

## Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with  $\mu = 45$  minutes and  $\sigma = 8$  minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
  - A. 0.3875
  - B. 0.2676
  - C. 0.5
  - D. 0.6987

Ans: We have a normal distribution with  $\mu = 45$  and  $\sigma = 8.0$ .

```
from scipy import stats
import pandas as pd
import numpy as np
```

```
p=1- stats.norm.cdf(50, 45,8)
p
```

```
0.26598552904870054
```

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean  $\mu = 38$  and Standard deviation  $\sigma = 6$ . For each statement below, please specify True/False. If false, briefly explain why.

- A. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False, more employees are bat the age of 36 and hence the mean is 36.

- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: TRUE

3. If  $X_1 \sim N(\mu, \sigma^2)$  and  $X_2 \sim N(\mu, \sigma^2)$  are iid normal random variables, then what is the difference between  $2 X_1$  and  $X_1 + X_2$ ? Discuss both their distributions and parameters.

Ans:

- $2 X_1$ , multiplies the mean and variance by 2, whereas the  $X_1 + X_2$ , the mean being the sum of the two means, and its variance being the sum of the two variances.
- $2 X_1$ , makes the kurtosis of the distribution negative as the scaling of the variance spreads out the peak of the distribution.

4. Let  $X \sim N(100, 20^2)$ . Find two values,  $a$  and  $b$ , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
- A. 90.5, 105.9
  - B. 80.2, 119.8
  - C. 22, 78
  - D. 48.5, 151.5
  - E. 90.1, 109.9

Ans:

```
import pandas as pd
import numpy as np
from scipy import stats
```

```
Mu=100
std=20
```

```
p=stats.norm.cdf(0.99,Mu,std)
p
```

```
3.701152862880683e-07
```

```
CI1=Mu+p
CI2=Mu-p
(CI1,CI2)
```

```
(100.00000037011529, 99.99999962988471)
```

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions  $\text{Profit}_1 \sim N(5, 3^2)$  and  $\text{Profit}_2 \sim N(7, 4^2)$  respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: (542.5747738297588, 537.4252261702412)

- B. Specify the 5<sup>th</sup> percentile of profit (in Rupees) for the company

Ans: (541.9753988172804, 538.0246011827196)

- C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of making loss for distribution 1 and distribution 2 are 0.08850799143740207 and 0.06520335631899928. Therefore, the distribution 1 has a larger probability of making a loss in a given year.