Lecture 11

September 30, 2020 2:30 PM

Distribution parameters

$X \sim \text{Bernoulli}(\theta)$

$\iff X = \begin{cases} 1 & \text{wp } \theta \\ 0 & \text{wp } 1-\theta \end{cases}$

$X \sim N(\mu, \sigma)$

$\iff \text{pdf}(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

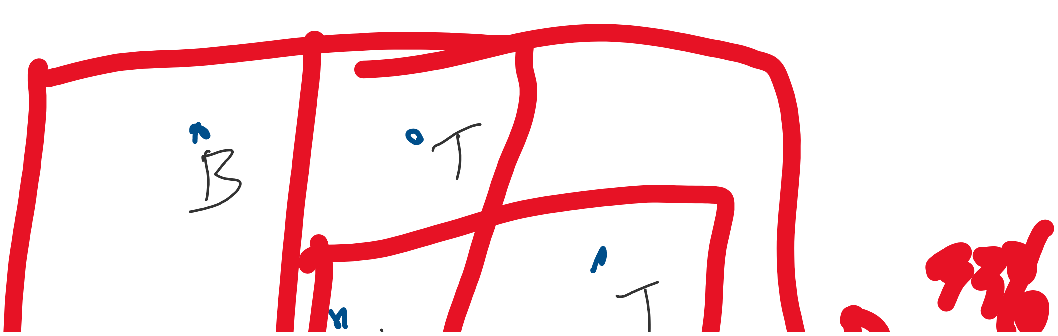
Big question:

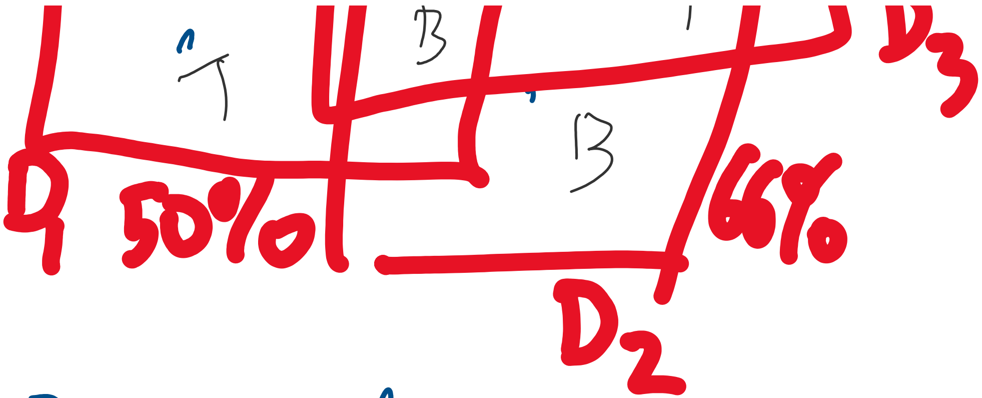
estimate θ, μ, σ

- Maximum likelihood
- with high probability
- biased/unbiased

X, x_1, x_2, ...

$\hat{\theta} = \frac{1}{m} \sum_{i=1}^{m} x_i$





$D_i \to \underbrace{\hat{\theta}_i}$ also random variables

also have mean, variance

$\hat{\theta}$ is <u>unbiased</u> if

$$\mathbb{E}_D[\hat{\theta}] = \theta$$

<u>biased</u> if not unbiased

- B, T, T, B, B

Suppose $x_1, \dots, x_m \sim \mathcal{N}(\mu, \sigma)$

$$\hat{\mu} = \frac{1}{m} \sum_{i=1}^{m} x_i$$

Is $\hat{\mu}$ an unbiased estimate of μ?

$$\mathbb{E}_D[\hat{\mu}_D] = \mathbb{E}_D\left[\frac{1}{|D|}\sum_{i\in D} x_i\right]$$

$$= \frac{1}{|D|}\mathbb{E}_D\left[\sum_{i\in D} x_i\right]$$

$$= \frac{1}{|D|}\sum_{i\in D}\mathbb{E}_D[x_i]$$

$$= \frac{1}{|D|}\cdot|D|\cdot\mu = \mu \checkmark$$

$$\hat{\sigma}^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i-\hat{\mu})^2$$

$$\mathbb{E}[\hat{\sigma}^2] = \frac{1}{m}\sum_{i=1}^{m}\mathbb{E}\left[(x_i-\hat{\mu}+\mu-\mu)^2\right]$$

$$= \frac{1}{m}\left(\underbrace{\sum_{i=1}^{m} E[(x-\mu)^2]}_{\sigma^2} + \sum_{i=1}^{m} E[(\mu-\hat{\mu})^2]\right.$$

$$\left. + \underbrace{\sum_{i=1}^{m} 2E\left[(x_i-\mu)(\mu-\hat{\mu})\right]}\right)$$

$x_1, x_2, \dots, x_m$

$\hat{x} = \frac{1}{m}\sum x_i,\ \text{median}(x_i),\ m$

Bayes $\min_{\hat{x}} \frac{1}{m}\sum_{i=1}^{m} (x_i - \hat{x})^2 = f_1(\hat{x})$ (1), f_1

$\min_{\hat{x}} \frac{1}{m}\sum_{i} |x_i - \hat{x}| = f_2(\hat{x})$ (2)

$\min_{\hat{x}} \max_{i} |x_i - \hat{x}| = f_3(\hat{x})$ (3)

	stay	leave
alien	-100	0
meteor	+10	-10

utility

pr(alien) $= \frac{1}{1000}$

pr(meteor) $= \frac{999}{1000}$

max risk$(\hat{y}) = \begin{cases} \text{leave} & -10 \\ \text{stay} & -100 \end{cases}$

maximin

minimax estimater : leave

Bayes$(\hat{y}) = \begin{cases} \text{leave} & 0 \cdot \frac{1}{1000} + \frac{999}{1000} \cdot (-10) = \\ \text{stay} & -100 \cdot \ldots \end{cases}$

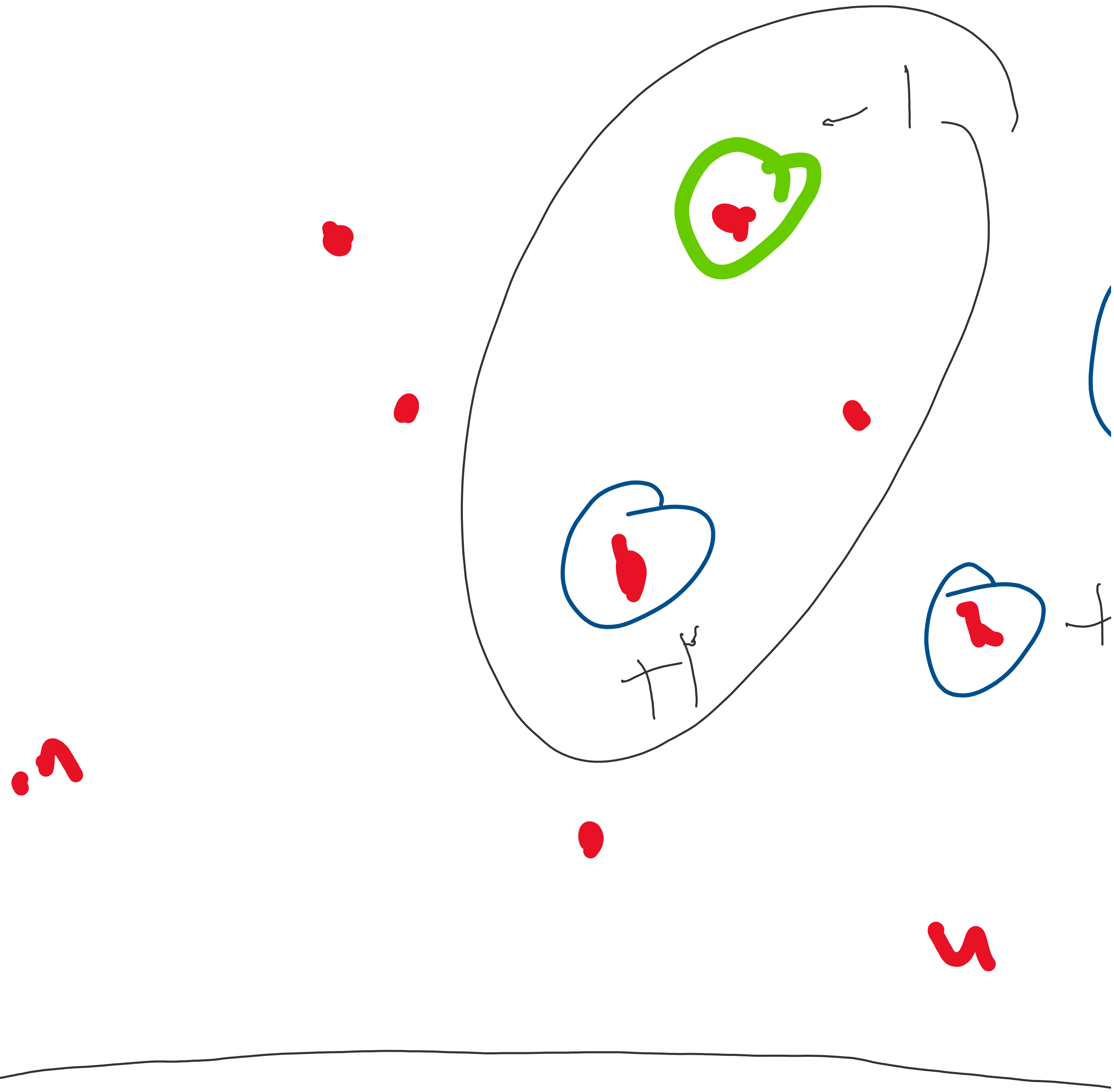
reward

utility

$\frac{\quad}{1000} + 10 \cdot \frac{\quad}{1000}$

Bayes estimator: stay

maximize



$$|\lambda_{max} - E[X]| = \varepsilon$$

$$|E[X] - \lambda_{max}| = \varepsilon$$

$$\frac{1}{\lambda}$$

guess $\lambda = \frac{1}{n} \sum_{i=1}^{n}$

(θ) $(\;)^{n+m}$

W = E cIK

m
constants

A

variab