CSE-512 HW-5

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Q1)

a) $y_i = x + z_i$ (x is constant)

$z_i = y_i - x$,

Given $z_i \sim N(0,1)$

$\Rightarrow$ As x is constant, $y_i - x$ will also be a $N(0,1)$.

$\therefore (y_i - x) \sim N(0,1)$

Normal distribution, $P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$(\mu = 0, \sigma = 1) \Rightarrow \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$.

Likelihood, $L(y_i) = \prod_{i=1}^{m} e^{\frac{-(y_i - x)^2}{2}}$

Taking log on both sides, we get,

$\log(L(y_i)) = \sum_{i=1}^{m} \frac{-(y_i - x)^2}{2}$

For maximum, equate derivative to 0.

$\frac{\partial}{\partial y}(\log(L(y_i))) = 0$

$\Rightarrow \frac{\partial}{\partial y}\left(\sum_{i=1}^{m} \frac{-(y_i - x)^2}{2}\right) = 0$

$\sum_{i=1}^{m} \frac{-2(y_i - x)}{2} = 0$

$-\left(\sum_{i=1}^{m} y_i - mx\right) = 0$

$$x_{MLE} = \frac{\sum_{i=1}^{m} y_i}{m}$$

Bias $= E[\hat{x}] - x$

$E[\hat{x}] = E\left[\frac{1}{m}\sum_{i=1}^{m} y_i\right]$

$= \frac{1}{m} E\left[\sum_{i=1}^{m} y_i\right]$

$= \frac{1}{m} \times m \times E[y_i]$

$= E[y_i]$ $\left\{ \begin{array}{l} y_i = x + 2i \\ y_i \sim N(x, 1) \end{array} \right\}$ → Shifted standard normal

$= x$

Bias $= x - x = 0$

Variance, $\hat{\sigma}^2$

It can also be written as

$\hat{\sigma}^2(\hat{\theta}) = \sigma^2\left(\frac{1}{m}\sum_{i=1}^{m} y_i\right)$

$= \frac{1}{m^2}\sigma^2\left(\sum_{i=1}^{m} y_i\right)$

$= \frac{1}{m^2} \times m \times 1 = \frac{1}{m}$

as $m \to \infty$, variance $= 0$

Bias :- 0

variance :- $\frac{1}{m}$

as $m \to \infty$

Bias: 0

variance: 0

(b) $\underset{x}{\text{minimize}} \; \frac{1}{m} \sum_{i=1}^{m} (y_i - x)^2 + \frac{\rho}{2} (x - \bar{x})^2$

For x_{MAP}, take derivative and equate to 0.

$$\Rightarrow \frac{1}{m} * 2 * \sum_{i=1}^{m} (y_i - x)(-1) + \frac{\rho}{2} \times 2 * (x - \bar{x})(1) = 0$$

$$\Rightarrow \frac{2}{m} \sum_{i=1}^{m} (y_i - x) = \rho (x - \bar{x})$$

$$\Rightarrow \frac{2}{m} \sum_{i=1}^{m} y_i - \frac{2}{m} \times m \times x = \rho x - \rho \bar{x}$$

$$\Rightarrow 2 \sum_{i=1}^{m} y_i - 2m x = \rho m x - \rho \bar{x} m$$

$$\boxed{x_{MAP} = \frac{2 \sum_{i=1}^{m} y_i + \rho m \bar{x}}{m(\rho + 2)}}$$

Bias, $\Rightarrow E[\hat{x}] - x$

$$= E\left[\frac{2 \sum_{i=1}^{m} y_i + \rho m \bar{x}}{m(\rho+2)} \right] - x$$

$$= \frac{\rho m \bar{x}}{m(\rho+2)} + \frac{2}{m(\rho+2)} E\left[\sum_{i=1}^{m} y_i \right] - x$$

$$= \frac{\rho m \bar{x}}{m(\rho+2)} + \frac{2}{m(\rho+2)} \times m \times E[y_i] - x$$

$$= \frac{\rho m\bar{x}}{m(\rho+2)} + \frac{2\mu}{\rho+2} - \mu$$

$$= \frac{\rho\bar{x} + 2\mu - \rho\mu - 2\mu}{\rho+2}$$

$$= \frac{\rho(\bar{x} - \mu)}{\rho+2}$$

$$\therefore \text{Bias} = \frac{\rho(\bar{x} - \mu)}{\rho+2}$$

Variance,

$$= \sigma^2 \left(\frac{2\Sigma y_i + \rho m\bar{x}}{(\rho+2)m} \right)$$

$$= \sigma^2 \left(\frac{\rho m\bar{x}}{(\rho+2)m} \right) + \sigma^2 \left(\frac{2\Sigma y_i}{(\rho+2)m} \right)$$

$$= 0 \left(\uparrow \text{constant}\right) + \frac{4}{(\rho+2)^2 m^2} \sigma^2(\Sigma y_i)$$

$$= \frac{4}{(\rho+2)^2 m^2} m \cdot 1 = \frac{4}{(\rho+2)^2 m}$$

Bias $= \frac{\beta(\bar{x} - x)}{\beta + 2}$

Variance: $\frac{4}{(\beta+2)^2 m}$

as $m \to \infty$

Bias $= \frac{\beta(\bar{x} - x)}{\beta + 2}$

Variance: 0

(c) $E[(x - \hat{x})^2] = E\left[\left(x - E(\hat{x}) + E(\hat{x}) - \hat{x}\right)^2\right]$

$= \left(x - E(\hat{x})\right)^2 - 2E\left[\left(x - E(\hat{x})\right)\left(E(\hat{x}) - \hat{x}\right)\right]$

$+ E\left[\left(E(\hat{x}) - \hat{x}\right)^2\right]$

$= \left(x - E(\hat{x})\right)^2 - 2\left(x - E(\hat{x})\right) E\left[E(\hat{x}) - \hat{x}\right]$

$+ E\left[\left(E(\hat{x}) - \hat{x}\right)^2\right]$

$E\left[E(\hat{x}) - \hat{x}\right] = 0$

$= \underbrace{\left(x - E(\hat{x})\right)^2}_{} + E\left[\left(E(\hat{x}) - \hat{x}\right)^2\right]$

$= B^2 + V$

Here, $B = x - E(\hat{x})$ → estimator bias

$$V = \left(E(\hat{x}) - \hat{x}\right)^2$$

(a) $\text{Bias}(B) = \dfrac{\rho(x-\bar{x})}{\rho+2}$, variance $(V) = \dfrac{4}{(\rho+2)^2 m}$

$$B^2 = \frac{\rho^2(x-\bar{x})^2}{(\rho+2)^2}, \quad V = \frac{4}{m(\rho+2)^2}$$

$$\frac{\partial}{\partial \rho}(B^2+V) = \frac{\partial}{\partial \rho}\left(\frac{\rho^2(x-\bar{x})^2}{(\rho+2)^2} + \frac{4}{m(\rho+2)^2}\right)$$

↑ Result from 1C

$$\Rightarrow \frac{(\rho+2)^2 2\rho(x-\bar{x})^2 - 2(\rho+2)\rho^2(x-\bar{x})^2}{(\rho+2)^4} + \frac{-4m2(\rho+2)}{m^2(\rho+2)^4}$$

$$\Rightarrow \frac{4\rho(x-\bar{x})^2}{(\rho+2)^3} - \frac{4}{m}\left(\frac{2}{(\rho+2)^3}\right) = 0$$

$$\Rightarrow \frac{4\rho D^2}{(\rho+2)^3} = \frac{4}{m}\left(\frac{2}{(\rho+2)^3}\right)$$

$$\boxed{\rho = \frac{2}{mD^2}}$$

```
In [1]: import numpy as np
        import pandas as pd
        from numpy import count_nonzero
        import matplotlib.pyplot as plt
        import scipy.sparse as ss
        from sklearn.metrics import accuracy_score
        from sklearn.metrics import f1_score
        from sklearn import tree
        from tabulate import tabulate
```

```
In [2]: #load data, in sparse format
        def load_data(filename):
            f = open('dorothea/%s.data' % filename)
            I,J = [],[]
            for k in range(100000):
                line = f.readline()
                if len(line) == 0:
                    print ('done')
                    break

                line = [int(x) for x in line.split()]
                J.extend(line)
                I.extend(k for i in range(len(line)))

            return ss.coo_matrix((np.ones(len(I)), (I,J)))


        def load_labels(filename):
            f = open('dorothea/%s.labels' % filename)
            y = []
            for k in range(100000):
                line = f.readline()

                if len(line) == 0:
                    print('done')
                    break
                y.append(int(line))
            return np.array(y)


        Xtrain = load_data('dorothea_train')
        ytrain = load_labels('dorothea_train')

        Xtest= load_data('dorothea_valid')
        ytest = load_labels('dorothea_valid')

        print(Xtrain.shape,  Xtest.shape, Xtrain.nnz)

        done
        done
        done
        done
        (800, 100001) (350, 100001) 727760
```

Q1) Sparsity

```
In [3]: xTrainNumpy = Xtrain.todense()
        print(ytrain.shape,  xTrainNumpy.shape, Xtrain.nnz)

        (800,) (800, 100001) 727760
```

```
In [4]: ss.coo_matrix(Xtrain)

Out[4]: <800x100001 sparse matrix of type '<class 'numpy.float64'>'
                with 727760 stored elements in COOrdinate format>
```

```
In [5]: xTrainNumpy = Xtrain.todense()
        sparsity = 1.0 - (Xtrain.nnz / float(xTrainNumpy.size) )
        print("Number of Zero Elements: "+ str(xTrainNumpy.size - count_nonzero(xTrainNumpy)))
        print("Number of Non Zero Elements: "+ str(Xtrain.nnz))
        print("Sparsity : % 0.3f" %(sparsity))

        Number of Zero Elements: 79273040
        Number of Non Zero Elements: 727760
        Sparsity :  0.991
```

```
In [6]: def get_class_balance(y):
            uniqueValues, counts = np.unique(y,return_counts=True)
            total = np.shape(y)[0]
            positiveValues = 0
            negativeValues = 0
            for index, value in enumerate(uniqueValues):
                if value == 1:
                    positiveValues = counts[index]
                else:
                    negativeValues = counts[index]
            assert(positiveValues+negativeValues == total), "Something wrong please check"

            return positiveValues/total, negativeValues/total

        print ('Training class balance is currently %.2f  +1, %.2f -1' % get_class_balance(ytrain))
        print ('Test class balance is currently %.2f  +1, %.2f -1' % get_class_balance(ytest))

        Training class balance is currently 0.10  +1, 0.90 -1
        Test class balance is currently 0.10  +1, 0.90 -1
```

Q2 comments on class_balance and sparsity

1) dorothea datset is highly imbalanced dataset with only 10% positives (rare events) and 90& normal events.

This is the same for both training and test sets.

2) The dataset is highly sparsed, it has 99.1 % of elements as zeros and only 0.9% non-zero elements.

3) The number of training examples in this dataset is almost of the same order as compared to covtype dataset (800 vs 468) but the number of features in dorothea dataset is 100,000 as compared to only 54 features in covtype dataset.

```
In [7]: def get_misclass(y,yhat):
            incorrect = 0
            totalSamples = len(y)
            for i in range(totalSamples):
                if y[i] != yhat[i]:
                    incorrect += 1
            return incorrect / totalSamples

        def get_f1(y,yhat):
            totalSamples = len(y)
            truePostives = 0
            trueNegatives = 0
            falsePositives = 0
            falseNegatives = 0

            for i in range(totalSamples):
                if y[i] == 1 and yhat[i] == 1:
                    truePostives +=1
                elif y[i] == -1 and yhat[i] == -1:
                    trueNegatives +=1
                elif y[i] == -1 and yhat[i] == 1:
                    falsePositives +=1
                elif y[i] == 1 and yhat[i] == -1:
                    falseNegatives +=1

            precision = truePostives/(truePostives + falsePositives)
            recall = truePostives/(truePostives + falseNegatives)

            assert((truePostives + trueNegatives + falsePositives + falseNegatives) == totalSamples),"something
         is wrong"
            return (2 * (precision* recall))/(precision + recall)

        depth = 3
        clf = tree.DecisionTreeClassifier(criterion='entropy',
                                          splitter='best',
                                          max_depth=depth,
                                          class_weight='balanced')
        clf = clf.fit(Xtrain, ytrain)

        print(get_misclass(ytrain, clf.predict(Xtrain) ), get_f1(ytrain, clf.predict(Xtrain)))

        0.04125 0.7724137931034484
```

Verification with in-built methods

```
In [8]: yPredTrain = clf.predict(Xtrain)
        yPredTest = clf.predict(Xtest)

        results = []
        results.append(("train",
                        get_misclass(ytrain, yPredTrain),
                        1 - accuracy_score(ytrain, yPredTrain),
                        get_f1(ytrain, yPredTrain),
                        f1_score(ytrain, yPredTrain)))

        results.append(("test",
                        get_misclass(ytest, yPredTest),
                        1 - accuracy_score(ytest, yPredTest),
                        get_f1(ytest, yPredTest),
                        f1_score(ytest, yPredTest)))

        columns = ['dataset',
                   'missclassification-score (custom)',
                   'missclassification-score (in-built)',
                   'F1-score (custom)',
                   'F1-score (in-built)',]

        pd.set_option("display.max_colwidth",2)
        df = pd.DataFrame(results, columns=columns)
        df.set_index('dataset')
        df
```

Out[8]:

	dataset	missclassification-score (custom)	missclassification-score (in-built)	F1-score (custom)	F1-score (in-built)
0	train	0.041250	0.041250	0.772414	0.772414
1	test	0.071429	0.071429	0.603175	0.603175

Comments about Misclassification score vs F1 Score

1) **F1 score** is a **better** metric than **missclassification score** for this task.

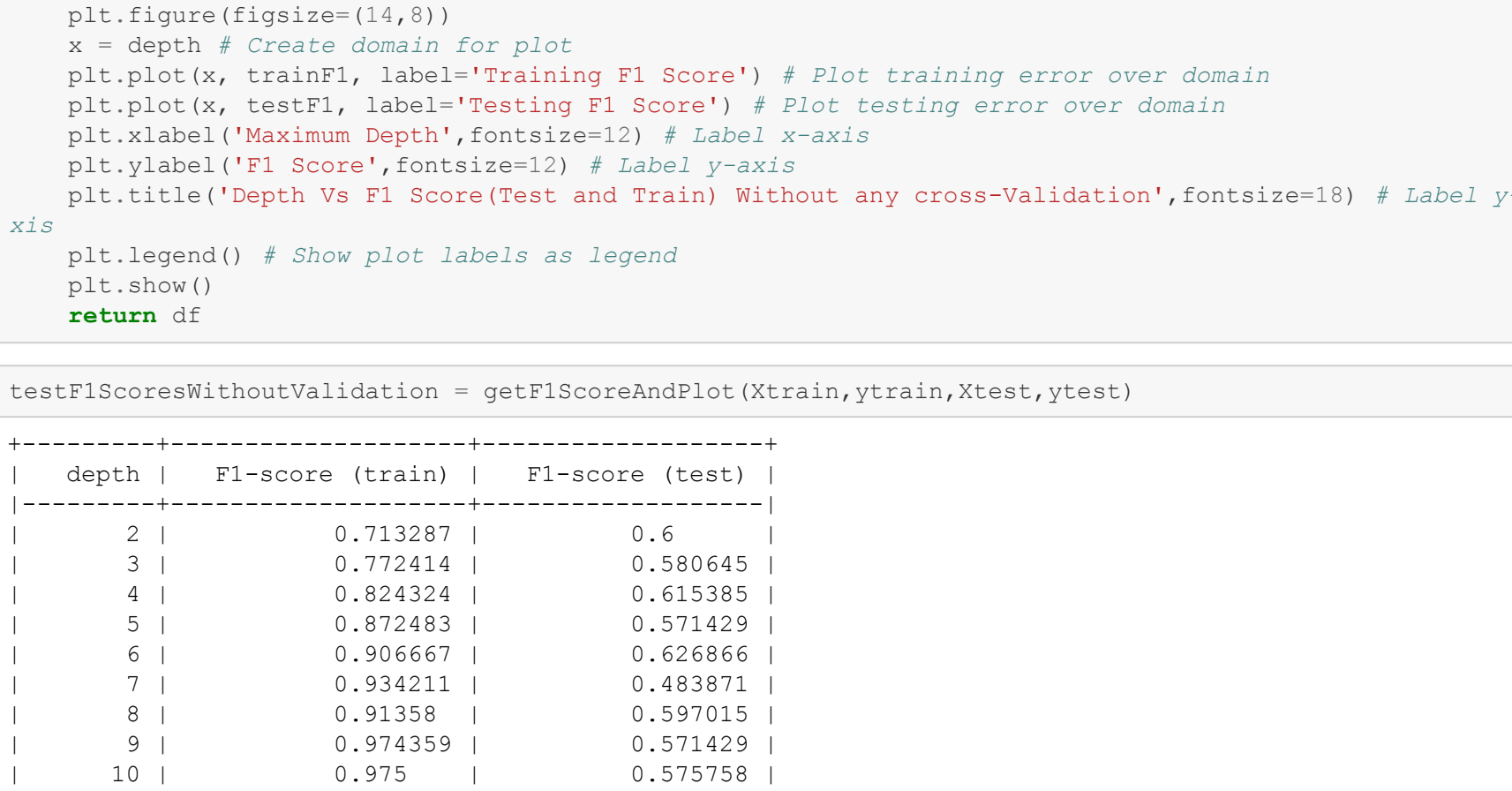
Reason: The classes are **highly imbalanced**, even if we say every event id not a rare-event, we would get a misclassification score of 0.1.

So, using F1 score is a better metric, as it gives weightage for both precision and recall and gives a good evaluation metric for our model,

Without any cross-validation

```
In [9]: def getF1ScoreAndPlot(Xtrain,ytrain,Xtest,ytest):
            depth   = list(range(2,11))
            results = []
            for d in depth:
                clf = tree.DecisionTreeClassifier(criterion='entropy',
                                                  splitter='best',
                                                  max_depth=d,
                                                  class_weight='balanced')
                clf = clf.fit(Xtrain, ytrain)
                yPredTrain = clf.predict(Xtrain)
                yPredTest = clf.predict(Xtest)
                results.append([d,
                               get_f1(ytrain, yPredTrain),
                               get_f1(ytest, yPredTest),
                               ])
            columns = ['depth',
                       'F1-score (train)',
                       'F1-score (test)',]

            pd.set_option("display.max_colwidth",2)
            df = pd.DataFrame(results, columns=columns)

            print(tabulate(df.set_index('depth'), headers='keys', tablefmt='psql'))

            trainF1 = [resultsAtAGivenDepth[1] for resultsAtAGivenDepth in results]
            testF1  = [resultsAtAGivenDepth[2] for resultsAtAGivenDepth in results]


            plt.figure(figsize=(14,8))
            x = depth # Create domain for plot
            plt.plot(x, trainF1, label='Training F1 Score') # Plot training error over domain
            plt.plot(x, testF1, label='Testing F1 Score') # Plot testing error over domain
            plt.xlabel('Maximum Depth',fontsize=12) # Label x-axis
            plt.ylabel('F1 Score',fontsize=12) # Label y-axis
            plt.title('Depth Vs F1 Score(Test and Train) Without any cross-Validation',fontsize=18) # Label y-a
        xis
            plt.legend() # Show plot labels as legend
            plt.show()
            return df
```

```
In [10]: testF1ScoresWithoutValidation = getF1ScoreAndPlot(Xtrain,ytrain,Xtest,ytest)

        +---------+--------------------+-------------------+
        |   depth |   F1-score (train) |   F1-score (test) |
        |---------+--------------------+-------------------|
        |       2 |           0.713287 |          0.6      |
        |       3 |           0.772414 |          0.580645 |
        |       4 |           0.824324 |          0.615385 |
        |       5 |           0.872483 |          0.571429 |
        |       6 |           0.906667 |          0.626866 |
        |       7 |           0.934211 |          0.483871 |
        |       8 |           0.91358  |          0.597015 |
        |       9 |           0.974359 |          0.571429 |
        |      10 |           0.975    |          0.575758 |
        +---------+--------------------+-------------------+
```



K-fold cross-validation (k=5)

```
In [11]: import random
         randomIndices = random.sample(range(800), 800)
         xSplit = []
         ySplit = []
         for i in range(5):
             randomRowIndices = randomIndices[(i*160):(i*160)+160]
             xSplit.append(xTrainNumpy[randomRowIndices])
             ySplit.append(ytrain[randomRowIndices])
```

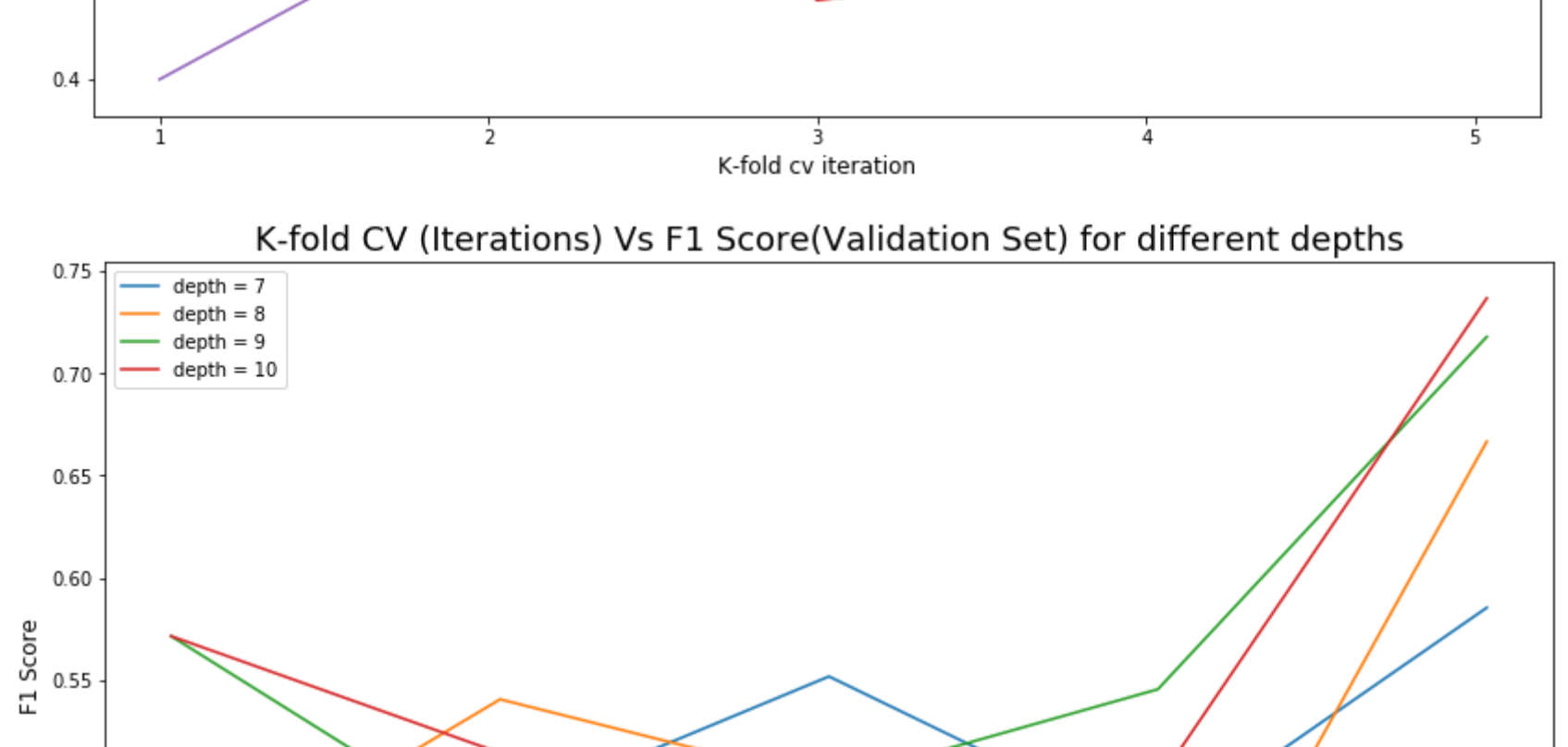
```
In [12]: def prepareTrainSetForKFoldCV(xTrain,yTrain,indexOfTheValidation):
             assert np.shape(xSplit) == (5,160,100001), "Check xTrain"
             assert np.shape(ySplit) == (5,160), "Check yTrain"
             xTrainWithoutValidationSet = []
             yTrainWithoutValidationSet = []
             for index in range(5):
                 if index!=i:
                     if len(xTrainWithoutValidationSet) == 0:
                         xTrainWithoutValidationSet = xTrain[index]
                         yTrainWithoutValidationSet = yTrain[index]
                     else:
                         xTrainWithoutValidationSet = np.concatenate([xTrainWithoutValidationSet,xTrain[index]])
                         yTrainWithoutValidationSet = np.concatenate([yTrainWithoutValidationSet,yTrain[index]])
             return xTrainWithoutValidationSet, yTrainWithoutValidationSet
```

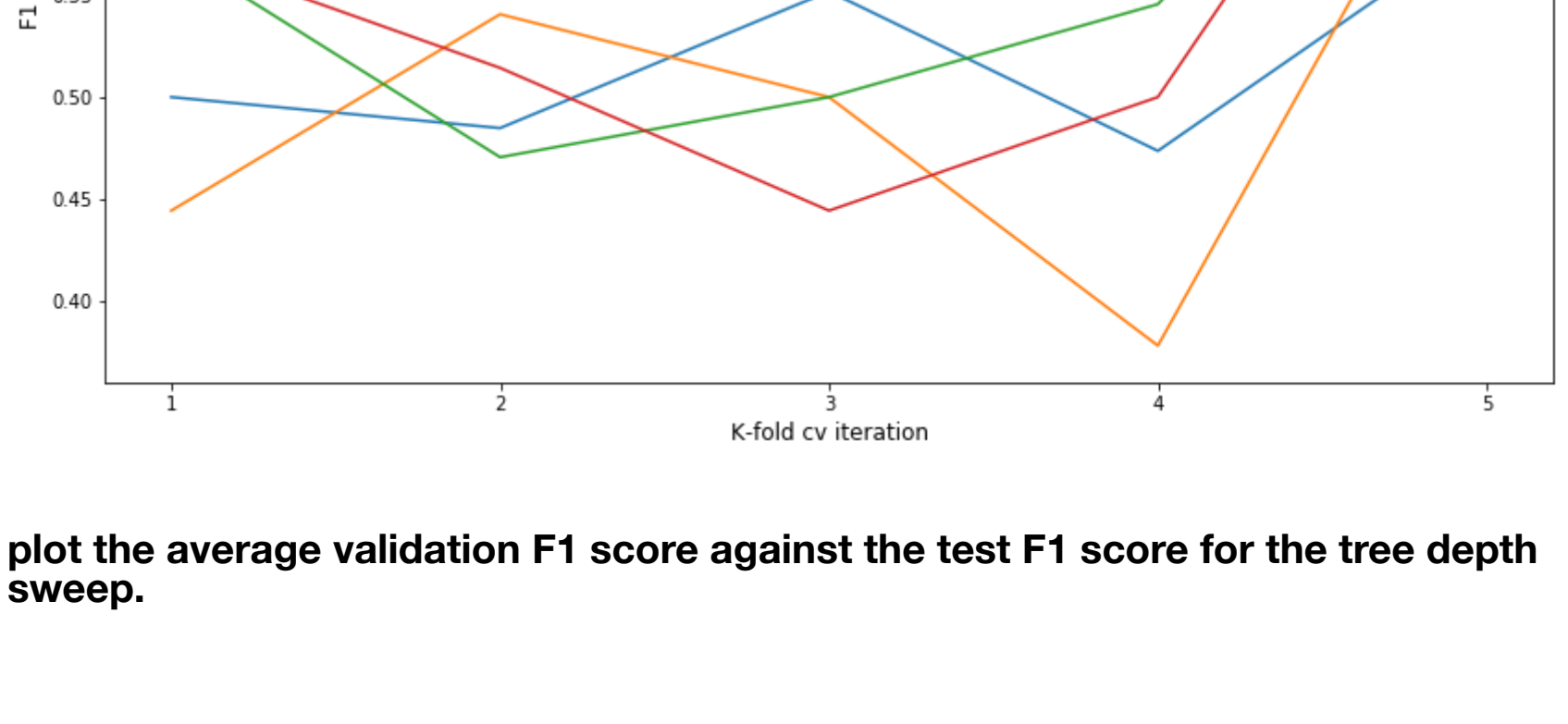
```
In [13]: def buildTreeAndgetF1Score(Xtrain,ytrain,
                                    XValidation,YValidation,
                                    i,d):

             clf = tree.DecisionTreeClassifier(criterion='entropy',
                                               splitter='best',
                                               max_depth=d,
                                               class_weight='balanced')
             clf = clf.fit(Xtrain, ytrain)
             yPredTrain      = clf.predict(Xtrain)
             yPredValidation = clf.predict(XValidation)
             return  get_f1(ytrain, yPredTrain), get_f1(YValidation, yPredValidation)
```

```
In [14]: depth   = list(range(2,11))
         allValidationF1Scores   = []
         averageValidationScores = []
         allResults = []
         for d in depth:
             results = []
             for i in range(5):
                 XTrain,yTrain = prepareTrainSetForKFoldCV(xSplit,ySplit,i)
                 XValidation = xSplit[i]
                 YValidation = ySplit[i]
                 trainF1Score, validationF1Score = buildTreeAndgetF1Score(XTrain,yTrain,
                                                                          XValidation,YValidation,

                                                                          i,d)
                 allValidationF1Scores.append(validationF1Score)
                 results.append([d,
                                 i+1,
                                 trainF1Score,
                                 validationF1Score])
                 allResults.append([d,
                                 i+1,
                                 trainF1Score,
                                 validationF1Score])
                 columns = ['depth',
                            'iteration in k-fold-cv',
                            'F1-score (train)',
                            'F1-score (validation)']
                 df = pd.DataFrame(results, columns=columns)
             averageValidationScores.append(df['F1-score (validation)'].mean())
             print(tabulate(df.set_index('depth'), headers='keys', tablefmt='psql'))

        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       2 |                        1 |           0.730435 |                0.571429 |
        |       2 |                        2 |           0.725664 |                0.545455 |
        |       2 |                        3 |           0.717949 |                0.608696 |
        |       2 |                        4 |           0.736842 |                0.62069  |
        |       2 |                        5 |           0.696429 |                0.774194 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       3 |                        1 |           0.793103 |                0.666667 |
        |       3 |                        2 |           0.810811 |                0.516129 |
        |       3 |                        3 |           0.789916 |                0.666667 |
        |       3 |                        4 |           0.803419 |                0.551724 |
        |       3 |                        5 |           0.739496 |                0.685714 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       4 |                        1 |           0.833333 |                0.666667 |
        |       4 |                        2 |           0.882883 |                0.606061 |
        |       4 |                        3 |           0.826446 |                0.64     |
        |       4 |                        4 |           0.845528 |                0.451613 |
        |       4 |                        5 |           0.806723 |                0.685714 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       5 |                        1 |           0.905983 |                0.62069  |
        |       5 |                        2 |           0.883333 |                0.588235 |
        |       5 |                        3 |           0.873016 |                0.444444 |
        |       5 |                        4 |           0.868217 |                0.466667 |
        |       5 |                        5 |           0.87931  |                0.810811 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       6 |                        1 |           0.918033 |                0.4      |
        |       6 |                        2 |           0.811594 |                0.5      |
        |       6 |                        3 |           0.926829 |                0.551724 |
        |       6 |                        4 |           0.929134 |                0.451613 |
        |       6 |                        5 |           0.915254 |                0.685714 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       7 |                        1 |           0.921875 |                0.5      |
        |       7 |                        2 |           0.966102 |                0.484848 |
        |       7 |                        3 |           0.943089 |                0.551724 |
        |       7 |                        4 |           0.885714 |                0.473684 |
        |       7 |                        5 |           0.897638 |                0.585366 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       8 |                        1 |           0.983871 |                0.444444 |
        |       8 |                        2 |           1        |                0.540541 |
        |       8 |                        3 |           0.818182 |                0.5      |
        |       8 |                        4 |           0.872483 |                0.378378 |
        |       8 |                        5 |           0.983333 |                0.666667 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |       9 |                        1 |           0.992    |                0.571429 |
        |       9 |                        2 |           1        |                0.470588 |
        |       9 |                        3 |           1        |                0.5      |
        |       9 |                        4 |           0.992248 |                0.545455 |
        |       9 |                        5 |           1        |                0.717949 |
        +---------+--------------------------+--------------------+-------------------------+
        +---------+--------------------------+--------------------+-------------------------+
        |   depth |   iteration in k-fold-cv |   F1-score (train) |   F1-score (validation) |
        |---------+--------------------------+--------------------+-------------------------|
        |      10 |                        1 |           1        |                0.571429 |
        |      10 |                        2 |           1        |                0.514286 |
        |      10 |                        3 |           0.977444 |                0.444444 |
        |      10 |                        4 |           1        |                0.5      |
        |      10 |                        5 |           1        |                0.736842 |
        +---------+--------------------------+--------------------+-------------------------+
```

Plot the F1 score over the validation set for all K trials.

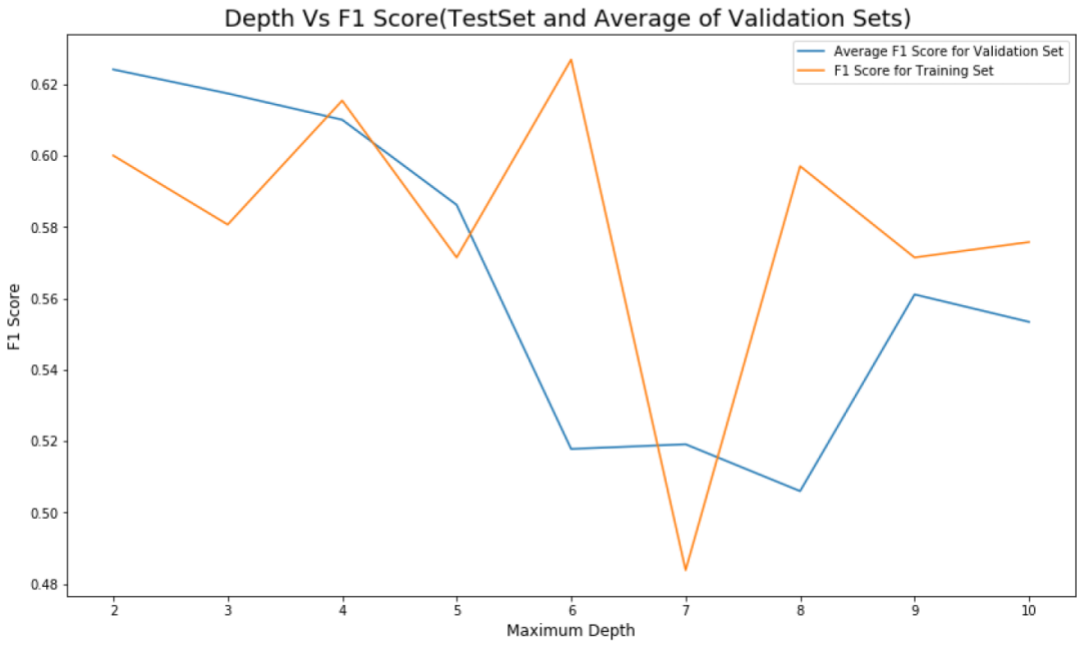
```
In [15]: validationScoresByDepth = [allValidationF1Scores[i:i + 5] for i in range(0, len(allValidationF1Scores),
          5)]

         plt.figure(figsize=(14,8))
         validation = [1,2,3,4,5]

         plt.plot(validation, validationScoresByDepth[0], label='depth = 2')
         plt.plot(validation, validationScoresByDepth[1], label='depth = 3')
         plt.plot(validation, validationScoresByDepth[2], label='depth = 4')
         plt.plot(validation, validationScoresByDepth[3], label='depth = 5')
         plt.plot(validation, validationScoresByDepth[4], label='depth = 6')
         plt.xlabel('K-fold cv iteration',fontsize=12) # Label x-axis
         plt.xticks(np.arange(1, 6, step=1))
         plt.ylabel('F1 Score',fontsize=12) # Label y-axis
         plt.title('K-fold CV (Iterations) Vs F1 Score(Validation Set) for different depths',fontsize=18) # Labe
         l y-axis
         plt.legend() # Show plot labels as legend
         plt.show()


         plt.figure(figsize=(14,8))
         validation = [1,2,3,4,5]
         plt.plot(validation, validationScoresByDepth[5], label='depth = 7')
         plt.plot(validation, validationScoresByDepth[6], label='depth = 8')
         plt.plot(validation, validationScoresByDepth[7], label='depth = 9')
         plt.plot(validation, validationScoresByDepth[8], label='depth = 10')
         plt.xlabel('K-fold cv iteration',fontsize=12) # Label x-axis
         plt.xticks(np.arange(1, 6, step=1))
         plt.ylabel('F1 Score',fontsize=12) # Label y-axis
         plt.title('K-fold CV (Iterations) Vs F1 Score(Validation Set) for different depths',fontsize=18) # Labe
         l y-axis
         plt.legend() # Show plot labels as legend
         plt.show()
```





plot the average validation F1 score against the test F1 score for the tree depth sweep.

In [16]:
```
plt.figure(figsize=(14,8))
depth = list(range(2,11))

plt.plot(depth, averageValidationScores, label='Average F1 Score for Validation Set') # Plot training e
rror over domain
testF1Scores = list(testF1ScoresWithoutValidation['F1-score (test)'])
plt.plot(depth, testF1Scores, label='F1 Score for Training Set') # Plot testing error over domain
plt.xlabel('Maximum Depth',fontsize=12) # Label x-axis
plt.ylabel('F1 Score',fontsize=12) # Label y-axis
plt.title('Depth Vs F1 Score(TestSet and Average of Validation Sets) ',fontsize=18) # Label y-axis
plt.legend() # Show plot labels as legend
plt.show()
```



Q3)

(a) $L(y, \hat{y}) = e^{-(y\hat{y})} = e^{-(y, H_t(x_i))}$

$\hat{y} = H_t(x_i)$

$$H_t(x) = \sum_{i=1}^{t} \alpha_i h_i(x)$$

$$= \sum_{i=1}^{t-1} \alpha_i h_i(x) + \alpha_t h_t(x)$$

$$H_t(x) = H_{t-1}(x) + \alpha_t h_t(x)$$

$$L(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^{n} e^{-y_i H(x_i)}$$

$$= \frac{1}{n} \sum_{i=1}^{n} e^{-y_i (H_{t-1} + \alpha_t h_t)}$$

$$= \frac{1}{n} \sum_{i=1}^{n} e^{-y_i H_{t-1}} \; e^{-y_i \alpha_t h_t}$$

$$= \frac{1}{n} \sum_{i=1}^{n} w_i^t \, e^{-y_i \alpha_t h_t}$$

To minimize loss, equate derivative to '0'.

$$\frac{\partial L(y, \hat{y})}{\partial \alpha_t} = \frac{\partial}{\partial \alpha} \left(\frac{1}{n} \sum_{i=1}^{n} w_i^t \, e^{-y_i \alpha_t h_t} \right) = 0$$

$$0 \Rightarrow \frac{-1}{n} \sum_{i=1}^{n} w_i^t y_i h_t e^{-y_i \alpha_t h_t}$$

$e^{x+y} = e^x e^y$

$$\Rightarrow \frac{-1}{n} \left[\sum_{i: h(x_i) = y_i}^{n} w_i^t e^{-\alpha_t} - \sum_{i: h(x_i) \neq y_i} w_i^t e^{\alpha_t} \right]$$

$y_i * h_t = 1$; $y_i * h_t = -1$

$$\Rightarrow -\frac{1}{n} \sum_{i: h(x_i) = y_i}^{n} w_i^t e^{-\alpha_t} + \frac{1}{n} \sum_{i: h(x_i) \neq y_i}^{n} w_i^t e^{\alpha_t}$$

$$\varepsilon_t = \sum_{i: h(x_i) \neq y_i}^{n} w_i^t \qquad 1 - \varepsilon_t = \sum_{i: h(x_i) = y_i}^{n} w_i^t$$

$$\Rightarrow 0 = (1 - \varepsilon_t) e^{-\alpha_t} - \varepsilon_t e^{\alpha_t}$$

$$\Rightarrow (1 - \varepsilon_t) e^{-\alpha_t} = \varepsilon_t e^{\alpha_t}$$

$$\Rightarrow \frac{1 - \varepsilon_t}{e^{\alpha_t}} = \varepsilon_t e^{\alpha_t}$$

$$\Rightarrow \frac{1 - \varepsilon_t}{\varepsilon_t} = e^{2\alpha_t}$$

$$\Rightarrow \boxed{\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_t}{\varepsilon_t} \right)}$$

In this we proved that the α for a given iteration is $\frac{1}{2} \ln \left(\frac{1 - \varepsilon_t}{\varepsilon_t} \right)$ and we proved that this α_t minimizes the loss function.

So, an update, i.e. at the next step the empirical risk will indeed be reduced.

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
from sklearn import tree
import scipy.io as sio
import pandas as pd
from numpy import count_nonzero
import matplotlib.pyplot as plt
import scipy.sparse as ss
from sklearn.metrics import accuracy_score
from sklearn.metrics import f1_score
from sklearn import tree
from tabulate import tabulate
```

```
In [2]: data = sio.loadmat('mnist.mat')

Xtrain = data['trainX'][:10000, :].astype(int)
Xtest = data['testX'].astype(int)
ytrain = data['trainY'][0, :10000].astype(int)
ytest = data['testY'][0, :].astype(int)

idx = np.logical_or(np.equal(ytrain, 4), np.equal(ytrain, 9))
Xtrain = Xtrain[idx, :]
ytrain = ytrain[idx]
ytrain[np.equal(ytrain, 4)] = 1
ytrain[np.equal(ytrain, 9)] = -1

idx = np.logical_or(np.equal(ytest, 4), np.equal(ytest, 9))
Xtest = Xtest[idx, :]
ytest = ytest[idx]
ytest[np.equal(ytest, 4)] = 1
ytest[np.equal(ytest, 9)] = -1

sio.savemat('mnist_binary_small.mat', {'Xtrain': Xtrain, 'ytrain': ytrain, 'Xtest': Xtest, 'ytest': yte
st})

data = sio.loadmat('mnist_binary_small.mat')

print(Xtrain.shape, Xtest.shape, ytrain.shape, ytest.shape)
```

```
(1958, 784) (1991, 784) (1958,) (1991,)
```

Helper Functions

```
In [3]: def get_misclass(y,yhat):
    incorrect = 0
    totalSamples = len(y)
    for i in range(totalSamples):
        if y[i] != yhat[i]:
            incorrect += 1
    return incorrect / totalSamples

def getExponentialLoss(y, yhat):
    return np.sum(np.exp(-y * yhat)) / len(y)


def plot(y, yLabel, title):
    x = list(range(len(y)))
    plt.figure(figsize=(14, 8))
    plt.plot(x, y)
    plt.xlabel('Iterations', fontsize=12)
    plt.ylabel(yLabel, fontsize=12)
    plt.title(title , fontsize=18)
    plt.show()
```

Without AdaBoost

```
In [4]: clf = tree.DecisionTreeClassifier(criterion='entropy',
                                  splitter='best',
                                  max_depth=1,
                                  class_weight='balanced')
w = [1/len(Xtrain)]*len(Xtrain)
clf = clf.fit(Xtrain, ytrain, sample_weight = w)

yPredTrain = clf.predict(Xtrain)
yPredTest = clf.predict(Xtest)
results = []
results.append([1,
               get_misclass(ytrain, yPredTrain),
               get_misclass(ytest, yPredTest),
               ])
columns = ['depth',
           'misclassification rate (train)',
           'misclassification rate (test)',]

pd.set_option("display.max_colwidth",2)
df = pd.DataFrame(results, columns=columns)

print("Train andf test misclassification rates")
print(tabulate(df.set_index('depth'), headers='keys', tablefmt='psql'))

print("\n Train Exponential Loss Value: "+str(getExponentialLoss(ytrain, yPredTrain)))
```

```
Train andf test misclassification rates
+---------+---------------------------------+--------------------------------+
|   depth |   misclassification rate (train) |   misclassification rate (test) |
|---------+---------------------------------+--------------------------------|
|       1 |                        0.109806 |                       0.152687 |
+---------+---------------------------------+--------------------------------+

 Train Exponential Loss Value: 0.6259675480492947
```

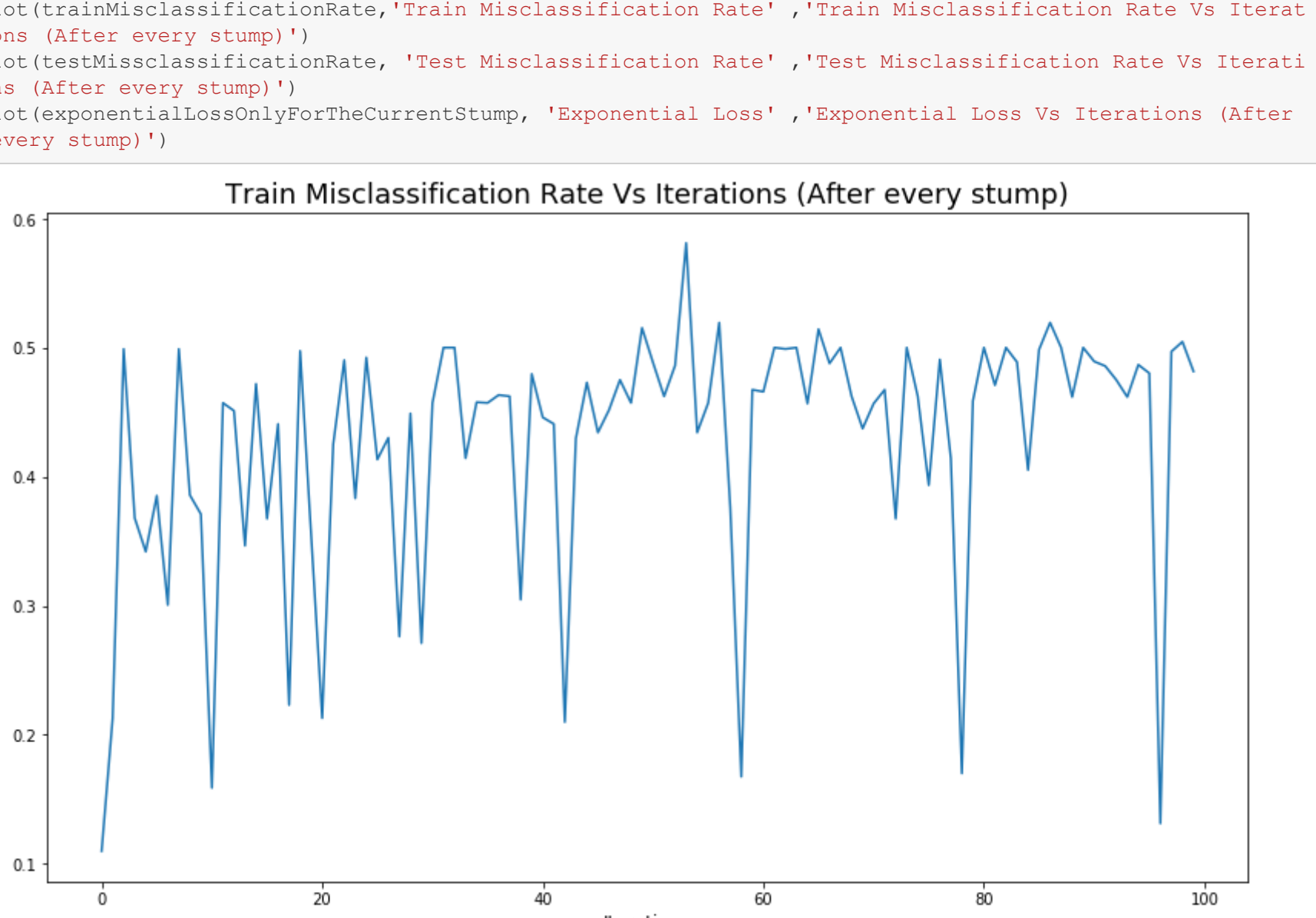
With AdaBoost

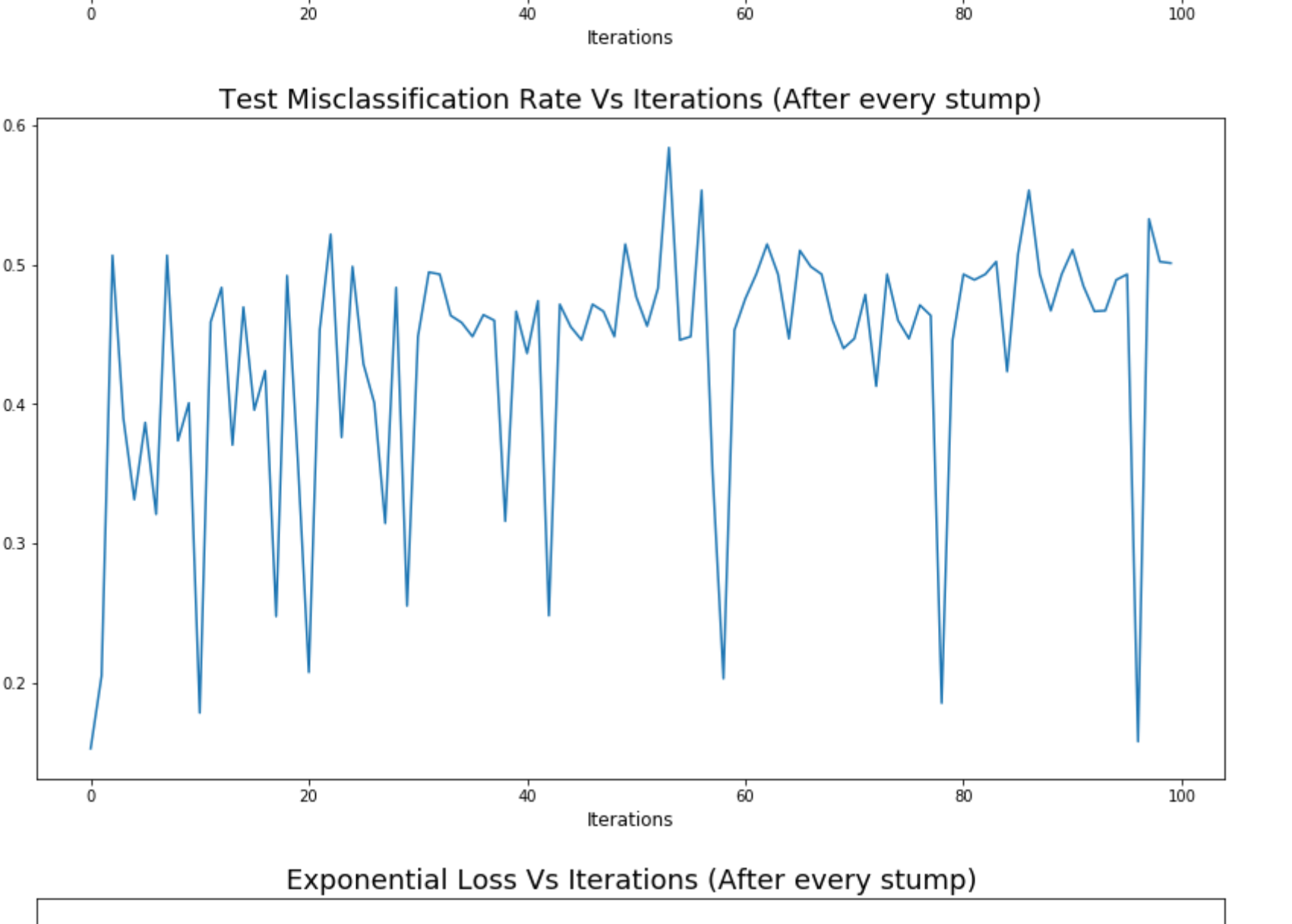
```
In [5]: def predictAdaboost(X, stumps, stump_weights):
    # Stumps will be of size = (iterations, ) -> For every iteration we get 1 stump
    # Stump weights will be of size = (iterations, ) -> For every iteration we get 1 stump

    allStumpPredictions = []
    for i in range(len(stumps)):
        allStumpPredictions.append((stumps[i].predict(X)))

    weightedStumpPredictions = allStumpPredictions * stump_weights[:, None]
    sumOfWeightedStumpPredictions = weightedStumpPredictions.sum(axis=0)
    finalStumpPrediction = np.sign(sumOfWeightedStumpPredictions)

    return finalStumpPrediction


def getErrorAndAlpha(stumpWeight, stumpPrediction, y):
    error = np.sum(stumpWeight[(stumpPrediction != y)])
    alpha = 1 / 2 * np.log((1 - error) / error)
    return error, alpha


def fitAdaboost(XTrain, yTrain, XTest, yTest, iterations):
    # Total samples
    numberOfSamples = XTrain.shape[0]

    # initialization
    exponentialLossOnlyForTheCurrentStump = np.zeros(shape=iterations)
    exponentialLossConsideringAllStumps = np.zeros(shape=iterations)
    trainMisclassificationRate = np.zeros(shape=iterations)
    testMissclassificationRate = np.zeros(shape=iterations)
    trainMisclassificationRateContinuous = np.zeros(shape=iterations)
    testMissclassificationRateContinuous = np.zeros(shape=iterations)
    weights = np.zeros(shape=(iterations, numberOfSamples))
    stumps = np.zeros(shape=iterations, dtype=object)
    stump_weights = np.zeros(shape=iterations)
    errors = np.zeros(shape=iterations)
    alphas = np.zeros(shape=iterations)

    # initialize weights uniformly
    # Initial weights will be 1/n
    # n -> number of samples
    weights[0] = np.ones(shape=numberOfSamples) / numberOfSamples

    for index in range(iterations):
        # We will create new stump for every iteration
        currentWeights = weights[index]
        stump = tree.DecisionTreeClassifier(criterion='entropy',
                                            splitter='best',
                                            max_depth=1,
                                            max_leaf_nodes=2,
                                            class_weight='balanced')
        stump = stump.fit(XTrain, yTrain, sample_weight=currentWeights)

        # Stump Predictions for Test Set and Train Set
        stumpTrainPrediction = stump.predict(XTrain)
        stumpTestPrediction = stump.predict(XTest)

        # Stump Train and Test Misclassification rates
        trainMisclassificationRate[index] = get_misclass(yTrain, stumpTrainPrediction)
        testMissclassificationRate[index] = get_misclass(yTest, stumpTestPrediction)
        exponentialLossOnlyForTheCurrentStump[index] = getExponentialLoss(stumpTrainPrediction, yTrain)

        error, alpha = getErrorAndAlpha(currentWeights, stumpTrainPrediction, yTrain)

        updatedSampleWeights = currentWeights * np.exp(-alpha * yTrain * stumpTrainPrediction)
        # Normalizing new sample weights
        updatedSampleWeights /= updatedSampleWeights.sum()

        # If not final iteration, update sample weights for t+1
        if index != iterations - 1:
            weights[index + 1] = updatedSampleWeights

        stumps[index] = stump
        stump_weights[index] = alpha

        # All the required results for plotting
        cummilative_stump_train_prediction = predictAdaboost(XTrain, stumps[:index + 1], stump_weights
[:index + 1])
        cummilative_stump_test_prediction = predictAdaboost(XTest, stumps[:index + 1], stump_weights[:i
ndex + 1])

        errors[index] = error
        alphas[index] = alpha
        exponentialLossConsideringAllStumps[index] = getExponentialLoss(cummilative_stump_train_predict
ion, ytrain)
        trainMisclassificationRateContinuous[index] = get_misclass(yTrain, cummilative_stump_train_pred
iction)
        testMissclassificationRateContinuous[index] = get_misclass(yTest, cummilative_stump_test_predic
tion)

    output = [stumps,
              stump_weights,
              exponentialLossOnlyForTheCurrentStump,
              trainMisclassificationRate,
              testMissclassificationRate,
              exponentialLossConsideringAllStumps,
              trainMisclassificationRateContinuous,
              testMissclassificationRateContinuous,
              errors,
              alphas]

    return output
```

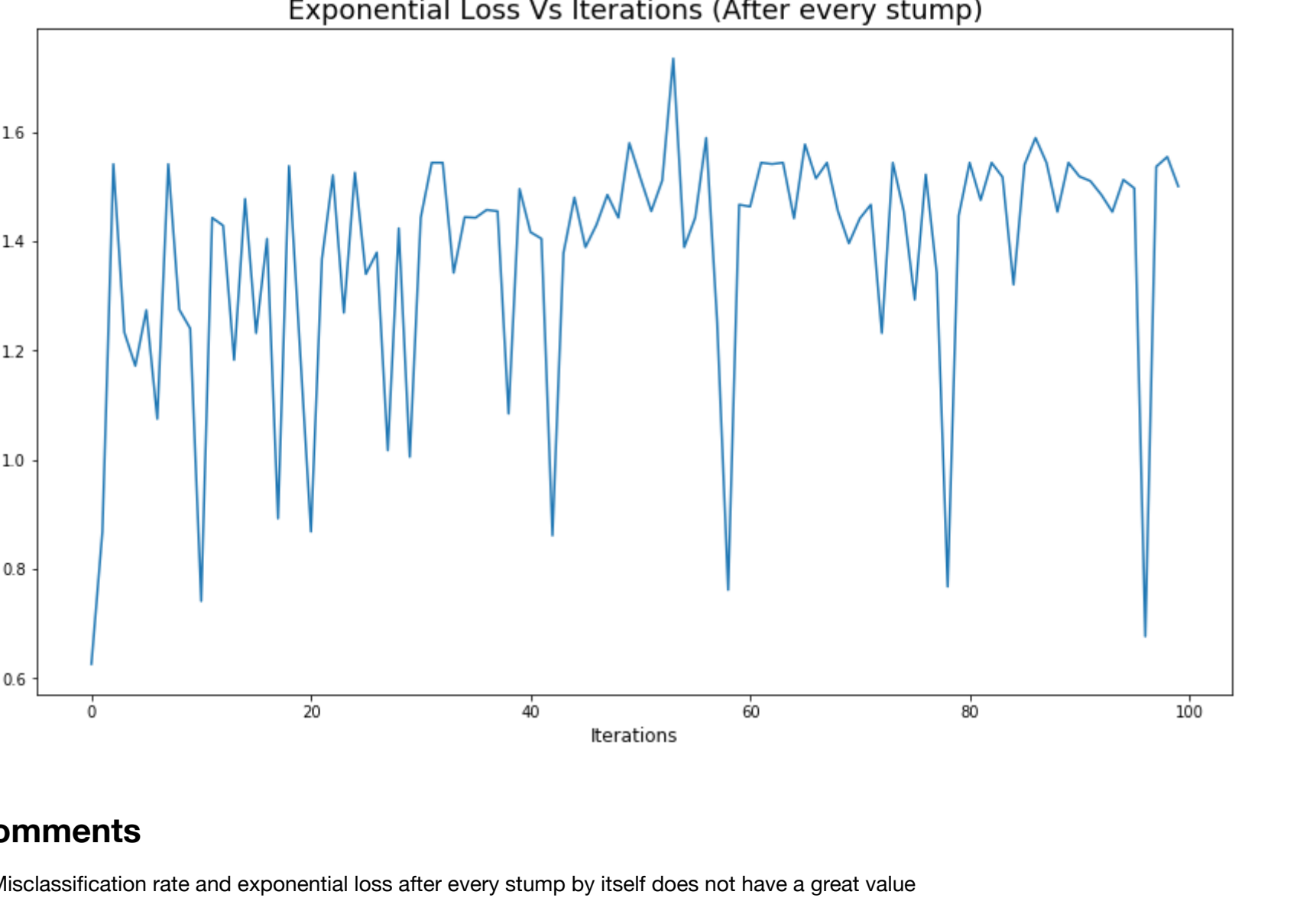
```
In [6]: iterations = 100
modelParameters = fitAdaboost(Xtrain, ytrain, Xtest, ytest, iterations)
stumps = modelParameters[0]
stump_weights = modelParameters[1]
exponentialLossOnlyForTheCurrentStump = modelParameters[2]
trainMisclassificationRate = modelParameters[3]
testMissclassificationRate = modelParameters[4]
exponentialLossConsideringAllStumps = modelParameters[5]
trainMisclassificationRateContinuous = modelParameters[6]
testMissclassificationRateContinuous = modelParameters[7]
errors = modelParameters[8]
alphas = modelParameters[9]
yPredTrain = predictAdaboost(Xtrain, stumps, stump_weights)
yPredTest = predictAdaboost(Xtest, stumps, stump_weights)
results = [[get_misclass(ytrain, yPredTrain),
            get_misclass(ytest, yPredTest),
            ]]
columns = ['misclassification rate (train)',
           'misclassification rate (test)', ]

pd.set_option("display.max_colwidth", 2)
df = pd.DataFrame(results, columns=columns)

print("Train andf test misclassification rates after " + str(iterations) + " iterations")
print(tabulate(df, headers='keys', tablefmt='psql'))

print("\n Train Exponential Loss Value after " + str(iterations) + " iterations: " + str(
    getExponentialLoss(ytrain, yPredTrain)))
```

```
Train andf test misclassification rates after 100 iterations
+----+---------------------------------+--------------------------------+
|    |   misclassification rate (train) |   misclassification rate (test) |
|----+---------------------------------+--------------------------------|
|  0 |                        0.014811 |                       0.048217 |
+----+---------------------------------+--------------------------------+

 Train Exponential Loss Value after 100 iterations: 0.4026913253549667
```

```
In [7]: plot(trainMisclassificationRate,'Train Misclassification Rate' ,'Train Misclassification Rate Vs Iterat
ions (After every stump)')
plot(testMissclassificationRate, 'Test Misclassification Rate' ,'Test Misclassification Rate Vs Iterati
ons (After every stump)')
plot(exponentialLossOnlyForTheCurrentStump, 'Exponential Loss' ,'Exponential Loss Vs Iterations (After
 every stump)')
```





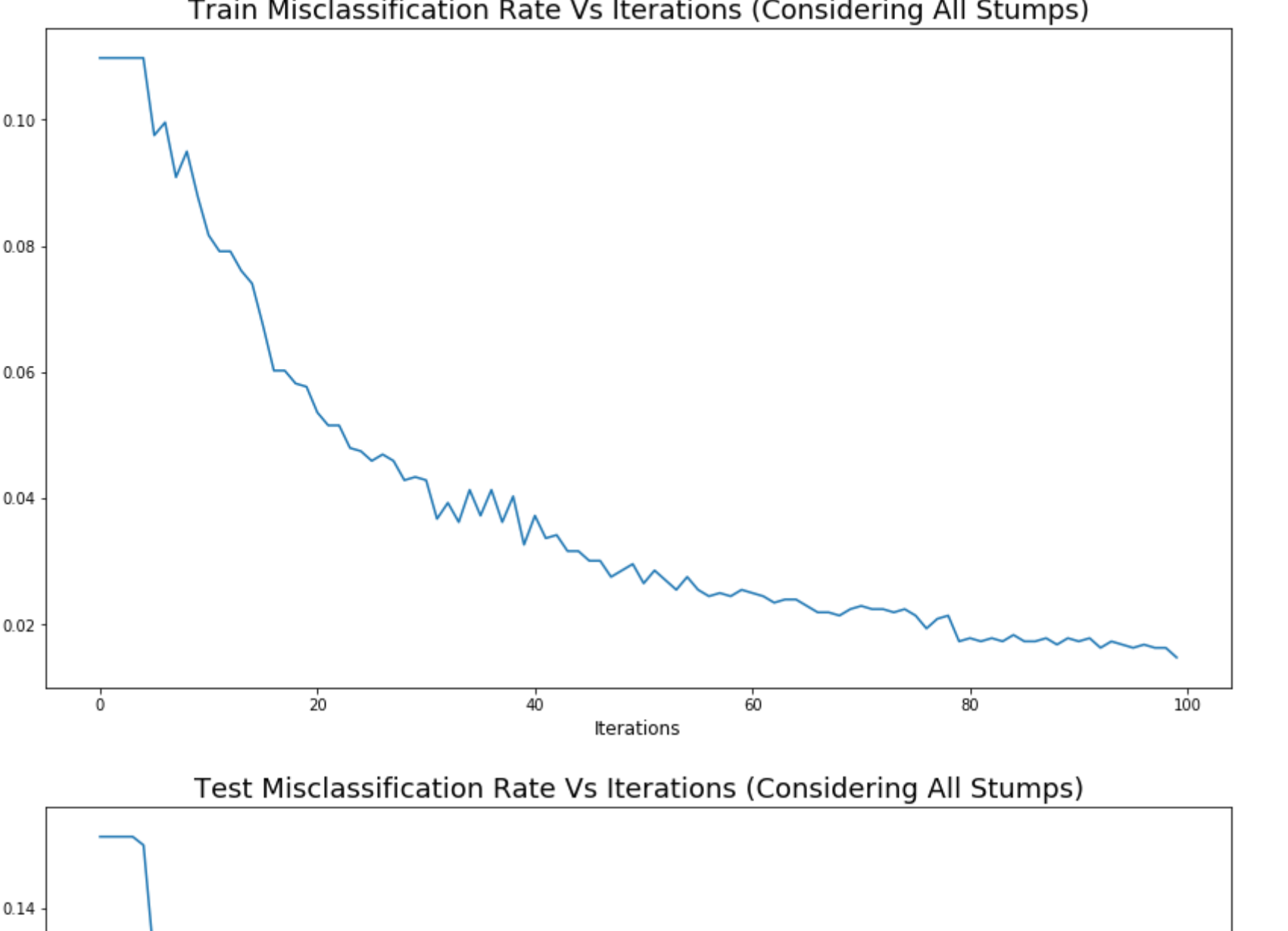


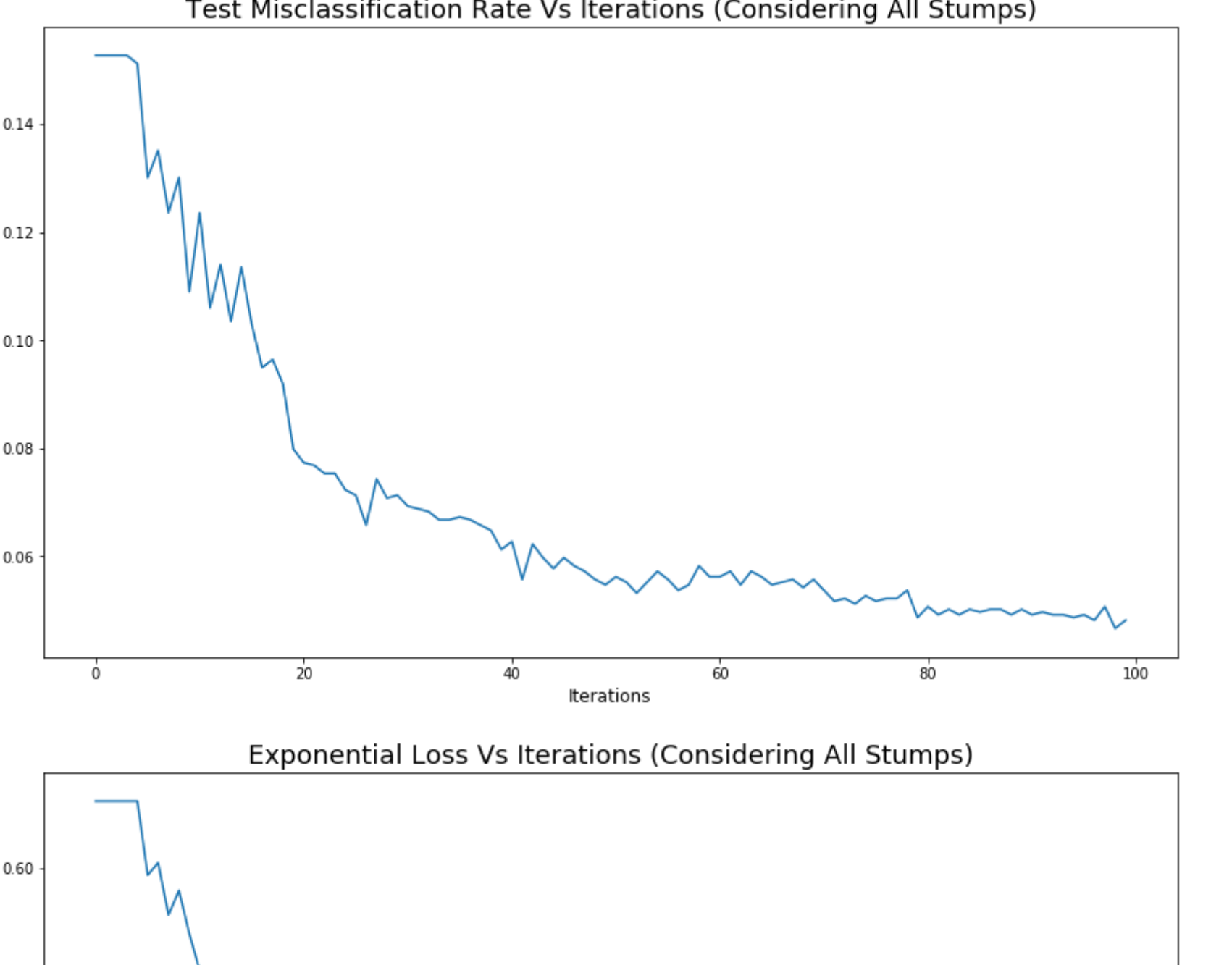
Comments

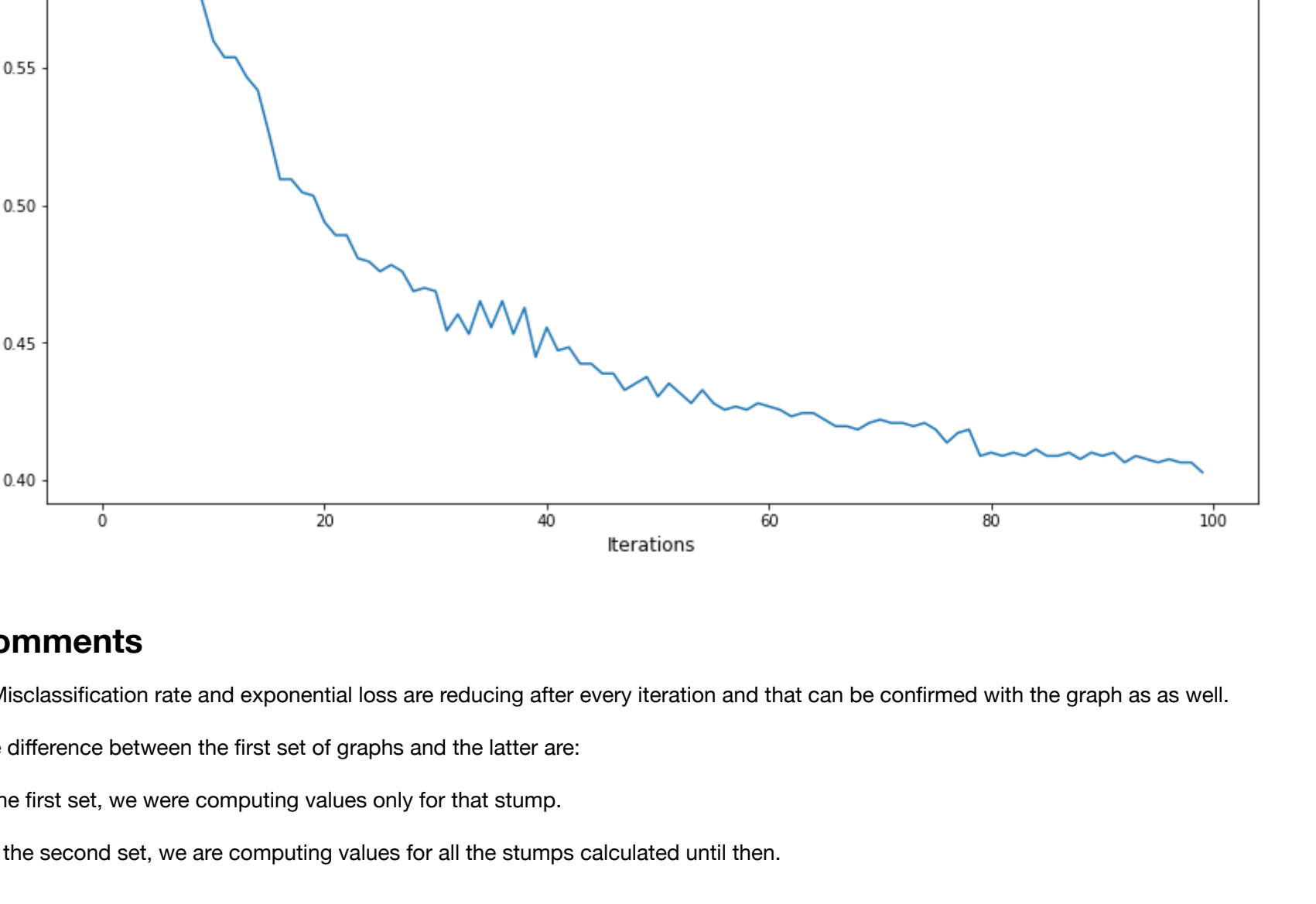
1) Misclassification rate and exponential loss after every stump by itself does not have a great value

by themselves and is quiet evident from the graph as each stump is a weak learner.

```
In [8]: plot(trainMisclassificationRateContinuous, 'Train Misclassification Rate' , 'Train Misclassification Ra
te Vs Iterations (Considering All Stumps)')
plot(testMissclassificationRateContinuous, 'Test Misclassification Rate'  , 'Test Misclassification Rat
e Vs Iterations (Considering All Stumps)')
plot(exponentialLossConsideringAllStumps , 'Exponential Loss'            , 'Exponential Loss Vs Iterat
ions (Considering All Stumps)')
```







Comments

1) Misclassification rate and exponential loss are reducing after every iteration and that can be confirmed with the graph as as well.

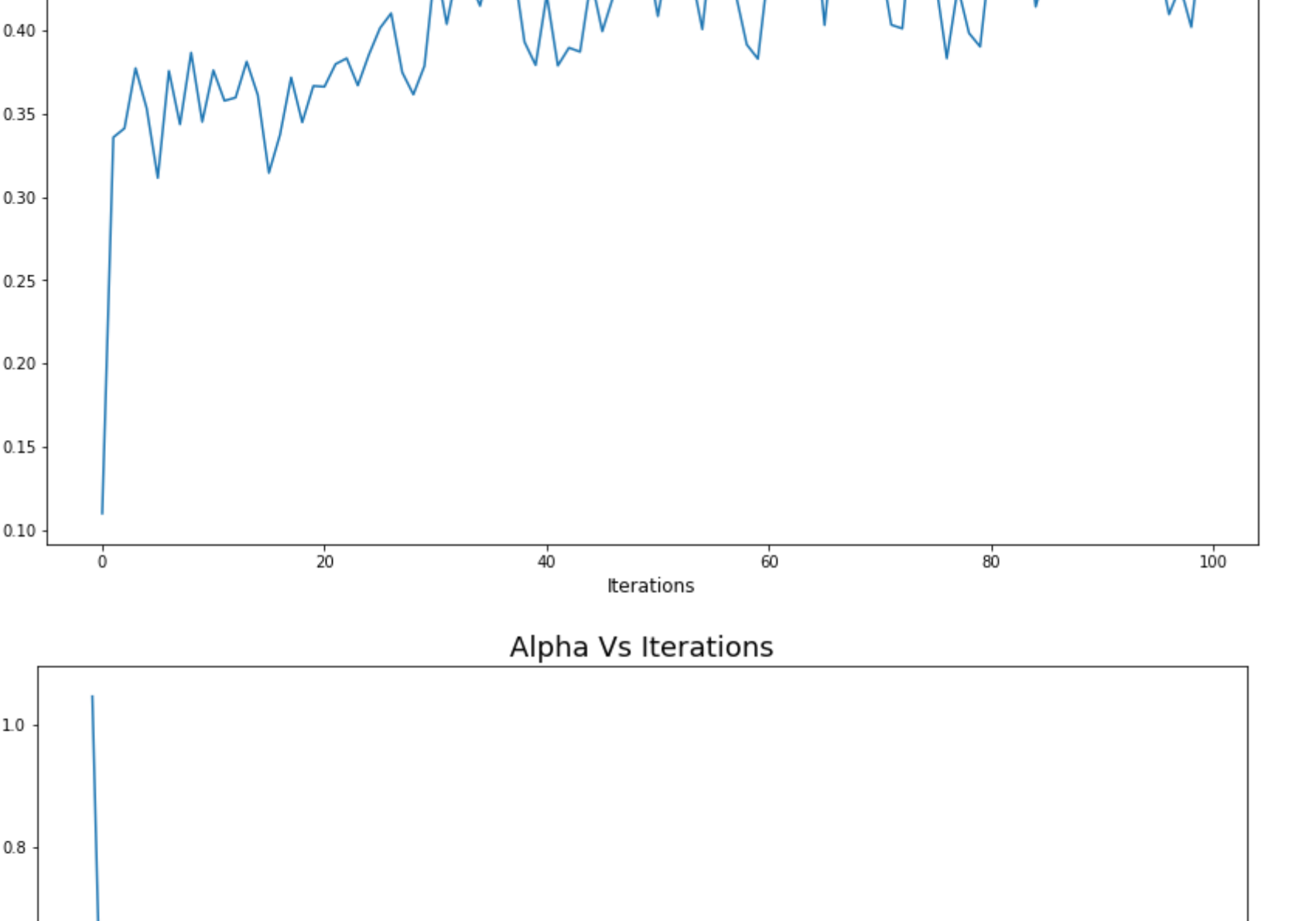
The difference between the first set of graphs and the latter are:

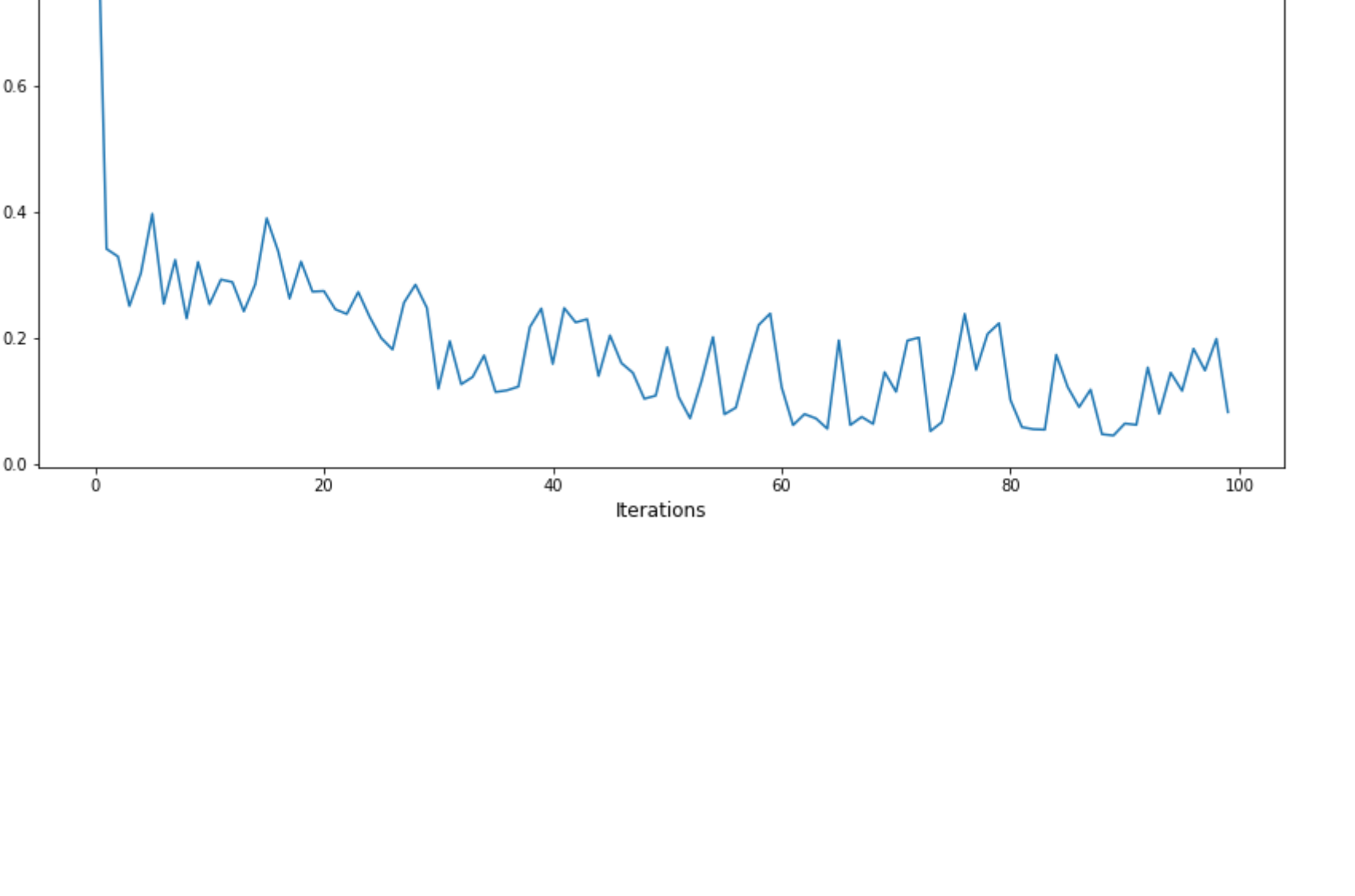
In the first set, we were computing values only for that stump.

For the second set, we are computing values for all the stumps calculated until then.

$\epsilon^{(t)}$, $\alpha^{(t)}$ as a function of t

```
In [9]: plot(errors , 'Error' , 'Error Vs Iterations')
plot(alphas , 'Alpha' , 'Alpha Vs Iterations')
```





Comments

1) Total error takes a value between 0 and 1.

0-> very good stump

1-> very bad stump

2) Alpha,or the amount of say a stump has is inversely proportional to the error.

More, the error, less alpha and vice-versa.

When total error is small, i.e. if a stump is good then alpha will be a large positive value.

Interpretations (in terms of weighted performance):

1) If a stump has a high weight and high alpha value, it's say in the final prediction would be high.

2) If a stump has a low alpha value, it's say would be very less.

3) If error is 0.5 (i.e. just a random guess), it's alpha value would be 0, i.e that stump would mean nothing in the final prediction.

Challenge

Q1) $H(X) = -P_X(x=\text{red}) \log_2 P_X(x=\text{red}) - P_X(x=\text{black}) \log_2 P_X(x=\text{black})$

(a) For maximum entropy, $H'(X) = 0$

we know that,

$P_X(x=\text{red}) + P_X(x=\text{black}) = 1$

$P_X(x=\text{black}) = 1 - P_X(x=\text{red})$

$H(X) = -P(x=\text{red}) \log_2 P_X(x=\text{red}) - (1 - P_X(x=\text{red})) \log_2 (1 - P_X(x=\text{red}))$

$$H'(x) = -\log P - \frac{P}{P} - \left(-\log(1-P) - \frac{(1-P)}{(1-P)} \right) = 0$$

$$\Rightarrow -\log P - 1 + \log(1-P) + 1 = 0$$

$$\Rightarrow \log\left(\frac{1-P}{P}\right) = 0$$

$1 - P = P \Rightarrow 2P = 1 \Rightarrow P = 1/2$

To maximize entropy, buy 5 red socks and 5 black socks

(b) $\max_P f(P) := -P \log_2(P) - (C-P) \log_2(C-P)$

$$f'(P) = -\log P - \frac{P}{P} - \left[-\log(C-P) + \frac{(C-P)}{C-P}(-1) \right] = 0$$

$$\Rightarrow -\log P - 1 + \log(C-P) + 1 = 0$$

$$\Rightarrow \log\left(\frac{c-p}{p}\right) = 0$$

$$\Rightarrow c-p = p \quad \Rightarrow \quad c = 2p \qquad p = c/2$$

$\therefore p = c/2$ for optimum value

(c) Assume we have 3 different colours,

$P(\text{colour } 1) = x$

$P(\text{colour } 2) = y$

$P(\text{colour } 3) = z = 1-x-y$

$$H(x,y) = -x\log x - y\log y - (1-x-y)\log(1-x-y)$$

$\frac{\partial H(x,y)}{\partial(x)} = 0$, $\frac{\partial H(x,y)}{\partial(y)} = 0$ } for maximum entropy

$\frac{\partial H(x,y)}{\partial(x)} = 0$, we get $2x + y = 1$ — (1)

$\frac{\partial H(x,y)}{\partial(y)} = 0$, we get $2y + x = 1$ — (2)

Solving (1) & (2), we get $x = \frac{1}{3}$, $y = \frac{1}{3}$, $z = \frac{1}{3}$

we can extend this to n-colours,

where $P(i) = \frac{1}{n} \quad (i = 1 \dots n)$

Part (a) of the question is when $n = 2$

Here, $n = 100$

$$P(i) = \frac{1}{100}$$

Total socks = 10,000

$\therefore$ we need to buy $\frac{10,000}{100} = 100$ socks

of each color.

Q2) $Pr(Y_i=1) = p$,

$Pr(Y_i=1 \mid Y_{i-1}=1) = c+p-cp$

$Pr(Y_i=-1 \mid Y_{i-1}=-1) = cp-p+1$

(a) we will prove this with induction.

for $k=1$:- $Pr(Y_1=1) = p$ → given

for $k=n-1$:- $Pr(Y_{n-1}=1) = p$ → Assumption

for $k=n$:- $Pr(Y_n=1) = Pr(Y_n=1 \mid Y_{n-1}=1)\, Pr(Y_{n-1}=1)$

$+ Pr(Y_n=1 \mid Y_{n-1}=-1)\, Pr(Y_{n-1}=-1)$

$= (c+p-cp)(p) + (1-(cp-p+1))(1-p)$

$= (c+p-cp)(p) + (p-cp)(1-p)$

$= p$

$\therefore Pr(Y_i=1) = p$ for all i

(b) $corr(U,V) = \dfrac{E[UV] - E[U]\,E[V]}{\sqrt{var(U)\, var(V)}}$

(Y_i, Y_{i-1})	$Y_i * Y_{i-1}$	$P(Y_i, Y_{i-1}) = P(Y_i \mid Y_{i-1})\, P(Y_{i-1})$
$(1, 1)$	1	$(1-p)\,(1-p)$
$(1, -1)$	-1	$p\,(1-p)$
$(-1, 1)$	-1	$(1-(1-p))\,(1-p)$
$(-1, -1)$	1	$(1-p)\,(1-(1-p))$

$$E[UV] = \sum_{i=1}^{n} (u_i v_i)\, P_X(u_i, v_i)$$

$$= 4p^2 - 4p^2(1-p) + 4p(1-p) - 4p$$

$$E[U] = 1 \cdot p - 1\,(1-p) = 2p - 1$$

$$E[U] = E[V] = 2p - 1$$

$$E[UV] - E[U]\,E[V] = 4p(1-p)$$

$$E[U^2] = 1^2 \cdot p + (-1)^2 (1-p) = p + 1 - p = 1$$

$$\mathrm{var}(U) = E[U^2] - E(U)^2 = 1 - (2p-1)^2 = 4p(1-p)$$

$$\mathrm{var}(U) = \mathrm{var}(V) = 4p(1-p)$$

$$\text{corr}(U,V) = \frac{4PC(1-P)}{\sqrt{4P(1-P) * 4P(1-P)}} = C$$

$$\therefore \text{corr}(U,V) = C$$

c) For , $m=3$, majority votes (atleast 2 1's) are possible in the following scenario

Y_1	Y_2	Y_3
1	1	1
1	1	-1
1	-1	1
-1	1	1

$P_r(Y_i = 1 \mid Y_{i-1} = 1) = P_{11}$ $\quad$ $P_r(Y_i = -1 \mid Y_{i-1} = -1) = P_{00}$

$P_r(Y_i = -1 \mid Y_{i-1} = 1) = 1 - P_{11}$ $\quad$ $P_r(Y_i = 1 \mid Y_{i-1} = -1) = 1 - P_{00}$

$$= P * P_{11} * P_{11}$$
$$+ P * P_{11} * (1-P_{11})$$
$$+ P * (1-P_{11}) * (1-P_{00})$$

$$+ (1-p) * (1-p_{00}) * p_{11}$$

$$= p * \cancel{(p_{11})^2} + p * p_{11} - p * \cancel{(p_{11})^2}$$

$$+ p - p p_{00} - \cancel{p * p_{11}} + p * p_{11} * p_{00}$$

$$+ p_{11} - p_{00} p_{11} - p p_{11} + p p_{00} p_{11}$$

$$= p - p p_{00} + 2(p p_{00} p_{11}) + p_{11} - p_{00} p_{11}$$

$$- p p_{11}$$

$$= p + p_{11} - p p_{11} - p_{00} p_{11} - p p_{00} + 2 (p p_{00} p_{11})$$

Challenge Q2

In [1]:
```
import numpy as np
import random
import itertools
from collections import Counter
from tabulate import tabulate
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

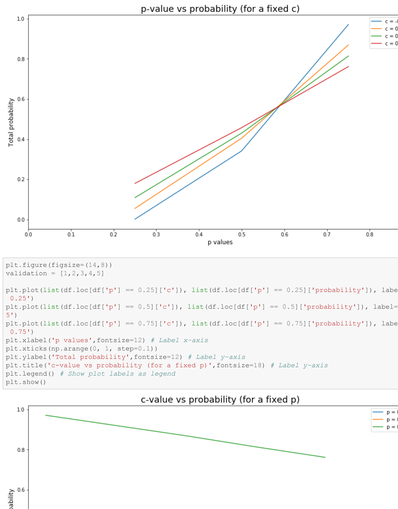
probability that I will buy a yacht, for m = 10

In [2]:
```
def getProbabilityFromPreviousConsultant(current, previous, p, c):
    if current == 1 and previous == 1: ## 11 -> 1
        return c + p -(c*p)
    elif current == -1 and previous == 1: ## 1 -1 ->0
        return 1 - (c + p -(c*p))
    elif current == -1 and previous == -1: #-1 -1 >1
        return (c*p) -p + 1
    elif current == 1 and previous == -1:
        return p -(c*p)
```

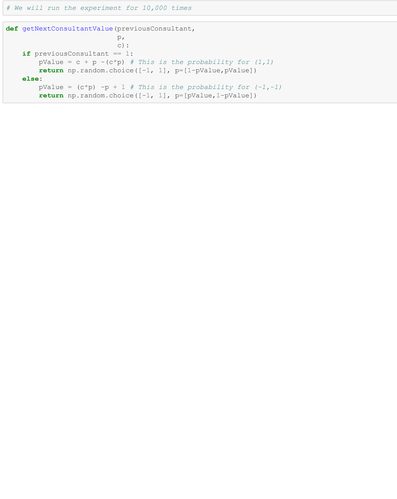
In [3]:
```
m = 10
allPermutations = list(itertools.product([-1, 1], repeat=m))

cpValues = [
    [0,0],
    [-0.25, 0.25],
    [0.25,0.25],
    [0.5,0.25],
    [0.75,0.25],
    [1,0.25],
    [-0.25, 0.5],
    [0.25,0.5],
    [0.5,0.5],
    [0.75,0.5],
    [1,0.5],
    [-0.25, 0.75],
    [0.25,0.75],
    [0.5,0.75],
    [0.75,0.75],
    [1,1],
]
results = []
for cpValue in cpValues:
    c,p = cpValue[0], cpValue[1]
    ## checking if probabilities lie in the range 0-1
    if 0<=c + p -(c*p)<=1 and 0<=1-(c + p -(c*p))<=1 and 0<=(c*p) -p + 1<=1 and 0<=p -(c*p)<=1:

        total = 0
        for permutation in allPermutations: #(1,1,1,1)
            frequencyMap = Counter(permutation)
            totalProbabilityForThisPermutation = 0
            if frequencyMap[1]>(m/2):
                for index in range(len(permutation)):
                    # For 1st advisor
                    if index == 0:
                        if permutation[index] == 1:
                            totalProbabilityForThisPermutation += p
                            continue
                        else:
                            totalProbabilityForThisPermutation += (1-p)
                            continue
                    # For the rest
                    totalProbabilityForThisPermutation *= getProbabilityFromPreviousConsultant(permutat
ion[index],
                                                                                               permutat
ion[index-1],
                                                                                               p,c)
            total += (totalProbabilityForThisPermutation)
        results.append([c,
                        p,
                        m,
                        total])
columns = ['c',
           'p',
           'm',
           'probability']
df = pd.DataFrame(results, columns=columns)
print(tabulate(df.set_index('c'), headers='keys', tablefmt='psql'))
```

```
+------+-----+---+---------------+
|    c |   p | m |   probability |
|------+-----+---+---------------|
|  0   | 0   | 10 |  0           |
| -0.25 | 0.25 | 10 |  0.00194145 |
|  0.25 | 0.25 | 10 |  0.0553765  |
|  0.5 | 0.25 | 10 |  0.108958   |
|  0.75 | 0.25 | 10 |  0.179941   |
|  1   | 0.25 | 10 |  0.25       |
| -0.25 | 0.5 | 10 |  0.34174    |
|  0.25 | 0.5 | 10 |  0.405044   |
|  0.5 | 0.5 | 10 |  0.430332   |
|  0.75 | 0.5 | 10 |  0.457382   |
|  1   | 0.5 | 10 |  0.5        |
| -0.25 | 0.75 | 10 |  0.970681   |
|  0.25 | 0.75 | 10 |  0.869105   |
|  0.5 | 0.75 | 10 |  0.813797   |
|  0.75 | 0.75 | 10 |  0.76134    |
|  1   | 1   | 10 |  1          |
+------+-----+---+---------------+
```

In [4]:
```
plt.figure(figsize=(14,8))
validation = [1,2,3,4,5]

plt.plot(list(df.loc[df['c'] == -0.25]['p']), list(df.loc[df['c'] == -0.25]['probability']), label='c =
 -0.25')
plt.plot(list(df.loc[df['c'] == 0.25]['p']), list(df.loc[df['c'] == 0.25]['probability']), label='c =
 0.25')
plt.plot(list(df.loc[df['c'] == 0.5]['p']), list(df.loc[df['c'] == 0.5]['probability']), label='c = 0.
5')
plt.plot(list(df.loc[df['c'] == 0.75]['p']), list(df.loc[df['c'] == 0.75]['probability']), label='c =
 0.75')
plt.xlabel('p values',fontsize=12) # Label x-axis
plt.xticks(np.arange(0, 1, step=0.1))
plt.ylabel('Total probability',fontsize=12) # Label y-axis
plt.title('p-value vs probability (for a fixed c)',fontsize=18) # Label y-axis
plt.legend() # Show plot labels as legend
plt.show()
```



In [5]:
```
plt.figure(figsize=(14,8))
validation = [1,2,3,4,5]

plt.plot(list(df.loc[df['p'] == 0.25]['c']), list(df.loc[df['p'] == 0.25]['probability']), label='p =
 0.25')
plt.plot(list(df.loc[df['p'] == 0.5]['c']), list(df.loc[df['p'] == 0.5]['probability']), label='p = 0.
5')
plt.plot(list(df.loc[df['p'] == 0.75]['c']), list(df.loc[df['p'] == 0.75]['probability']), label='p =
 0.75')
plt.xlabel('p values',fontsize=12) # Label x-axis
plt.xticks(np.arange(0, 1, step=0.1))
plt.ylabel('Total probability',fontsize=12) # Label y-axis
plt.title('c-value vs probability (for a fixed p)',fontsize=18) # Label y-axis
plt.legend() # Show plot labels as legend
plt.show()
```



Comments

We have generated all the 2^{10} = 1024 possibilities.

We have only considered cases where there > 5(10/2) 1's.

Probability increases as p increases for a fixed c

Simulate the sequential advisers,

In [6]:
```
# We will run the experiment for 10,000 times
```

In [7]:
```
def getNextConsultantValue(previousConsultant,
                           p,
                           c):
    if previousConsultant == 1:
        pValue = c + p -(c*p) # This is the probability for (1,1)
        return np.random.choice([-1, 1], p=[1-pValue,pValue])
    else:
        pValue = (c*p) -p + 1 # This is the probability for (-1,-1)
        return np.random.choice([-1, 1], p=[pValue,1-pValue])
```

In [8]:
```
numberOftrials = 10000

cpValues = [
    [0,0],
    [0.25,0.25],
    [0.5,0.5],
    [0.75,0.25],
    [0.75,0.75],
    [1,1],
]

mValues = [25,100]

for m in mValues:
    results = []
    for cpValue in cpValues:
        cValue,pValue = cpValue[0], cpValue[1]
        if (0<= cValue+pValue-(cValue*pValue)<=1 and
            0<= 1-(cValue+pValue-(cValue*pValue))<=1 and
            0<=(cValue*pValue)-pValue+1 <=1 and
            0<= pValue-(cValue*pValue)<= 1):

            permutations = []
            buyAYacht = 0

            for i in range(numberOftrials):

                permutations.append([])
                for index in range(m):
                    if index == 0:
                        firstConsultant = np.random.choice([-1, 1], p=[1-pValue,pValue])
                        permutations[i].append(firstConsultant)
                    else:
                        previousConsultant = permutations[i][index-1]
                        nextConsultant = getNextConsultantValue(previousConsultant,
                                                                pValue,
                                                                cValue)

                        permutations[i].append(nextConsultant)
                frequencyMap = Counter(permutations[i])
                if frequencyMap[1] >= m/2:
                    buyAYacht += 1
            results.append([cValue,
                            pValue,
                            m,
                            numberOftrials,
                            buyAYacht/numberOftrials])


    columns = ['c',
               'p',
               'm',
               'numberOfTrials',
               'probability of buying a yacht'
               ]
    df = pd.DataFrame(results, columns=columns)
    print(tabulate(df.set_index('c'), headers='keys', tablefmt='psql'))
```

```
+------+------+-----+-----------------+---------------------------------+
|    c |    p |   m |   numberOfTrials |   probability of buying a yacht |
|------+------+-----+-----------------+---------------------------------|
| 0    | 0    |  25 |           10000 |                          0      |
| 0.25 | 0.25 |  25 |           10000 |                          0.0168 |
| 0.5  | 0.5  |  25 |           10000 |                          0.5019 |
| 0.75 | 0.25 |  25 |           10000 |                          0.1399 |
| 0.75 | 0.75 |  25 |           10000 |                          0.8638 |
| 1    | 1    |  25 |           10000 |                          1      |
+------+------+-----+-----------------+---------------------------------+
+------+------+-----+-----------------+---------------------------------+
|    c |    p |   m |   numberOfTrials |   probability of buying a yacht |
|------+------+-----+-----------------+---------------------------------|
| 0    | 0    | 100 |           10000 |                          0      |
| 0.25 | 0.25 | 100 |           10000 |                          0      |
| 0.5  | 0.5  | 100 |           10000 |                          0.5233 |
| 0.75 | 0.25 | 100 |           10000 |                          0.0252 |
| 0.75 | 0.75 | 100 |           10000 |                          0.9789 |
| 1    | 1    | 100 |           10000 |                          1      |
+------+------+-----+-----------------+---------------------------------+
```

Resources:

1. https://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeClassifier.html
2. https://mitpress.mit.edu/sites/default/files/titles/content/boosting_foundations_algorithms/chapter007.html
3. https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.AdaBoostClassifier.html
4. https://geoffruddock.com/adaboost-from-scratch-in-python/
5. https://www.cs.toronto.edu/~mbrubake/teaching/C11/Handouts/AdaBoost.pdf
6. https://jeremykun.com/2015/05/18/boosting-census/
7. https://xavierbourretsicotte.github.io/AdaBoost.html
8. https://www.youtube.com/watch?v=LsK-xG1cLYA&list=PLblh5JKOoLUICTaGLRoHQDuF_7q2GfuJF&index=42&ab_channel=StatQuestwithJoshStarmer