (we accept the change)
P.18, line +1	Editor's intervention	<b>A ring</b> involution ...	An involution of the ring ...	As in the proofs (we accept the change)
P.19, line -1	Editor's intervention	$\text{GL}_2(\mathcal{K})$ is contained in $\text{var } \mathcal{S}$ but not in $\text{var } H(\mathcal{S})$	$\text{GL}_2(\mathcal{K})$ is contained in $\text{var } \mathcal{S}$ but is not contained in $\text{var } H(\mathcal{S})$	As in the proofs (we accept the change)
P.20, line -11	Editor's intervention	... onto $F(A)$	... to the space $F(A)$	As in the proofs (we accept the change)
P.20, line -11	Editor's intervention	... onto $N(A)^\perp$ .	... to the space $N(A)^\perp$ .	As in the proofs (we accept the change)
P.20, lines -8 and -7	Editor's intervention	since $A = (P_1 P_2)^\dagger$ (see [38, Exercise 5.15.9a]).	since $A = (P_1 P_2)^\dagger$ , see [38, Exercise 5.15.9a].	As in the proofs (we accept the change)
P.21, line +13	Editor's intervention	This might <b>incline</b> one ...	This might have provoked one ...	As in the proofs (we accept the change)
P.21, line +19	Editor's intervention	The characteristic of $\mathcal{K}$ is not <b>2</b> , whence the group	The characteristic of $\mathcal{K}$ is not 2 whence the group	As in the proofs (we accept the change)
P.22, lines 1-2	Editor's intervention	<b>and</b> the desired conclusion follows by reasoning as in step 1	whence the desired conclusion follows by the reasoning as in Step 1	As in the proofs (we accept the change)
P.23, line +8	Editor's intervention	Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a typical matrix in Asc, <b>and</b> $\begin{pmatrix} d & b \\ c & a \end{pmatrix}$ is <b>one</b> in Desc	Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a typical matrix in Asc, $\begin{pmatrix} d & b \\ c & a \end{pmatrix}$ is such in Desc	As in the proofs (we accept the change)
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Location	Type	In the proofs	In the original	Should be
P.23, line −6	Editor's intervention	$\text{Asc} \cdot A^n \cap \text{Desc} \cdot B^m = \emptyset$ .	$\text{Asc} \cdot A^n \cap \text{Desc} \cdot B^m = \emptyset$ .	As in the proofs (we accept the change)
P.24, line +2	Editor's intervention	that still <b>satisfy</b> $u'(A, B) = v'(A, B)$ . Observe that <b>neither</b> $u'$ <b>nor</b> $v'$ is empty ...	that still fulfil $u'(A, B) = v'(A, B)$ . Observe that none of the words $u'$ and $v'$ are empty ...	As in the proofs (we accept the change)
P.24, line +3	Editor's intervention	...the polynomial ring $\mathcal{K}[x]$ , which is not true.	...the polynomial ring $\mathcal{K}[x]$ which is not true.	As in the proofs (we accept the change)
P.24, line +6	Editor's intervention	$b^{\ell_1} a^{k_1} \dots b^{\ell_t} a^{k_t}$ ,	$b^{\ell_1} a^{k_1} \dots b^{\ell_t} a^{k_t}$	As in the proofs (we accept the change)
P.24, line +7	Typo (our fault) + Editor's intervention	$b^{\ell_1} a^{k_1} \dots b^{\ell_{t-1}} a^{k_{t-1}} b^{k_t}$ ,	$b^{\ell_1} a^{k_1} \dots b^{\ell_{t-1}} a^{k_{t-1}} b^{k_t}$	$b^{\ell_1} a^{k_1} \dots b^{\ell_{t-1}} a^{k_{t-1}} b^{\ell_t}$ ,
P.24, line +9	Editor's intervention	Desc, while in case (3.15), ...	Desc while in case (3.15) ...	As in the proofs (we accept the change)
P.24, line +18	Editor's intervention	say $z$ , contains ...	$z$ , say, contains ...	As in the proofs (we accept the change)
P.24, lines −12 and −11	Editor's intervention	...distinct, <b>so</b> $u_{ij} - v_{ij}$ is a non-zero polynomial. Now take any $\lambda \in \mathcal{K}$ and set $z(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda^2 & \lambda \end{pmatrix}$ . If	...distinct whence $u_{ij} - v_{ij}$ is a non-zero polynomial. Now take any $\lambda \in \mathcal{K}$ and set $z(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda^2 & \lambda \end{pmatrix}$ . If the equality	As in the proofs (we accept the change)
P.24, line −9	Editor's intervention	then $\lambda$ must ...	holds then $\lambda$ must ...	As in the proofs (we accept the change)
P.24, line −8	Editor's intervention	finitely many $\lambda$ <b>in</b> $\mathcal{K}$ .	finitely many elements $\lambda$ of $\mathcal{K}$ .	As in the proofs (we accept the change)
P.25, line +8	Editor's intervention	We start <b>by</b> considering ...	We start with considering ...	As in the proofs (we accept the change)
P.25, line −8	Editor's intervention	...and <b>yields</b> , in fact, a stronger	...and proves, in fact, a stronger	As in the proofs (we accept the change)
P.26, line −11	Editor's intervention	'only if' part of item 2,	'only if' part of item (2),	As in the proofs (we accept the change)
P.27, line +2	Editor's intervention	...the exponent $d$ of $\text{GL}_2(\mathcal{K})$ , <b>so</b> $\alpha^d = 1$ .	...the exponent $d$ of $\text{GL}_2(\mathcal{K})$ whence $\alpha^d = 1$ .	As in the proofs (we accept the change)
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Location	Type	In the proofs	In the original	Should be
P.28, line –16	Editor’s intervention	The $2 \times 2$ -matrices over an infinite field <b>satisfy</b> non-trivial ...	The $2 \times 2$ -matrices over an infinite field fulfill non-trivial ...	As in the proofs (we accept the change)
P.29, lines 4–5	Editor’s intervention	... is closed under multiplication and transposition, <b>so</b> ...	... is closed under multiplication and transposition whence ...	As in the proofs (we accept the change)
P.29, line +16	Editor’s intervention	... a set of edges <b>such</b> that every vertex ...	... a set of edges so that every vertex ...	As in the proofs (we accept the change)
P.29, line +17	Editor’s intervention	<b>then</b> $A$ is said to be a <i>Hall matrix</i>	$A$ is said to be a <i>Hall matrix</i>	As in the proofs (we accept the change)
P.30, line +6	Editor’s intervention	(from the top right to the bottom left corner).	(the diagonal from the top right to the bottom left corner).	As in the proofs (we accept the change)
P.31, line +12	Editor’s intervention	... in the latter.	... in the latter one.	As in the proofs (we accept the change)
P.31, Acknowledgements, line –1	Typo (our fault)	... grants <b>s</b> 10-01-00524.	... grants <b>s</b> 10-01-00524.	... grant 10-01-00524.
P.32, item [27], line +1	Typo (our fault)	The semigroups <b>s</b> of Hall relations.	The semigroups <b>s</b> of Hall relations.	The semigroup of Hall relations.