Logistic Regression

In this assignment, you will implement logistic regression and apply it to a dataset.

1 - Packages

First, let's run the cell below to import all the packages that you will need during this assignment.

- numpy is the fundamental package for scientific computing with Python.
- matplotlib is a famous library to plot graphs in Python.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import copy
import math
```

2 - Logistic Regression

In this part of the assignment, you will build a logistic regression model to predict whether a patient gets breast cancer.

2.1 Problem Statement

Suppose that you want to determine each patient's chance of getting breast cancer.

- You have historical data from previous patients that you can use as a training set for logistic regression.
- For each training example, you have the applicant's tumor radius and texture as well as the dignosis.
  - radius - mean of distances from center to points on the perimeter
  - texture - standard deviation of gray-scale values
- Your task is to build a classification model that estimates a patient's probability of getting breast cancer based on the tumor radius and texture.

2.2 Loading and visualizing the data

You will start by loading the dataset for this task.

- `X_train` contains tumor radius and texture of patients
- `y_train` is the dignosis decision
  - `y_train = 1` if the patient got breast cancer
  - `y_train = 0` if the patient did not get breast cancer

- Both `X_train` and `y_train` are numpy arrays.

In [2]:
```
def load_data(filename):
    data = np.loadtxt(filename)
    X = data[:,:2]
    y = data[:,2]

    return X, y
```

In [3]:
```
# load dataset

X_train, y_train = load_data("./data/breastcancer.txt")
```

View the variables

Let's get more familiar with your dataset.

- A good place to start is to just print out each variable and see what it contains.

The code below prints the first five values of `X_train` and the type of the variable.

In [4]:
```
print("First five elements in X_train are:\n", X_train[:5])
print("Type of X_train:",type(X_train))
```

```
First five elements in X_train are:
 [[17.99 10.38]
 [20.57 17.77]
 [19.69 21.25]
 [11.42 20.38]
 [20.29 14.34]]
Type of X_train: <class 'numpy.ndarray'>
```

Now print the first five values of `y_train`

In [5]:
```
print("First five elements in y_train are:\n", y_train[:5])
print("Type of y_train:",type(y_train))
```

```
First five elements in y_train are:
 [1. 1. 1. 1. 1.]
Type of y_train: <class 'numpy.ndarray'>
```

Check the dimensions of your variables

Another useful way to get familiar with your data is to view its dimensions. Let's print the shape of `X_train` and `y_train` and see how many training examples we have in our dataset.

In [6]:
```
print ('The shape of X_train is: ' + str(X_train.shape))
print ('The shape of y_train is: ' + str(y_train.shape))
print ('We have m = %d training examples' % (len(y_train)))
```

```
The shape of X_train is: (569, 2)
The shape of y_train is: (569,)
We have m = 569 training examples
```

Visualize your data

Before starting to implement any learning algorithm, it is always good to visualize the data if possible.

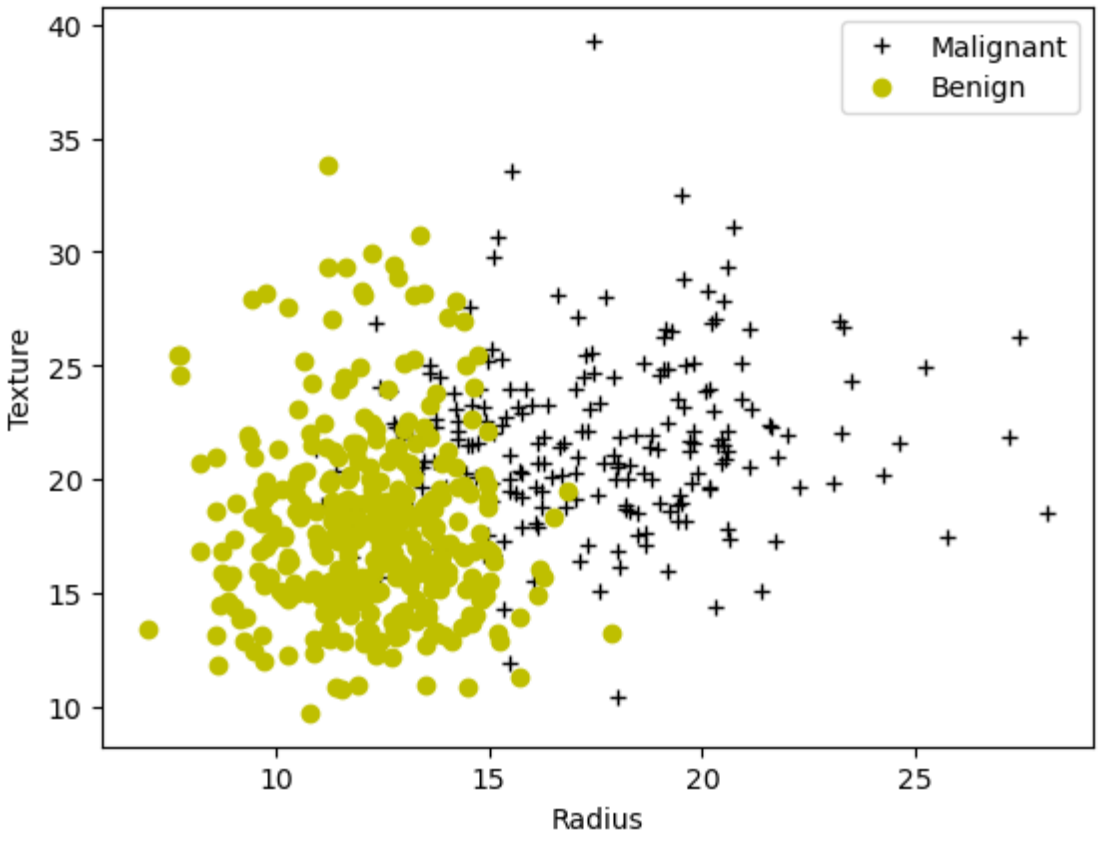
- The code below displays the data on a 2D plot, where the axes are the tumor radius and texture, and the positive and negative examples are shown with different markers.

In [7]:
```
def plot_data(X, y, pos_label="y=1", neg_label="y=0"):
    positive = y == 1
    negative = y == 0

    # Plot examples
    plt.plot(X[positive, 0], X[positive, 1], 'k+', label=pos_label)
    plt.plot(X[negative, 0], X[negative, 1], 'yo', label=neg_label)
```

In [8]:
```
# Plot examples
plot_data(X_train, y_train[:], pos_label="Malignant", neg_label="Benign")

# Set the y-axis label
plt.ylabel('Texture')
# Set the x-axis label
plt.xlabel('Radius')
plt.legend(loc="upper right")
plt.show()
```



Your goal is to build a logistic regression model to fit this data.

2.3 Sigmoid function

Recall that for logistic regression, the model is represented as

$$f_{\mathbf{w},b}(x) = g(\mathbf{w} \cdot \mathbf{x} + b)$$

where function g is the sigmoid function. The sigmoid function is defined as:

$$g(z) = \frac{1}{1+e^{-z}}$$

Let's implement the sigmoid function first, so it can be used by the rest of this assignment.

Please complete the `sigmoid` function to calculate

$$g(z) = \frac{1}{1+e^{-z}}$$

Note that

- `z` is not always a single number, but can also be an array of numbers.
- If the input is an array of numbers, we'd like to apply the sigmoid function to each value in the input array.
- `numpy` has a function called `np.exp()`, which offers a convinient way to calculate the exponential (e^z) of all elements in the input array (`z`).

In [9]:
```
# GRADED FUNCTION: sigmoid
def sigmoid(z):
    """
    Compute the sigmoid of z

    Args:
        z (ndarray): A scalar, numpy array of any size.

    Returns:
        g (ndarray): sigmoid(z), with the same shape as z

    """

    ### START CODE HERE ###

    g = 1 / (1 + np.exp(-1 * z))

    ### END SOLUTION ###

    return g
```

When you are finished, try testing a few values by calling `sigmoid(x)` in the cell below.

- For large positive values of x, the sigmoid should be close to 1, while for large negative values, the sigmoid should be close to 0.
- Evaluating `sigmoid(0)` should give you exactly 0.5.

In [10]:
```
print ("sigmoid(0) = " + str(sigmoid(0)))
```

```
sigmoid(0) = 0.5
```

Expected Output:

sigmoid(0)	0.5

- As mentioned before, your code should also work with vectors and matrices. For a matrix, your function should perform the sigmoid function on every element.

In [11]:
```
print ("sigmoid([ -1, 0, 1, 2]) = " + str(sigmoid(np.array([-1, 0, 1, 2]))))
```

```
sigmoid([ -1, 0, 1, 2]) = [0.26894142 0.5        0.73105858 0.88079708]
```

Expected Output:

sigmoid([-1, 0, 1, 2])	[0.26894142 0.5 0.73105858 0.88079708]

2.4 Cost function for logistic regression

In this section, you will implement the cost function for logistic regression.

Please complete the `compute_cost` function using the equations below.

Recall that for logistic regression, the cost function is of the form

$$J(\mathbf{w},b) = \frac{1}{m}\sum_{i=0}^{m-1}\left[loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}),y^{(i)})\right] \tag{1}$$

where

- m is the number of training examples in the dataset
- $loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}),y^{(i)})$ is the cost for a single data point, which is -

$$loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}),y^{(i)}) = (-y^{(i)}\log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)}\right)\right) - \left(1-y^{(i)}\right)\log\left(1-f_{\mathbf{w},b}\left(\mathbf{x}^{(i)}\right)\right) \quad (2$$

- $f_{\mathbf{w},b}(\mathbf{x}^{(i)})$ is the model's prediction, while $y^{(i)}$, which is the actual label
- $f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = g(\mathbf{w}\cdot\mathbf{x^{(i)}}+b)$ where function g is the sigmoid function.
    - It might be helpful to first calculate an intermediate variable $z_{\mathbf{w},b}(\mathbf{x}^{(i)}) = \mathbf{w}\cdot\mathbf{x^{(i)}}+b = w_0x_0^{(i)}+\ldots+w_{n-1}x_{n-1}^{(i)}+b$ where n is the number of features, before calculating $f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = g(z_{\mathbf{w},b}(\mathbf{x}^{(i)}))$

Note:

- As you are doing this, remember that the variables `X_train` and `y_train` are not scalar values but matrices of shape (m,n) and $(m,1)$ respectively, where n is the number of features and m is the number of training examples.
- You can use the sigmoid function that you implemented above for this part.

In [12]:
```
# GRADED FUNCTION: compute_cost
def compute_cost(X, y, w, b, lambda_= 1):
    """
    Computes the cost over all examples
    Args:
      X : (ndarray Shape (m,n)) data, m examples by n features
      y : (array_like Shape (m,)) target value
      w : (array_like Shape (n,)) Values of parameters of the model
      b : scalar Values of bias parameter of the model
      lambda_: unused placeholder
    Returns:
      total_cost: (scalar)         cost
    """

    m, n = X.shape
    total_cost = 0

    ### START CODE HERE ###
    for i in range(0,m):
        z = np.dot(w, X[i,:]) + b
        g = sigmoid(z)
        total_cost += -1*y[i]*math.log(g) - (1-y[i])*math.log(1-g)

    total_cost /= m

    ### END CODE HERE ###

    return total_cost
```

Run the cells below to check your implementation of the `compute_cost` function with two different initializations of the parameters w

In [13]:
```
m, n = X_train.shape

# Compute and display cost with w initialized to zeroes
initial_w = np.zeros(n)
initial_b = 0.
cost = compute_cost(X_train, y_train, initial_w, initial_b)
print('Cost at initial w (zeros): {:.3f}'.format(cost))
```

```
Cost at initial w (zeros): 0.693
```

Expected Output:

Cost at initial w (zeros)	0.693

In [14]:
```
# Compute and display cost with non-zero w
test_w = np.array([0.2, 0.2])
test_b = -24.
cost = compute_cost(X_train, y_train, test_w, test_b)

print('Cost at test w,b: {:.3f}'.format(cost))
```

```
Cost at test w,b: 6.031
```

Expected Output:

2.5 Gradient for logistic regression

In this section, you will implement the gradient for logistic regression.

Recall that the gradient descent algorithm is:

$$\begin{aligned}
&\text{repeat until convergence: } \{ \\
&b := b - \alpha \frac{\partial J(\mathbf{w},b)}{\partial b} \\
&w_j := w_j - \alpha \frac{\partial J(\mathbf{w},b)}{\partial w_j} \qquad \text{for j := 0..n-1} \\
&\}
\end{aligned} \tag{1}$$

where, parameters b, w_j are all updated simultaniously

Please complete the `compute_gradient` function to compute $\frac{\partial J(\mathbf{w},b)}{\partial w}$, $\frac{\partial J(\mathbf{w},b)}{\partial b}$ from equations (2) and (3) below.

$$\frac{\partial J(\mathbf{w},b)}{\partial b} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{\mathbf{w},b}(\mathbf{x}^{(i)}) - \mathbf{y}^{(i)}) \tag{2}$$

$$\frac{\partial J(\mathbf{w},b)}{\partial w_j} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{\mathbf{w},b}(\mathbf{x}^{(i)}) - \mathbf{y}^{(i)}) x_j^{(i)} \tag{3}$$

- m is the number of training examples in the dataset
- $f_{\mathbf{w},b}(x^{(i)})$ is the model's prediction, while $y^{(i)}$ is the actual label
- **Note**: While this gradient looks identical to the linear regression gradient, the formula is actually different because linear and logistic regression have different definitions of $f_{\mathbf{w},b}(x)$.

In [15]:
```
# GRADED FUNCTION: compute_gradient
def compute_gradient(X, y, w, b, lambda_=None):
    """
    Computes the gradient for logistic regression

    Args:
      X : (ndarray Shape (m,n)) variable such as house size
      y : (array_like Shape (m,1)) actual value
      w : (array_like Shape (n,1)) values of parameters of the model
      b : (scalar)                 value of parameter of the model
      lambda_: unused placeholder.
    Returns
      dj_dw: (array_like Shape (n,1)) The gradient of the cost w.r.t. the parameters w
      dj_db: (scalar)                The gradient of the cost w.r.t. the parameter b.
    """
    m, n = X.shape
    dj_dw = np.zeros(w.shape)
```

```
    dj_db = 0.

    ### START CODE HERE ###
    for i in range(0, m):
        z = np.dot(w, X[i,:]) + b
        g = sigmoid(z)

        dj_dw += np.dot(g - y[i], X[i,:])
        dj_db += g - y[i]

    dj_dw /= m
    dj_db /= m
    ### END CODE HERE ###


    return dj_db, dj_dw
```

Run the cells below to check your implementation of the `compute_gradient` function with two different initializations of the parameters w

In [16]:
```
# Compute and display gradient with w initialized to zeroes
initial_w = np.zeros(n)
initial_b = 0.

dj_db, dj_dw = compute_gradient(X_train, y_train, initial_w, initial_b)
print(f'dj_db at initial w (zeros):{dj_db}' )
print(f'dj_dw at initial w (zeros):{dj_dw.tolist()}' )
```

```
dj_db at initial w (zeros):0.1274165202108963
dj_dw at initial w (zeros):[0.557283831282953, 1.5951933216168726]
```

Expected Output:

dj_db at initial w (zeros)	0.1274165202108963
dj_dw at initial w (zeros):	[0.557283831282953, 1.5951933216168726]

In [17]:
```
# Compute and display cost and gradient with non-zero w
test_w = np.array([ 0.2, -0.5])
test_b = -24
dj_db, dj_dw  = compute_gradient(X_train, y_train, test_w, test_b)

print('dj_db at test_w:', dj_db)
print('dj_dw at test_w:', dj_dw.tolist())
```

```
dj_db at test_w: -0.3725834797889203
dj_dw at test_w: [-6.506362038661654, -8.04963093145612]
```

Expected Output:

dj_db at initial w (zeros)	-0.3725834797889203
ddj_dw at initial w (zeros):	[-6.506362038661654, -8.04963093145612]

2.6 Learning parameters using gradient descent

Similar to the previous assignment, you will now find the optimal parameters of a logistic regression model by using gradient descent.

- A good way to verify that gradient descent is working correctly is to look at the value of $J(\mathbf{w}, b)$ and check that it is decreasing with each step.
- Assuming you have implemented the gradient and computed the cost correctly, your value of $J(\mathbf{w}, b)$ should never increase, and should converge to a steady value by the end of the algorithm.

In [18]:
```
def gradient_descent(X, y, w_in, b_in, cost_function, gradient_function, alpha, num_it
    """
    Performs batch gradient descent to learn theta. Updates theta by taking
    num_iters gradient steps with learning rate alpha

    Args:
      X :    (array_like Shape (m, n)
      y :    (array_like Shape (m,))
      w_in : (array_like Shape (n,))  Initial values of parameters of the model
      b_in : (scalar)                 Initial value of parameter of the model
      cost_function:                  function to compute cost
      alpha : (float)                 Learning rate
      num_iters : (int)               number of iterations to run gradient descent
      lambda_ (scalar, float)         regularization constant

    Returns:
      w : (array_like Shape (n,)) Updated values of parameters of the model after
          running gradient descent
      b : (scalar)                Updated value of parameter of the model after
          running gradient descent
    """

    # number of training examples
    m = len(X)

    # An array to store cost J and w's at each iteration primarily for graphing later
    J_history = []
    w_history = []

    for i in range(num_iters):

        # Calculate the gradient and update the parameters
        dj_db, dj_dw = compute_gradient(X, y, w_in, b_in, lambda_)

        # Update Parameters using w, b, alpha and gradient
        w_in = w_in - alpha * dj_dw
        b_in = b_in - alpha * dj_db

        # Save cost J at each iteration
        if i<100000:      # prevent resource exhaustion
            cost =  cost_function(X, y, w_in, b_in, lambda_)
            J_history.append(cost)

        # Print cost every at intervals 10 times or as many iterations if < 10
        if i% math.ceil(num_iters/10) == 0 or i == (num_iters-1):
            w_history.append(w_in)
            print(f"Iteration {i:4}: Cost {float(J_history[-1]):8.2f}   ")
```

```
    return w_in, b_in, J_history, w_history #return w and J,w history for graphing
```

Now let's run the gradient descent algorithm above to learn the parameters for our dataset.

Note

The code block below takes a couple of minutes to run, especially with a non-vectorized version.

In [19]:
```
np.random.seed(1)
intial_w = 0.01 * (np.random.rand(2).reshape(-1,1) - 0.5)
initial_b = -8


# Some gradient descent settings
iterations = 10000
alpha = 0.001

w,b, J_history,_ = gradient_descent(X_train ,y_train, initial_w, initial_b,
                                    compute_cost, compute_gradient, alpha, iterations,
```

```
Iteration    0: Cost     2.87
Iteration 1000: Cost     0.33
Iteration 2000: Cost     0.33
Iteration 3000: Cost     0.33
Iteration 4000: Cost     0.33
Iteration 5000: Cost     0.33
Iteration 6000: Cost     0.33
Iteration 7000: Cost     0.33
Iteration 8000: Cost     0.33
Iteration 9000: Cost     0.33
Iteration 9999: Cost     0.33
```

▶ **Expected Output: Cost 0.33, (Click to see details):**

2.7 Evaluating logistic regression

We can evaluate the quality of the parameters we have found by seeing how well the learned model predicts on our training set.

You will implement the `predict` function below to do this.

Please complete the `predict` function to produce `1` or `0` predictions given a dataset and a learned parameter vector w and b.

- First you need to compute the prediction from the model $f(x^{(i)}) = g(w \cdot x^{(i)})$ for every example
  - You've implemented this before in the parts above
- We interpret the output of the model ($f(x^{(i)})$) as the probability that $y^{(i)} = 1$ given $x^{(i)}$ and parameterized by w.
- Therefore, to get a final prediction ($y^{(i)} = 0$ or $y^{(i)} = 1$) from the logistic regression model, you can use the following heuristic -

if $f(x^{(i)}) >= 0.5$, predict $y^{(i)} = 1$

if $f(x^{(i)}) < 0.5$, predict $y^{(i)} = 0$

If you get stuck, you can check out the hints presented after the cell below to help you with the implementation.

In [20]:
```
# GRADED FUNCTION: predict

def predict(X, w, b):
    """
    Predict whether the label is 0 or 1 using learned logistic
    regression parameters w

    Args:
    X : (ndarray Shape (m, n))
    w : (array_like Shape (n,))      Parameters of the model
    b : (scalar, float)              Parameter of the model

    Returns:
    p: (ndarray (m,1))
        The predictions for X using a threshold at 0.5
    """
    # number of training examples
    m, n = X.shape
    p = np.zeros(m)

    ### START CODE HERE ###
    for i in range(0, m):
        z = np.dot(w, X[i,:]) + b
        g = sigmoid(z)

        p[i] = 1 if g >= 0.5 else 0

    ### END CODE HERE ###
    return p
```

Once you have completed the function `predict`, let's run the code below to report the training accuracy of your classifier by computing the percentage of examples it got correct.

In [21]:
```
# Test your predict code
np.random.seed(1)
tmp_w = np.random.randn(2)
tmp_b = 0.3
tmp_X = np.random.randn(4, 2) - 0.5

tmp_p = predict(tmp_X, tmp_w, tmp_b)
print(f'Output of predict: shape {tmp_p.shape}, value {tmp_p}')
```

```
Output of predict: shape (4,), value [0. 1. 1. 1.]
```

Expected output

Output of predict: shape (4,),value [0. 1. 1. 1.]

Now let's use this to compute the accuracy on the training set

In [22]:
```
#Compute accuracy on our training set
p = predict(X_train, w,b)
print('Train Accuracy: %f'%(np.mean(p == y_train) * 100))
```

```
Train Accuracy: 89.279438
```

Train Accuracy (approx): 89.279438

In [ ]: