Achievable virtual system.

Looking at force as input and disp. as output

$$H^{AVS} = H_{zw} + \underbrace{H_{zu}}_{\text{relative degree 4 (or more)}} \underbrace{H_{uw}}_{\text{relative degree 0 (highest posible for causality)}}$$

relative degree 4 (or more)

relative degree 0 (highest posible for causality)

$H_{zw}$: relative degree 2 (Newton's law)

$\therefore$ relative degree 2.

From force to velocity

$sH^{AVS}$ : relative degree 1 - ok for passivity

: $s=0$ is a zero - needed so that constant force does not produce a velocity.

Other requirements on $H^{AVS}$:

1. Has factor $(s^2 + 2\zeta\omega s + \omega^2)$ in the denominator
2. $H^{AVS}(0) = \frac{1}{K^{VS}}$

(1–2: close to $H^{DVS}$)

3. Relative degree of $(H^{AVS} - H_{zw})$ $\geq$ relative degree $H_{zu}$, so that relative degree of $H_{uw} \geq 0$
4. $sH^{AVS}$ is positive real.

ST model

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{m} \\ 0 & -k_a & -\beta \end{bmatrix} x + \begin{pmatrix} 0 \\ 0 \\ d \end{pmatrix} x^v + \begin{pmatrix} 0 \\ 1/m \\ 0 \end{pmatrix} w$$

From Mathematica,

$$H_{xx_v} = \frac{d}{\underbrace{s(ms^2 + m\beta s + K_a)}_{D(s)}} \qquad H_{xw} = \frac{(s+\beta)}{d} H_{xx_v}$$

$$H_{Fx_v} = ms^2 H_{xx_v} \qquad H_{Fw} = -\frac{k_a s}{d} H_{xxv}$$

$$\begin{cases} X = H_{xx_v} x_v + H_{xw} w \\ F = H_{Fx_v} x_v + H_{Fw} w \end{cases}$$

Valve dynamics: $x_v = H^v u_v$

Set $H_v = \frac{N^v}{D^v}$ strictly proper with $D^v$ monic

$$\begin{cases} X = H_{xx_v} H^v u_v + H_{xw} w \\ F = H_{Fx_v} H^v u_v + H_{Fw} w \end{cases}$$

Feedback: $u_v = K_e(u - x) - K_p \cdot F$

Solve with Mathematica.

$$H_{xu}^{CL} = \frac{dK_e N_v}{(ms^3 + ms^2\beta + K_a s)D_v + d(ms^2 K_p + K_e)N_v}$$

$$H_{xw}^{CL} = - \frac{D_v(s+\beta) + dK_p N_v}{\text{same denominator.}}$$

NOTE on this minus sign:

$$\text{Try } H^{AVS} = \text{Const} \frac{\overbrace{N_1(s)}^{= s^n + a_{n-1}s^{n-1} + \dots + a_0 \text{ — degree } \deg(D_v)+3 = n}}{\underbrace{(\text{actuator den. poly})}_{\text{order} = \text{degree}(D_v)+3} \underbrace{(s^2 + 2\zeta\omega s + \omega^2)}_{D_1(s)}}$$

Relative degree 2 ✓

From $H^{AVS} = H_{xw} + H_{xu} H_{ux}$

- Makes sense for act. den. poly. to be in $H^{AVS}$
- Controller $H_{ux}$ is going to be second order.

Notation: $H^{AVS} = \text{const} \cdot \frac{N_1}{DD_1}$ ; $H_{xw}^{CL} = \frac{N}{D}$

$$H^{AVS} - H_{xw}^{CL} = \frac{1}{\underbrace{DD_1}_{\text{degree}(D_v)+5}} \Big[ \text{const} \cdot N_1 - \underbrace{ND_1}_{\text{degree}(D_v)+3} \Big]$$

The highest $(\deg(D_v)+1)$ order terms of $(\text{const.}N_1 - ND_1)$ must be zero for $H_{uw}$ to be causal.

- const $=1$ for the highest power term to be 0
- let $D_v = s^{\square} + \alpha_{\square-1} s^{\square-1} + \cdots + \alpha_0$

$$N_v = \gamma_{\otimes} s^{\otimes} + \gamma_{\otimes-1} s^{\otimes-1} + \cdots + \gamma_0$$

with $\alpha_0 = \gamma_0$

without restriction let $\otimes = \square - 1$ some of the $\gamma$'s may be 0

$$N_1 - ND_1$$

$$= (s^n + a_{n-1}s^{n-1} + \cdots + a_0) - \Big[(s+\beta)(s^{n-3} + \alpha_{n-4}s^{n-4} + \cdots + \alpha_0) + dK_p(\gamma_{n-4}s^{n-4} + \cdots + \gamma_0)\Big](s^2 + 2\zeta\omega s + \omega^2)$$

$$= (s^n + a_{n-1}s^{n-1} + \cdots + a_0)$$

Let the conditions be met for the highest $(\text{degree}(D_v)+1) = (n-2)$ order terms to be zero

What remains?

$$(a_2 s^2 + a_1 s + a_0) - [$$

## Going off on a different path

Want to see if actuator can have force $\rightarrow$ displacement causality.

$$\dot{F} = -\beta F + d x_v - k_a \dot{x}$$
$$\dot{x}_v = -\alpha x_v + \alpha u_v$$

$$x_v = \frac{\alpha}{s+\alpha} u_v$$

$$(s+\beta)F = \frac{d\alpha}{s+\alpha} u_v - k_a s x$$

$$① \left[ F = \frac{d\alpha}{(s+\alpha)(s+\beta)} u_v - \frac{k_a s}{(s+\beta)} x \right.$$

alternatively, $② \left[ x = \frac{d\alpha}{k_a s(s+\alpha)} u_v - \frac{s+\beta}{k_a s} F \right.$

$$u_v = K_e(u - x) - K_p \cdot F$$

Then ① becomes

$$F = \frac{d\alpha}{(s+\alpha)(s+\beta)} K_e u - \frac{d\alpha}{(s+\alpha)(s+\beta)} K_e x - \frac{d\alpha}{(s+\alpha)(s+\beta)} K_p F - \frac{k_a s}{s+\beta} x$$

$$\left(1 + \frac{d\alpha}{(s+\alpha)(s+\beta)} K_p\right) F = \frac{d\alpha}{(s+\alpha)(s+\beta)} K_e u - \left[\frac{d\alpha K_e + K_a s(s+\alpha)}{(s+\alpha)(s+\beta)}\right] x$$

$$F = \frac{d\alpha K_e}{(s+\alpha)(s+\beta) + d\alpha K_p} u - \left[\frac{K_a s(s+\alpha) + d\alpha K_e}{(s+\alpha)(s+\beta) + d\alpha K_p}\right] x$$

(2) becomes

$$X = -\frac{(s+\alpha)(s+\beta) + d\alpha K_p}{K_a s(s+\alpha) + d\alpha K_e} F + \frac{d\alpha K_e}{K_a s(s+\alpha) + d\alpha K_e} u$$

$$F = -w \quad \left[\text{since } \cancel{m}\overset{0}{\ddot{x}} = F + w\right]$$

$$X = \frac{(s+\alpha)(s+\beta) + d\alpha K_p}{K_a s(s+\alpha) + d\alpha K_e} w + \frac{d\alpha K_e}{K_a s(s+\alpha) + d\alpha K_e} u.$$

mHiL control design with force → displacement causality for the actuator

$$\dot{F} = -\beta F - k_a \dot{x} + d x_v$$
$$\dot{x}_v = -\alpha x_v + \alpha u_v$$

Feedback: $u_v = u - K_x x - K_F F$

$$\dot{F} = -\beta F - k_a \dot{x} + d x_v$$
$$\dot{x}_v = -\alpha x_v + \alpha u - \alpha K_x x - \alpha K_F F$$

State transformation.

$z = F + k_a x$ ; then. $x = \dfrac{z - F}{k_a}$

$$\dot{z} = d x_v - \beta F$$
$$\dot{x}_v = -\alpha x_v + \alpha u - \alpha \frac{K_x}{k_a} z + \alpha\left(\frac{K_x}{k_a} - K_F\right) F$$

$$\begin{pmatrix} \dot{z} \\ \dot{x}_v \end{pmatrix} = \begin{bmatrix} 0 & d \\ -\alpha \frac{K_x}{k_a} & -\alpha \end{bmatrix} \begin{pmatrix} z \\ x_v \end{pmatrix} + \begin{bmatrix} -\beta \\ \alpha\left(\frac{K_x}{k_a} - K_F\right) \end{bmatrix} F + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u$$

eigenvalues: $\lambda^2 + \underbrace{\alpha}_{>0} \lambda + \alpha \frac{k_x}{k_a} d = 0$ so stable

Set $F = -w$

$$\begin{pmatrix}\dot{z}\\ \dot{x}_v\end{pmatrix} = \begin{bmatrix}0 & d\\ -\alpha\frac{K_x}{K_a} & -\alpha\end{bmatrix}\begin{pmatrix}z\\ x_v\end{pmatrix} + \begin{bmatrix}\beta\\ \alpha\left(K_F - \frac{K_x}{K_a}\right)\end{bmatrix}w + \begin{bmatrix}0\\ \alpha\end{bmatrix}u$$

$$y = x = \frac{z-F}{K_a} = \frac{z}{K_a} + \frac{w}{K_a}.$$

$$y = \begin{bmatrix}1/K_a & 0\end{bmatrix}\begin{pmatrix}z\\ x_v\end{pmatrix} + \frac{1}{K_a}w$$

$$u_v = u - K_x x + K_F w$$

$$= u - \frac{K_x}{K_a}z - \frac{K_x}{K_a}w + K_F w$$

$$= -\frac{K_x}{K_a}z + \left(K_F - \frac{K_x}{K_a}\right)w + u$$

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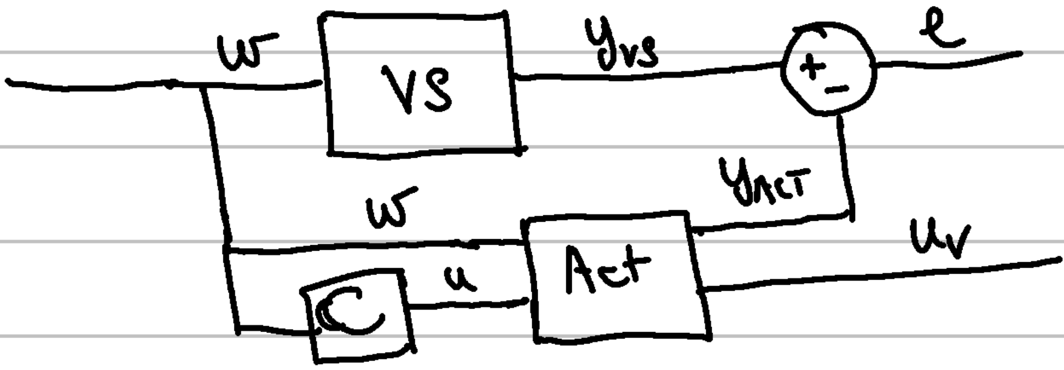
VS model:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{bmatrix}0 & 1\\ -k/m & -c/m\end{bmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{bmatrix}0\\ 1/m\end{bmatrix}w$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$



# PASSIVITY OF THE ACTUATOR.

$$\dot{x} = v$$
$$\dot{F} = -\beta F - k_a v + d x_v$$
$$\dot{x}_v = -\alpha x_v + \alpha u_v$$
$$u_v = u - K_x x - K_F F$$

$$X = \begin{pmatrix} x \\ F \\ x_v \end{pmatrix}$$

$$\dot{x} = v$$
$$\dot{F} = -\beta F + d x_v - k_a v$$
$$\dot{x}_v = -\alpha x_v + \alpha u - \alpha K_x x - \alpha K_F F$$
$$= -\alpha K_x x - \alpha K_F F - \alpha x_v + \cancel{\alpha u}.$$

$=0$ for passivity analysis

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\beta & d \\ -\alpha K_x & -\alpha K_F & -\alpha \end{bmatrix} \qquad B = \begin{pmatrix} 1 \\ -k_a \\ 0 \end{pmatrix} \text{ for input} = v$$

$$C = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \text{ for output} = -F$$

$$H = C(sI - A)^{-1} B$$

$$= \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} s & 0 & 0 \\ 0 & s+\beta & -d \\ \alpha K_x & \alpha K_F & s+\alpha \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ -k_a \\ 0 \end{pmatrix}$$

$$= \frac{\begin{bmatrix} 0 & -1 & 0 \end{bmatrix}}{s\left[(s+\alpha)(s+\beta) + d\alpha K_F\right]} \begin{bmatrix} \cdot & \cdot & \cdot \\ -d\alpha K_x & s(s+\alpha) & ds \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{pmatrix} 1 \\ -k_a \\ 0 \end{pmatrix}$$

What is the zero eigenvalue mode of A?

$$-\beta F + d x_v = 0$$
$$-\alpha K_x x - \alpha K_F F - \alpha x_v = 0$$

$$F = \frac{d}{\beta} x_v$$
$$x = -\frac{1}{K_x}\left[\frac{d K_F}{\beta} + 1\right] x_v$$

$$= \frac{1}{s}\,\frac{k_a s(s+\alpha) + d\alpha K_x}{(s+\alpha)(s+\beta) + d\alpha K_F}$$

$$= \frac{k_a}{s}\,\frac{\left(s^2+\alpha s + d\alpha \frac{K_x}{K_a}\right)}{\left(s^2+(\alpha+\beta)s+\alpha(\beta+dk_F)\right)}$$

$$= \frac{K_a}{s}\,\frac{\left(s^2+\alpha s+\alpha d\bar{K}_x\right)}{\left(s^2+(\alpha+\beta)s+\alpha(\beta+dK_F)\right)}$$

$$m_1 = s^2+\alpha d\bar{K}_x \qquad m_2 = (\alpha+\beta)s^2$$

$$n_1 = \alpha s \qquad n_2 = s^3+\alpha(\beta+dK_F)s$$

Passivity condition

$$\frac{m_1+n_1}{m_2+n_2} = \frac{(m_1+n_1)(n_2-n_2)}{m_2^2-n_2^2}$$

$$= \frac{m_1m_2-n_1n_2}{m_2^2-n_2^2} + \frac{m_2n_1-m_1n_2}{m_2^2-n_2^2}$$

$$m_1m_2 = (\alpha+\beta)s^4 + (\alpha+\beta)\alpha d\bar{K}_x s^2$$

$$n_1n_2 = \alpha s^4 + \alpha^2(\beta+dK_F)s^2$$

$$m_1m_2-n_1n_2 = \beta s^4 + \left[\alpha^2 d\bar{K}_x + \alpha\beta d\bar{K}_x - \alpha^2\beta - \alpha^2 dK_F\right]s^2$$

$$= \beta s^4 + \left[\alpha^2 d(\bar{K}_x-K_F) + \alpha\beta(d\bar{K}_x-\alpha)\right]s^2$$

$$p(x) = x\underbrace{\left(\beta x + \boxed{\phantom{x}}\right)}_{\text{need } \geq 0}$$

need $\alpha d(\bar{K}_x - K_F) + \beta(d\bar{K}_x - \alpha) \geq 0$

Going back to this version,

$$d\bar{K}_x(\alpha+\beta) - \alpha(\beta+dK_F) \geq 0$$

$$\frac{K_x}{K_a} \geq \frac{\alpha(\beta+dK_F)}{d(\alpha+\beta)}$$

Not clear what I wrote before. Start from scratch.

$$\begin{cases} \dot{F} = -\beta F - k_a \dot{x} + d x_v \\ \dot{x}_v = -\alpha x_v + \alpha u_v \end{cases}$$

$$\begin{cases} \dot{x} = -\frac{1}{k_a}\dot{F} - \frac{\beta}{k_a} F + \frac{d x_v}{k_a} \\ \dot{x}_v = -\alpha x_v + \alpha u_v \end{cases}$$

$$\left[ s x = -\frac{(s+\beta)}{k_a} F + \frac{d}{k_a} x_v = -\frac{(s+\beta)}{k_a} F + \frac{d\alpha}{k_a(s+\alpha)} u_v \right.$$

$$\begin{cases} x = -\frac{(s+\beta)}{k_a s} F + \frac{d\alpha}{k_a s(s+\alpha)} u_v \\ u_v = u - K_x x - K_F F \end{cases}$$

$$x = -\frac{(s+\beta)}{k_a s} F + \frac{d\alpha}{k_a s(s+\alpha)} u - \frac{K_x d\alpha}{k_a s(s+\alpha)} x - \frac{K_F d\alpha}{k_a s(s+\alpha)} F$$

$$\left[1 + \frac{K_x d\alpha}{k_a s(s+\alpha)}\right] x = -\left[\frac{(s+\alpha)(s+\beta) + K_F d\alpha}{k_a(s+\alpha)}\right] F + \frac{d\alpha}{k_a s(s+\alpha)} u$$

$$\star \quad x = -\frac{(s+\alpha)(s+\beta) + K_F d\alpha}{k_a s(s+\alpha) + K_x d\alpha} F + \frac{d\alpha}{k_a s(s+\alpha) + K_x d\alpha} u$$

NOTE: $F = -w$ from equilibrium.

$$x = \frac{s^2 + (\alpha+\beta)s + \alpha(\beta + K_F d)}{K_a\left(s^2 + \alpha s + \frac{K_x d\alpha}{K_a}\right)} w + \frac{d\alpha}{\cdots} u$$

$$x = H_{xu} u + H_{xw} w$$

$$= \frac{d\alpha}{K_a D(s)} u + \frac{N(s)}{K_a D(s)} w$$

Try $H_{xw}^{AVS}$ of the form

$$\frac{(s^2 + a_1 s + a_0)(s^2 + b_1 s + b_0)}{K_a\left(s^2 + \alpha s + \frac{K_x d\alpha}{K_a}\right)(s^2 + 2\zeta\omega s + \omega^2)}$$

① Want $H^{AVS}(0) = \frac{1}{K_{VS}}$ $\Rightarrow$ $\frac{a_0 b_0}{K_x d\alpha\omega^2} = \frac{1}{K_{VS}}$

$$\Rightarrow \boxed{a_0 b_0 = \frac{K_x d\alpha}{m_{VS}}}$$

② Want $H^{AVS} - H_{xw}$ to have relative degree 2.

Numerator of $H^{AVS} - H_{xw}$

$$= (s^2 + a_1 s + a_0)(s^2 + b_1 s + b_0) - (s^2 + (\alpha+\beta)s + \alpha(\beta + K_F d))(s^2 + 2\zeta\omega s + \omega^2)$$

$$= \left[(a_1 + b_1) - (2\zeta\omega + \alpha + \beta)\right] s^3 + \cdots$$

$$\boxed{a_1 + b_1 = 2\zeta\omega + \alpha + \beta}$$

③ Want $\mathbf{s}\cdot H^{\mathrm{Avs}}$ to be positive real.

Do $m_1 m_2 - n_1 n_2$ in Mathematica.

Now go the other way.
Inverting ✱

$$F = -\frac{k_a s(s+\alpha) + K_x d\alpha}{(s+\alpha)(s+\beta) + K_F d\alpha} x + \frac{d\alpha}{(s+\alpha)(s+\beta) + K_F d\alpha} u.$$

Set $w = -x$ (these signs must be considered/justified more carefully based on what is input and what is output)

Try $H^{AVS}_{Fw}$ to be of the form.

$$H^{AVS} = k_a \frac{(s^2 + 2\zeta\omega s + \omega^2)(s^2 + a_1 s + a_0)}{[s^2 + (\alpha+\beta)s + \alpha(\beta + dK_F)](s^2 + b_1 s + b_0)}$$

① Want $H^{AVS}(0) = k_{vs} \Rightarrow k_a \cdot \frac{\omega^2 a_0}{\alpha(\beta + dK_F) b_0} = k_{vs}$

$$\Rightarrow \boxed{b_0 = \frac{k_a}{m_{vs}\alpha(\beta + dK_F)} a_0}$$

② Want $H^{AVS} - H_{Fw}$ to have relative degree 2.

$$\text{Numerator} = (s^2 + 2\zeta\omega s + \omega^2)(s^2 + a_1 s + a_0) - \left(s^2 + \alpha s + \frac{dK_x\alpha}{k_a}\right)(s^2 + b_1 s + b_0)$$

$$= s^3 [a_1 + 2\zeta\omega - b_1 - \alpha] + \cdots$$

$$\Rightarrow \boxed{b_1 = a_1 + 2\zeta\omega - \alpha.}$$

Want $\frac{1}{s} H^{AVS}$ to be positive real.

Compute $m_1 m_2 - n_1 n_2$ using Mathematica.