## **Proof reading for "Matrix identities involving multiplication and transposition" by Auinger et al**

Location	Type	In the proofs	In the original	Should be
Throughout	Editor's in-	non-finitely based	nonfinitely based	As in the proofs
the text	tervention			(we accept the change)
P.1, Back-	Editor's in-	much attention as well:	much attention as well,	As in the proofs
ground and	tervention	see, for instance	see, for instance	(we accept the change)
Motivation,				
line +9				
P.1, footnote,	Update	21000	21000	21101
line +4				
P.1, footnote,	Update	Faculty of Mathematics	Faculty of Mathematics	Institute of Mathematics
line +5		and Mechanics, Ural State	and Mechanics, Ural State	and Computer Science,
		University	University	Ural Federal University
P.1, footnote,	Update	620083	620083	620000
line +6				
P.2, line +21	Typo (our	may be a summarized	may be a summarized	may be summarized
	fault)			
P.2, Theorem,	Editor's in-	None of the following sets	Each of following sets of	As in the proofs
line +1	tervention	of matrix identities admits	matrix identities admits no	(we accept the change)
		a finite identity basis:	finite identity basis:	
P.2, Theorem,	Editor's in-	the identities for	the identities of	As in the proofs
lines $+2$ , $+4$ ,	tervention			(we accept the change)
+6, +9 (4				
times)				
P.3, line +5	Editor's in-	(displayed formula)	(inline formula)	As in the proofs
	tervention			(we accept the change)
P.3, line +18	Editor's in-	then so is $u^*$ .	then so is $(u)^*$ .	As in the original (we do
	tervention			<b>not</b> accept the change)
P.3, line +20	Editor's in-	$u \mapsto u^*$ .	$u \mapsto (u)^*$ .	As in the original (we do
	tervention			<b>not</b> accept the change)
P.3, line −3	Typo (our	A variety is is said to be	A variety is is said to be	A variety is said to be
	fault)			
P.4, lines 1–2	Editor's in-	forming direct products	forming direct products,	As in the proofs
	tervention	and taking unary subsemi-	taking unary subsemi-	(we accept the change)
		groups	groups	
				continued on next page

continued from Location	Type	In the proofs	In the original	Should be
P.4, line -2	Editor's in-	$if p_{ik} = 0,$	$if p_{jk} = 0;$	As in the proofs
r.4, iiiie – 2	tervention	$\prod_{i} p_{jk} = 0,$	$n p_{jk} = 0,$	(we accept the change)
P.5, line +4	Editor's in-	If the group 9 involved	If the involved group 9	As in the proofs
F.3, IIIIe +4	tervention	If the group 9 involved	If the involved group 9	(we accept the change)
D5 diamlar	Editor's in-	oth omico	oth america.	As in the proofs
P.5, display		otherwise,	otherwise;	•
(1.1)	tervention		that will be	(we accept the change)
P.5, line +1	Editor's in-	semigroup that will be	semigroup that will be	As in the proofs
after display	tervention	quite useful is	quite useful in the sequel is	(we accept the change)
(1.1)				
P.5, line $-5$	Editor's in-	has dimension $n-1$ ,	has dimension $n-1$	As in the proofs
	tervention	whence	whence	(we accept the change)
P.6, line –16	Editor's in-	The following easy obser-	The following easy obser-	As in the proofs
	tervention	vation will be useful as it	vation will be useful in the	(we accept the change)
		helps	sequel as it helps	
P.6, line $-8$	Editor's in-	$H(\mathfrak{T}) \in var H(S)$ , and so	$H(\mathfrak{T}) \in var H(\mathfrak{S})$ . Since	As in the proofs
	tervention	$H(var S) \subseteq var H(S).$	this holds for an arbitrary	(we accept the change)
			$\mathfrak{T} \in var S, we conclude$	
			that $H(var S) \subseteq var H(S)$ .	
P.7, line +1	Editor's in-	there exists a group $\mathfrak{G} \in$	there exists a group $\mathfrak{G} \in$	As in the proofs
	tervention	$\mathbf{V} \setminus \mathrm{H}(\mathbf{V})$	<b>V</b> for which $\mathfrak{G} \notin H(\mathbf{V})$ .	(we accept the change)
P.7, line –10	Editor's in-	denotes the $n \times n$ -matrix	denotes the $n \times n$ -matrix of	As in the proofs
	tervention		the form	(we accept the change)
P.7, matrix	Editor's in-	:(produced by \vdots)	·. (produced by \ddots)	
·	tervention	. (produced by \vdots)	(produced by \aaocs)	As in the original (we do
$M_n(g)$ , entry	tervention			<b>not</b> accept the change)
(4,4)	Editor's in-	(This construction is in	(This construction is in	A signature and affective
P.7, line $-8$		(This construction is in	(This construction is in	As in the proofs
	tervention	a sense a combination of	a sense a combination of	(we accept the change)
		those of [3] and [53].)	those of the first and the	
			third authors' papers [3]	
DO 1' (	0 0 11		and [53].)	
P.8, line +6	Overfull	1	The row of dots is too long	
P.8, line $-9$	Editor's in-	As $2k < n$ according to	Using that $2k < n$ accord-	As in the proofs
	tervention		ing to	(we accept the change)
P.9, line +3	Editor's in-	For each <i>i</i> with	For each $i$ such that	As in the proofs
	tervention			(we accept the change)
				continued on next page

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Location	Type	In the proofs	In the original	Should be	
P.10, line +11	Editor's in-	such that $\mathfrak{G} \in \mathbf{V} \setminus \mathrm{P}_d(\mathbf{V})$	such that $\mathcal{G} \in \mathbf{V}$ but $\mathcal{G} \notin$	As in the proofs	
	tervention		$P_d(\mathbf{V})$	(we accept the change)	
P.10, line +18	Editor's in-	These words have already	These words already have	As in the proofs	
	tervention	been used	been used	(we accept the change)	
P.10, line -14	Editor's in-	Let $x_1, x_2, \ldots$ be a se-	Let $x_1, x_2, \ldots, x_n, \ldots$ be a	As in the proofs	
	tervention	quence of letters.	sequence of letters.	(we accept the change)	
P.10, line -8	Editor's in-	Aiming at a contradiction,	Arguing by contradiction,	As in the proofs	
	tervention	suppose	suppose	(we accept the change)	
P.11, line +2	Editor's in-	in Fig. 1 (left).	shown in the left hand	As in the proofs	
	tervention		part of Fig. 1	(we accept the change)	
P.11, lines 2–	Editor's in-	All odd-numbered columns	All odd columns	As in the proofs	
3	tervention			(we accept the change)	
P.11, line +4	Editor's in-	All even-numbered	All even columns	As in the proofs	
	tervention	columns		(we accept the change)	
P.11, line +5	Editor's in-	to	to the trans-	We do not accept the	
	tervention	$(1,2,\ldots,r,\ldots,1,2,\ldots,r)^t$	pose of the row	change in the proposed	
		where the block $1, 2, \ldots, r$	$(1,2,\ldots,r,\ldots,1,2,\ldots,r)$	form. The notation $()^t$	
		occurs $r$ times.	in which the block	for the transpose is incon-	
			$1, 2, \ldots, r$ occurs $r$	sistent with the notation	
			times.	elsewhere in the paper. We	
				suggest:	
				to the transpose of	
				$(1,2,\ldots,r,\ldots,1,2,\ldots,r)$	
				where the block $1, 2, \ldots, r$	
				occurs $r$ times.	
P.11, line +8	Editor's in-	(shown in Fig. 1, right)	(shown in the right hand	As in the proofs	
	tervention		part of Fig. 1)	(we accept the change)	
P.11, line +11	Editor's in-	Let $v_t$ be the word in the $t^{\rm th}$	Let $v_t$ be the word in the $t^{\rm th}$	As in the proofs	
	tervention	row of $M_A$ .	row of the matrix $M_A$ .	(we accept the change)	
P.12, line +13	Editor's in-	$\varphi(q)$ is not 0; say $\varphi(p) \neq$	$\varphi(q)$ is not equal to 0;	As in the proofs	
	tervention	0.	(without loss of generality)	(we accept the change)	
			assume that $\varphi(p) \neq 0$ .		
P.12, line -15	Editor's in-	which may	that may	As in the proofs	
	tervention			(we accept the change)	
P.13, foot-	Editor's in-	the expression that follows	the following expression is	As in the proofs	
note, line +1	tervention	is not	not	(we accept the change)	
				continued on next page	

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		-	- J		
P.14, line +14	Editor's in-	the six matrices	the 6 matrices	As in the proofs	
	tervention			(we accept the change)	
P.15, line +10	Editor's in-	have recently been ob-	have been recently ob-	As in the proofs	
	tervention	tained	tained	(we accept the change)	
P.15, line -15	Editor's in-	$\dots$ for each $a \in S$ .	for each element $a \in S$ .	As in the proofs	
	tervention			(we accept the change)	
P.16, line +3	Editor's in-	This implies that such a T	This implies that such T	As in the proofs	
	tervention	•••	_	(we accept the change)	
P.16, line -8	Editor's in-	For every $g \in \mathcal{F} \dots$	For every element $g \in \mathcal{F}$	As in the proofs	
·	tervention			(we accept the change)	
P.17, line +4	Editor's in-	hence belongs to var 9 and	whence this group belongs	As in the proofs	
•	tervention	so is locally finite.	to var 9 and so is locally fi-	(we accept the change)	
		January January	nite.	(	
P.17, line -2	Editor's in-	Then for $a \in \mathbb{S}$	Then for an arbitrary $a \in S$	As in the proofs	
,	tervention			(we accept the change)	
P.18, line +1	Editor's in-	A ring involution	An involution of the ring	As in the proofs	
,	tervention	· ·	J	(we accept the change)	
P.19, line –1	Editor's in-	$\operatorname{GL}_2(\mathfrak{K})$ is contained in	$\operatorname{GL}_2(\mathfrak{K})$ is contained in	As in the proofs	
,	tervention	var S but not in var H(S)	var 8 but is not contained in	(we accept the change)	
			$\operatorname{var} \mathrm{H}(\mathbb{S})$	(	
P.20, line -11	Editor's in-	$\dots$ onto $F(A)$	to the space $F(A)$	As in the proofs	
,	tervention	( )	( )	(we accept the change)	
P.20, line -11	Editor's in-	onto $N(A)^{\perp}$ .	to the space $N(A)^{\perp}$ .	As in the proofs	
1.20, 1110	tervention	6116 11 (11)	to the space 14 (11)	(we accept the change)	
P.20, lines -8	Editor's in-	since $A = (P_1P_2)^{\dagger}$ (see	since $A = (P_1 P_2)^{\dagger}$ , see	As in the proofs	
and $-7$	tervention	[38, Exercise 5.15.9a]).	[38, Exercise 5.15.9a].	(we accept the change)	
P.21, line +13	Editor's in-	This might incline one	This might have provoked	As in the proofs	
1.21, IIIIC 113	tervention	This hight menne one	one	(we accept the change)	
P.21, line +19	Editor's in-	The characteristic of $K$ is	The characteristic of $K$ is	As in the proofs	
1.21, IIIIC +19	tervention	not 2, whence the group	not 2 whence the group	(we accept the change)	
P.22, lines 1–	Editor's in-	and the desired conclusion	whence the desired conclu-		
2 nines 1–	tervention	follows by reasoning as in	sion follows by the reason-	As in the proofs (we accept the change)	
4	tervention		· · · · · · · · · · · · · · · · · · ·	(we accept the change)	
D22 line + 9	Editor's in	step 1 Then $\begin{pmatrix} a & b \end{pmatrix}$ is a typical ma	ing as in Step 1  Then $\begin{pmatrix} a & b \end{pmatrix}$ is a typical ma	As in the proofs	
P.23, line +8	Editor's in-	Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a typical matrix in Associated $\begin{pmatrix} d & b \\ d & b \end{pmatrix}$ is one	Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a typical matrix in $A$ as $\begin{pmatrix} d & b \\ d & d \end{pmatrix}$ is such in	As in the proofs	
	tervention	trix in Asc, and $\begin{pmatrix} d & b \\ c & a \end{pmatrix}$ is one	trix in Asc, $\begin{pmatrix} d & b \\ c & a \end{pmatrix}$ is such in	(we accept the change)	
		in Desc	Desc		
				continued on next page	

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Location	Type	In the proofs	In the original	Should be	
P.23, line -6	Editor's in-	$\operatorname{Asc} \cdot A^n \cap \operatorname{Desc} \cdot B^m = \emptyset.$	$\operatorname{Asc} \cdot A^n \cap \operatorname{Desc} \cdot B^m = \emptyset.$	As in the proofs	
	tervention			(we accept the change)	
P.24, line +2	Editor's in-	that still satisfy $u'(A, B) =$	that still fulfil $u'(A, B) =$	As in the proofs	
	tervention	v'(A,B). Observe that nei-	v'(A, B). Observe that	(we accept the change)	
		ther $u'$ nor $v'$ is empty	none of the words $u'$ and $v'$		
			are empty		
P.24, line +3	Editor's in-	the polynomial ring	the polynomial ring	As in the proofs	
	tervention	$\mathcal{K}[x]$ , which is not true.	$\mathcal{K}[x]$ which is not true.	(we accept the change)	
P.24, line +6	Editor's in-	$b^{\ell_1}a^{k_1}\cdots b^{\ell_t}a^{k_t}$ ,	$b^{\ell_1}a^{k_1}\cdots b^{\ell_t}a^{k_t}$	As in the proofs	
	tervention			(we accept the change)	
P.24, line +7	Typo (our	$b^{\ell_1}a^{k_1}\cdots b^{\ell_{t-1}}a^{k_{t-1}}b^{k_t}$ ,	$b^{\ell_1}a^{k_1}\cdots b^{\ell_{t-1}}a^{k_{t-1}}b^{k_t}$	$b^{\ell_1}a^{k_1}\cdots b^{\ell_{t-1}}a^{k_{t-1}}b^{\ell_t}$ ,	
	fault) +				
	Editor's in-				
	tervention				
P.24, line +9	Editor's in-	Desc, while in case (3.15),	Desc while in case (3.15)	As in the proofs	
	tervention	•••	•••	(we accept the change)	
P.24, line +18	Editor's in-	say $z$ , contains	z, say, contains	As in the proofs	
	tervention			(we accept the change)	
P.24, lines	Editor's in-	distinct, so $u_{ij} - v_{ij}$ is a	$\dots$ distinct whence $u_{ij} - v_{ij}$	As in the proofs	
-12  and  -11	tervention	non-zero polynomial. Now	is a non-zero polynomial.	(we accept the change)	
		take any $\lambda \in \mathcal{K}$ and set	Now take any $\lambda \in \mathcal{K}$ and		
		$z(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda^2 & \lambda \end{pmatrix}$ . If	set $z(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda^2 & \lambda \end{pmatrix}$ . If the		
			equality		
P.24, line –9	Editor's in-	then $\lambda$ must	holds then $\lambda$ must	As in the proofs	
20111	tervention			(we accept the change)	
P.24, line –8	Editor's in-	finitely many $\lambda$ in $K$ .	finitely many elements $\lambda$ of	As in the proofs	
	tervention		<b></b>	(we accept the change)	