Challenge: +1.75

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CSE-512 (HW 4)

1)

(a) $P(X = \text{red}) = \frac{1}{2}$ $\quad P(X = \text{yellow}) = \frac{1}{5}$

$P(X = \text{blue}) = \frac{1}{4}$ $\quad P(X = \text{black}) = \frac{1}{20}$

$$\boxed{H(X) = -\sum_{X=x} Pr(X = x) \log_2 (Pr(X = x))}$$

$$= -\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{5}\log_2\left(\frac{1}{5}\right) + \frac{1}{20}\log_2\left(\frac{1}{20}\right)\right)$$

$= 1.68$ bits 1.0

b) $X$ = color of a sock randomly picked

$Y$ = which drawer (up, or down)

$P(\text{top drawer}) = 2\,P(\text{bottom drawer})$ — 1

$P(\text{top drawer}) + P(\text{bottom drawer}) = 1$ — 2

Solving the above 2 equations, we get

$$\boxed{\begin{aligned} P(\text{top drawer}) &= 2/3 \\ P(\text{bottom drawer}) &= 1/3 \end{aligned}}$$

$$\boxed{H(X|Y) = - \sum_{X=x, Y=y} Pr(X=x, Y=y) \log_2 (Pr(X=x|Y=y))}$$

Calculating relevant Probabilities

$Pr(X=x, Y=y) = Pr(X|Y) * Pr(Y)$

$Pr(X=red | Y=top) = 1$

$Pr(x=red, y=top) = 1 \times \frac{2}{3} = \frac{2}{3}$

$Pr(X=red | Y=bottom) = 0$

$Pr(X=blue | Y=top) = 0$

$Pr(x=blue | Y=below) = \frac{1}{2}$

$Pr(x=blue, Y=below) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$Pr(x = \text{yellow} \mid y = \text{top}) = 0$

$Pr(x = \text{yellow} \mid y = \text{below}) = 2/5$

$Pr(x = \text{yellow}, y = \text{below}) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

$Pr(x = \text{black} \mid y = \text{top}) = 0$

$Pr(x = \text{black} \mid y = \text{below}) = \frac{1}{10}$

$Pr(x = \text{black}, y = \text{below}) = \frac{1}{10} \times \frac{1}{3} = \frac{1}{30}$

we, will assume, $0 \log_2(0)$ as $0$

$$H(X \mid Y) = -\left[\frac{2}{3}\log_2(1) + \frac{1}{6}\log_2\left(\frac{1}{2}\right) + \frac{2}{15}\log_2\left(\frac{2}{5}\right) + \frac{1}{30}\log_2\left(\frac{1}{10}\right)\right]$$

$= 0.45$ bits 1.0

c) Information gain:-

$I(X : Y) = H(X) - H(X \mid Y)$

$= 1.68 - 0.45$

$= 1.23$ 1.0

2)

a) $p(\text{word} = \text{the}) = 6/141$

$p(\text{word} = \text{rabbit}) = 3/141$

$p(\text{word} = a) = 5/141$

i) $Pr(\text{current word} = \text{rabbit} \mid \text{Previous word} = \text{the})$

$\Rightarrow \frac{\cancel{2}\,1}{\cancel{6}\,3} = \frac{1}{3}$

ii) $Pr(\text{current word} = a \mid \text{Previous word} = \text{rabbit})$

$= \frac{1}{3}$

iii) $Pr(\text{current word} = \text{the} \mid \text{Previous word} = \text{rabbit})$

$= \frac{2}{3}$

iv) $Pr(\text{current word} = \text{the or } a \mid \text{previous word} = \text{rabbit})$

$= 1$

v) $P(A \cap B) = P(A) * P(B) \rightarrow$ condition for naive Baye's

$p(\text{current word} = \text{the} \cap \text{previous\_word} = \text{rabbit})$

$= \frac{2}{141}$

$p(\text{word} = \text{the}) = \frac{6}{141}$

$p(\text{word} = \text{rabbit}) = \frac{3}{141}$

$\Rightarrow \frac{2}{141} \neq \frac{6}{141} \times \frac{3}{141}$

$\therefore$ Naive Bayes assumption is not valid here

1.5

In [1]:
```python
import numpy as np
import pickle
import matplotlib.pyplot as plt
import copy
import random
```

## Loading data from pkl file

In [2]:
```python
with open('alice_parsed.pkl','rb') as f:
    u = pickle._Unpickler(f)
    u.encoding = 'latin1'
    data = u.load()
count, next_word_count = data[0], data[1]
```

## Q2(b) i

In [3]:
```python
def getWordProbability(word, count=count, next_word_count = next_word_count):
    return count[word]/sum(count.values())

# Testing
getWordProbability('rabbit')
```

Out[3]:
```
0.0016590000754090944
```

## Q2(b) ii

Conditional Probability

In [4]:
```python
def getConditionalProbability(x, y, count=count, next_word_count = next_word_count):
    word = x
    nextWord = y

    if nextWord not in next_word_count[word]:
        return 0

    nextWordGivenWordCount = next_word_count[word][nextWord]
    nextWordAll = sum(next_word_count[word].values())
    return nextWordGivenWordCount/nextWordAll

# Testing
getConditionalProbability('rabbit','just')
```

Out[4]:
```
0.022727272727272728
```

## Q2(c) iii

From Bayes' theorem

$$(A \mid B) = \frac{(B|A)\cdot \ (A)}{(B)}$$

$A, B =$ events

$(A \mid B) \quad =$ probability of A given B

$(B \mid A) \quad =$ probability of B given A

$(A), \quad (B) =$ the independent probabilities of A and B

Here,

A = nextWord

B = word

$$(nextWord \mid word) = \frac{(word|nextWord)\cdot \ (nextWord)}{(word)}$$

In [7]:
```python
def predict(word, topk, count=count, next_word_count = next_word_count):

    possibleNextWords = next_word_count[word]
    ans = []
    pWord = getWordProbability(word)
    for nextWord in possibleNextWords.keys():
        pNextWord = getWordProbability(nextWord)
        bayesEstimate =  getConditionalProbability(word, nextWord) * getWordProbability(nextWord)/ pWor
d
        ans.append((nextWord, bayesEstimate))
    topk = min(len(possibleNextWords), topk)
    return [(k,v) for k, v in sorted(ans, key=lambda item: item[1], reverse = True)][:topk]
#     return [(k) for k, v in sorted(ans, key=lambda item: item[1], reverse = True)][:topk]

print ("word most likely to follow 'a' is: " ,predict('a',1)[0])
print ("word most likely to follow 'the' is: " ,predict('the',1)[0])
print ("word most likely to follow 'splendidly' is: " ,predict('splendidly',1)[0])
print ("word most likely to follow 'exclaimed' is: " ,predict('exclaimed',1)[0])
```

```
word most likely to follow 'a' is:  ('little', 0.019377904182022114)
word most likely to follow 'the' is:  ('queen', 0.001647382599763552)
word most likely to follow 'splendidly' is:  ('dressed', 1.0)
word most likely to follow 'exclaimed' is:  ('alice', 32.083333333333336)
```

1.5

```
In [1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

import scipy.io as sio
from pyitlib import discrete_random_variable as drv

from tabulate import tabulate

from graphviz import Source

import os
import warnings
warnings.filterwarnings('ignore')

from sklearn.datasets import load_iris
from sklearn.metrics import accuracy_score
from sklearn.tree import export_graphviz
from sklearn.tree import DecisionTreeClassifier
```

## Loading Data

no IG reported -0.5
no discussion reported -0.5

```
In [2]: data = sio.loadmat('covtype_reduced.mat')
X_train = data['X_train']
X_test = data['X_test']
y_train = data['y_train'][0]
y_test = data['y_test'][0]

print (X_train.shape, X_test.shape, y_train.shape, y_test.shape)

data = sio.loadmat('covtype_reduced.mat')
X_train = data['X_train']
X_test = data['X_test']
y_train = data['y_train'].T
y_test = data['y_test'].T

print (X_train.shape, X_test.shape, y_train.shape, y_test.shape)
```

```
(468, 54) (116202, 54) (468,) (116202,)
(468, 54) (116202, 54) (468, 1) (116202, 1)
```

## computing Entropy and conditionalEntropy

```
In [3]: def entropy(y):
    if len(y) == 0:
        return 0
    unique, count = np.unique(y, return_counts=True, axis=0)
    individualProbabilities = count/len(y)
    entropy = -np.sum(individualProbabilities*np.log2(individualProbabilities))
    return entropy


#Additional Helper function
def jointEntropy(y,x):
    yx = np.c_[y,x]
    return entropy(yx)

def cond_entropy(y,yhat):
    return jointEntropy(y,yhat) - entropy(yhat)

random_sequences = sio.loadmat('random_sequences.mat')

s1 = random_sequences['s1'][0]
s2 = random_sequences['s2'][0]

print ('entropy = ', entropy(s1))
print ('conditional entropy = ', cond_entropy(s1,s2))

print("\n====Verification with in-built functions")
print("verification entropy = :",drv.entropy(s1))
print("verification conditional entropy = :",drv.entropy_conditional(s1,s2))
```

```
entropy =  3.3141823231610834
conditional entropy =  3.3029598816135173

====Verification with in-built functions
verification entropy = : 3.3141823231610834
verification conditional entropy = : 3.3029598816135173
```

## Tree Class and its functions

```
In [4]: class Tree:
    def __init__(self,
                 max_depth = 10,
                 minimum_gain = 1e-7,
                 min_samples_split = 2):
        self.max_depth = max_depth
        self.minimum_gain = minimum_gain
        self.min_samples_split = min_samples_split

    def fit(self, X, y):
        self.numberOfClasses = np.unique(y).shape[0]
        self.feature_importance = np.zeros(X.shape[1])
        # 1st call to create the Decision Tree
        self.tree = getDecisionTree(X, y,
                                    self.max_depth,
                                    self.minimum_gain,
                                    self.min_samples_split,
                                    self.numberOfClasses,
                                    self.feature_importance,
                                    X.shape[0])
        self.feature_importance /= np.sum(self.feature_importance)
        return self

    def predict(self,X):
        """
        Traverse each row and predict the relevant class.
        """
        predictions =[]
        for i in range(X.shape[0]):
            temp = self.classifyExample(X[i, :], self.tree)
            predictions.append(temp)
        return predictions

    def classifyExample(self,example, tree):
        """
        classification is done recurssively until you reach the leaf node
        """

        # base case
        if tree['is_leaf']:
            return np.argmax(tree['prob'])

        # recurssion
        else:
            featureName, value = tree['split_col'], tree['threshold']

            splitColumn = tree['split_col']
            featureType = FEATURE_TYPES[splitColumn]

            # Differentiating between continuous and categorical values

            if featureType == "continuous":
                if example[featureName] <= value:
                    return self.classifyExample(example, tree['left'])
                else:
                    return self.classifyExample(example, tree['right'])
            else:
                if example[featureName] == value:
                    return self.classifyExample(example, tree['left'])
                else:
                    return self.classifyExample(example, tree['right'])

    def printTree(self):
        """
        Helper function to print the tree for debugging.
        """
        print ('printing tree...')
        def printNode(parent, tree, childType):
            if not tree:
                return
            if parent is  None:
                print(', ROOT', )
                # Differentiating between continuous and categorical values
                if tree['featureType'] == "continuous":
                    print(', ROOT, Condition: ' +str(tree['colName'])+ '<= '+ str(tree['threshold']))

                else:
                    print(', ROOT, Condition: ' +str(tree['colName'])+ '== '+ str(tree['threshold']))

            # Differentiating between Leaf Node and Non-Leaf Node
            if tree['is_leaf']:
                print(', LEAF, ', 'Total Samples '+str(sum(tree['counts'])) + ' , Distribution' + str(t
ree['counts']))
            else:
                # Differentiating between continuous and categorical values
                if tree['featureType'] == "continuous":
                    if childType == "left":
                        print(', NONLEAF, Condition: ' +str(tree['colName'])+ '<= '+ str(tree['threshol
d']))
                    else:
                        print(', NONLEAF, Condition: ' +str(tree['colName'])+ '> '+ str(tree['threshol
d']))
                else:
                    if childType == "left":
                        print(', NONLEAF, Condition: ' +str(tree['colName'])+ '== '+ str(tree['threshol
d']))
                    else:
                        print(', NONLEAF, Condition: ' +str(tree['colName'])+ '!= '+ str(tree['threshol
d']))

                printNode(tree, tree['left'], "left")
                printNode(tree, tree['right'], "right")

        printNode(None, tree.tree,"")
```

## Decision Tree helper functions

```
In [5]: def getDecisionTree(X, y, max_depth,
                    minimum_gain,
                    min_samples_split, numberOfClasses,
                    feature_importance, n_row):
    """
    Recusively constructs a Decision tree
    1) Determine if we can split the tree (or) is it a leaf node.
    2) If we can split:
        i) Find best split
        ii) Recursivel call Decision tree on both the splits (in our case, it is binary decision tree)
    3) If we cannot split, i.e. it is a Leaf Node
        i) We store the distribution of the data('label') at the leaf node and use this information in
 our prediction.
    """

    if max_depth>0 and X.shape[0] > min_samples_split :
        column, value, informationGain = findBestSplit(X, y)
        if informationGain > minimum_gain:
            feature_importance[column] += (X.shape[0] / n_row) * informationGain

        # computing left and right child
        left_X, right_X, left_y, right_y = splitData(X, y, column, value)
        left_child = getDecisionTree(left_X, left_y,
                                     max_depth - 1,
                                     minimum_gain,
                                     min_samples_split,
                                     numberOfClasses,
                                     feature_importance,
                                     n_row)
        right_child = getDecisionTree(right_X, right_y,
                                      max_depth - 1,
                                      minimum_gain,
                                      min_samples_split,
                                      numberOfClasses,
                                      feature_importance,
                                      n_row)
        nonLeafNode = {
        'is_leaf': False,
        'split_col': column,
        'colName': COLUMN_HEADERS[column],
        'featureType': FEATURE_TYPES[column],
        'threshold': value,
        'left': left_child,
        'right': right_child
        }
        return nonLeafNode
    elif X.shape[0] >0 :
        counts = np.bincount(y, minlength = numberOfClasses)
        prob = counts / y.shape[0]
        leafNode = {'is_leaf': True, 'prob': prob,'counts':counts}
        return leafNode


def findBestSplit(X, y):
    """
    We try to determine which is the best split for the data based on the Information gain.
    1) We find all the unique values for every column, be it continuous (or) categorical
    2) We split the data at every unique value for every column and find the informationGain at that
       value for that column.
    3) we pick the column and corresponding split value where we get the maximum information gain.
    """
    bestSplitColumn, bestSplitValue, maxInformationGain = None, None, None
    existingEntropy = entropy(y)
    totalFeatures   = X.shape[1]
    for column in range(totalFeatures):
        splitValues = np.unique(X[:, column])
        for value in splitValues:
            splits = splitData(X, y, column, value, return_X = False)
            informationGain = existingEntropy - computeEntropyAfterSplit(y, splits)
            if maxInformationGain is None or informationGain > maxInformationGain:
                bestSplitColumn, bestSplitValue, maxInformationGain = column, value, informationGain
    return bestSplitColumn, bestSplitValue, maxInformationGain

def splitData(X, y, splitColumn, splitValue, return_X=True):
    """
    1) Takes the input data (X,Y).
    2) Uses the splitColumn and splitValue to split the data into 2 halves:
        i) In case of continuous data
            a) We split them as rowsBelowThreshold and rowsAboveThreshold
        ii) In case of categorical data (Here we have binary data)
            a) We split them as rowsWhereValueIsZero and rowsWhereValueIsOne
    """

    type_of_feature = FEATURE_TYPES[splitColumn]
    splitColumnValues = X[:, splitColumn]
    if type_of_feature == "continuous":
        rowsBelowThreshold =  splitColumnValues <= splitValue
        rowsAboveThreshold =  splitColumnValues >  splitValue

    else:#categorical
        rowsBelowThreshold =  splitColumnValues == splitValue
        rowsAboveThreshold =  splitColumnValues !=  splitValue

    XBelowThreshold    = X[rowsBelowThreshold]
```

[illegible]

```
In [6]: data = sio.loadmat('covtype_reduced.mat')
X_train = data['X_train'].astype(float)
X_test = data['X_test'].astype(float)
y_train = data['y_train'][0].astype(int)
y_test = data['y_test'][0].astype(float)

global COLUMN_HEADERS, FEATURE_TYPES
COLUMN_HEADERS = ['Elevation', 'Aspect', 'Slope', 'Horizontal_Distance_To_Hydrology', 'Vertical_Distanc
e_To_Hydrology', 'Horizontal_Distance_To_Roadways', 'Hillshade_9am', 'Hillshade_Noon', 'Hillshade_3pm',
 'Horizontal_Distance_To_Fire_Points', 'Wilderness_Area1', 'Wilderness_Area2', 'Wilderness_Area3', 'Wil
derness_Area4', 'Soil_Type1', 'Soil_Type2', 'Soil_Type3', 'Soil_Type4', 'Soil_Type5', 'Soil_Type6', 'So
il_Type7', 'Soil_Type8', 'Soil_Type9', 'Soil_Type10', 'Soil_Type11', 'Soil_Type12', 'Soil_Type13', 'Soi
l_Type14', 'Soil_Type15', 'Soil_Type16', 'Soil_Type17', 'Soil_Type18', 'Soil_Type19', 'Soil_Type20', 'S
oil_Type21', 'Soil_Type22', 'Soil_Type23', 'Soil_Type24', 'Soil_Type25', 'Soil_Type26', 'Soil_Type27',
'Soil_Type28', 'Soil_Type29', 'Soil_Type30', 'Soil_Type31', 'Soil_Type32', 'Soil_Type33', 'Soil_Type34'
, 'Soil_Type35', 'Soil_Type36', 'Soil_Type37', 'Soil_Type38', 'Soil_Type39', 'Soil_Type40', 'label']
COLUMN_HEADERS = COLUMN_HEADERS
FEATURE_TYPES = determineTypeOfFeature(X_train)

results = []
for depth in range(1,6):
    tree = Tree(max_depth=depth, min_samples_split=2)
    tree.fit(X_train, y_train)
    y_pred = tree.predict(X_train)
    y_pred2 = tree.predict(X_test)
    results.append((2**depth,
                    "{0:.2f}".format(100*accuracy_score(y_train, y_pred))+' %',
                    "{0:.2f}".format(100*(1-accuracy_score(y_train, y_pred)))+' %',
                    "{0:.2f}".format(100*accuracy_score(y_test, y_pred2))+' %',
                    "{0:.2f}".format(100*(1-accuracy_score(y_test, y_pred2)))+' %'))

columns = ['No of Splits','Train Accuracy','Train Error', 'Test Accuracy', 'Test Error']
df = pd.DataFrame(results, columns=columns)
df.reset_index()
print(tabulate(df, headers='keys', tablefmt='psql'))
```

```
+----+----------------+------------------+---------------+-----------------+--------------+
|    |   No of Splits | Train Accuracy   | Train Error   | Test Accuracy   | Test Error   |
|----+----------------+------------------+---------------+-----------------+--------------|
|  0 |              2 | 49.79 %          | 50.21 %       | 50.40 %         | 49.60 %      |
|  1 |              4 | 62.82 %          | 37.18 %       | 73.32 %         | 26.68 %      |
|  2 |              8 | 63.68 %          | 36.32 %       | 70.29 %         | 29.71 %      |
|  3 |             16 | 67.31 %          | 32.69 %       | 73.01 %         | 26.99 %      |
|  4 |             32 | 72.65 %          | 27.35 %       | 63.50 %         | 36.50 %      |
+----+----------------+------------------+---------------+-----------------+--------------+
```

## Leaf Nodes Info

```
In [7]: getLeafNodeInfo(tree.tree)
```

```
Leaf Node 01 Total Samples 001 , Distribution [0 0 0 0 1 0 0]
Leaf Node 02 Total Samples 003 , Distribution [0 0 2 1 0 0 0]
Leaf Node 03 Total Samples 003 , Distribution [0 0 0 3 0 0 0]
Leaf Node 04 Total Samples 001 , Distribution [0 0 1 0 0 0 0]
Leaf Node 05 Total Samples 004 , Distribution [0 0 0 4 0 0 0]
Leaf Node 06 Total Samples 006 , Distribution [0 0 0 1 0 0 5]
Leaf Node 07 Total Samples 019 , Distribution [ 0  0  0 15  0  0  4]
Leaf Node 08 Total Samples 009 , Distribution [0 0 6 0 0 0 3]
Leaf Node 09 Total Samples 004 , Distribution [0 0 0 4 0 0 0]
Leaf Node 10 Total Samples 026 , Distribution [ 0  4 22  0  0  0  0]
Leaf Node 11 Total Samples 009 , Distribution [0 1 4 0 0 0 4]
Leaf Node 12 Total Samples 001 , Distribution [0 0 1 0 0 0 0]
Leaf Node 13 Total Samples 009 , Distribution [0 0 9 0 0 0 0]
Leaf Node 14 Total Samples 007 , Distribution [0 2 4 0 0 1 0]
Leaf Node 15 Total Samples 014 , Distribution [0 0 4 6 0 4 0]
Leaf Node 16 Total Samples 003 , Distribution [0 3 0 0 0 0 0]
Leaf Node 17 Total Samples 055 , Distribution [ 0 11 42  0  0  2  0]
Leaf Node 18 Total Samples 001 , Distribution [0 0 1 0 0 0 0]
Leaf Node 19 Total Samples 015 , Distribution [ 0  0 15  0  0  0  0]
Leaf Node 20 Total Samples 015 , Distribution [ 0 13  2  0  0  0  0]
Leaf Node 21 Total Samples 121 , Distribution [ 0 62 59  0  0  0  0]
Leaf Node 22 Total Samples 001 , Distribution [0 0 1 0 0 0 0]
Leaf Node 23 Total Samples 009 , Distribution [0 0 9 0 0 0 0]
Leaf Node 24 Total Samples 015 , Distribution [ 0 15  0  0  0  0  0]
Leaf Node 25 Total Samples 050 , Distribution [ 0 38  6  0  0  0  6]
Leaf Node 26 Total Samples 013 , Distribution [ 0 13  0  0  0  0  0]
Leaf Node 27 Total Samples 021 , Distribution [ 0  7 14  0  0  0  0]
Leaf Node 28 Total Samples 005 , Distribution [0 2 0 0 0 0 3]
Leaf Node 29 Total Samples 014 , Distribution [ 0 12  2  0  0  0  0]
Leaf Node 30 Total Samples 002 , Distribution [0 2 0 0 0 0 0]
Leaf Node 31 Total Samples 012 , Distribution [ 0  2  0  0  0  0 10]
```

## Printing a sample tree for visualisation

```
In [8]: tree = Tree(max_depth=2, min_samples_split=2)
tree.fit(X_train, y_train)
tree.printTree()
```

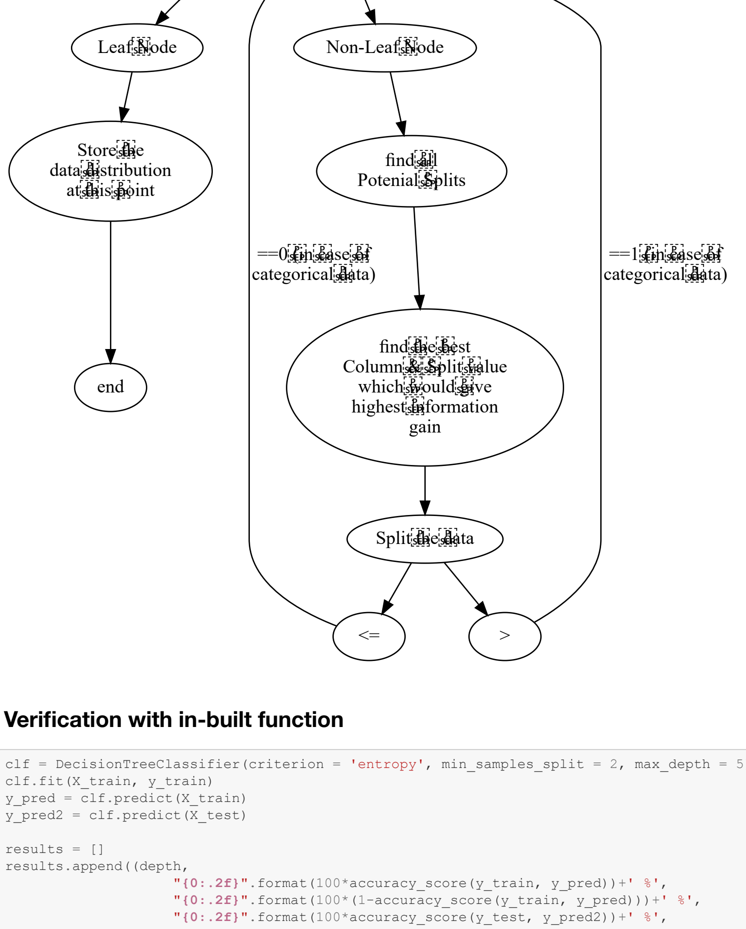
```
printing tree...
, ROOT
, ROOT, Condition: Elevation<= 2843.0
, NONLEAF, Condition: Elevation> 2843.0
, NONLEAF, Condition: Elevation<= 2524.0
, LEAF,  Total Samples 37 , Distribution[ 0  0  3 24  1  0  9]
, LEAF,  Total Samples 79 , Distribution[ 0  7 50 10  0  5  7]
, NONLEAF, Condition: Elevation> 3170.0
, LEAF,  Total Samples 220 , Distribution[  0  89 129   0   0   2   0]
, LEAF,  Total Samples 132 , Distribution[ 0 91 22  0  0  0 19]
```

## Algorithm

```
In [9]: from graphviz import Digraph
```

```
In [10]: g = Digraph('G')

g.edge('Read data', 'Create Decison Tree', label='Training data')
g.edge('Create Decison Tree','Start')
g.edge('Start', 'Is the data seperable?')
g.edge('Is the data seperable?', 'Leaf Node', label='yes')
g.edge('Leaf Node', 'Store the \ndata distribution\n at this point')
g.edge('Store the \ndata distribution\n at this point','end')
g.edge('Is the data seperable?', 'Non-Leaf Node', label='no')
g.edge('Non-Leaf Node', 'find all \n Potenial Splits')
g.edge('find all \n Potenial Splits', 'find the best \n Column & Split value \n which would give \n hig
hest information \n gain')
g.edge('find the best \n Column & Split value \n which would give \n highest information \n gain', 'Spl
it the data')
g.edge('Split the data', '<=')
g.edge('Split the data', '>')
g.edge('<=','Is the data seperable?',label='==0 (in case of \n categorical data)')
g.edge('>','Is the data seperable?' ,label='==1 (in case of \n categorical data)')

g
```

Out[10]:

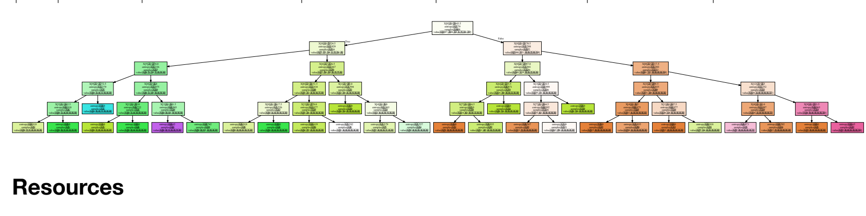



## Verification with in-built function

```
In [11]: clf = DecisionTreeClassifier(criterion = 'entropy', min_samples_split = 2, max_depth = 5)
clf.fit(X_train, y_train)
y_pred = clf.predict(X_train)
y_pred2 = clf.predict(X_test)

results = []
results.append((depth,
                "{0:.2f}".format(100*accuracy_score(y_train, y_pred))+' %',
                "{0:.2f}".format(100*(1-accuracy_score(y_train, y_pred)))+' %',
                "{0:.2f}".format(100*accuracy_score(y_test, y_pred2))+' %',
                "{0:.2f}".format(100*(1-accuracy_score(y_test, y_pred2)))+' %'))

columns = ['Depth','Train Accuracy','Train Error', 'Test Accuracy', 'Test Error']
df = pd.DataFrame(results, columns=columns)
df.reset_index()
print(tabulate(df, headers='keys', tablefmt='psql'))

export_graphviz(clf, filled = True,
                out_file = 'tree.dot')

os.system('dot -Tpng tree.dot -o tree.jpeg')
with open('tree.dot') as f:
    dot_graph = f.read()

Source(dot_graph)
```

```
+----+---------+------------------+---------------+-----------------+--------------+
|    |   Depth | Train Accuracy   | Train Error   | Test Accuracy   | Test Error   |
|----+---------+------------------+---------------+-----------------+--------------|
|  0 |       5 | 72.65 %          | 27.35 %       | 64.95 %         | 35.05 %      |
+----+---------+------------------+---------------+-----------------+--------------+
```

Out[11]:



## Resources

1) https://piazza.com/class/kdhx0iapnk06la?cid=347

2) https://www.youtube.com/watch?v=y6DmpG_PtN0&list=PLPOTBrypY74xS3WD0G_uzqPjCQfU6IRK-&index=1&ab_channel=SebastianMantey

```
In [ ]:
```

# Challenge

## Q1

In [4]:
```
def predict(word, topk, count=count, next_word_count = next_word_count):

    possibleNextWords = next_word_count[word]
    ans = []
    pWord = getWordProbability(word)
    for nextWord in possibleNextWords.keys():
        pNextWord = getWordProbability(nextWord)
        bayesEstimate =  getConditionalProbability(word, nextWord) * getWordProbability(nextWord)/ pWor
d
        ans.append((nextWord, bayesEstimate))
    topk = min(len(possibleNextWords), topk)
    return [(k,v) for k, v in sorted(ans, key=lambda item: item[1], reverse = True)][:topk]
```

In [99]:
```
seedWord = 'alice'
prev = seedWord
paragraph = []
for i in range(100):
    k = 3
    nextWordsPossible = predict(prev,k)
    if len(nextWordsPossible) < k:
        k = len(nextWordsPossible)

    nextWord = nextWordsPossible[random.randint(0, k-1)][0]
    paragraph.append(prev)
    prev = nextWord
print(" ".join(paragraph))
```

```
alice the queen the mock turtles all said the queen and a little the king said to alice to alice and
she had the king said to be the queen said to the mock turtle and the queen and she said to be the ki
ng the queen said alice to be said to the queen said to the mock turtle to alice and the queen said t
he queen the queen the queen the king said the queen and she said the king the queen and a great or t
he king said to the mock turtles heavy sobbing of the mock
```

This is an interesting approach, but not sampled based on  probability of next word
+0.5

## Challenge

+0.5

Q3) $E[x] = \sum_{i=1}^{n} x_i p_i$

$E[f(x)] = \sum_{i=1}^{n} p_i f(x_i)$

$f(E[x]) = f(x_1 p_1 + x_2 p_2 \ldots \ldots x_n p_n)$

$f(E(x)) = f\left(p_1 x_1 + (1-p_1)\left[\frac{p_2}{1-p_1} x_2 + \frac{p_3}{1-p_1} x_3 \cdots \frac{p_n}{1-p_n} x_n\right]\right)$

$< p_1 f(x_1) + (1-p_1) f\left[\frac{p_2}{1-p_1} x_2 + \frac{p_3}{1-p_1} x_3 \cdots \frac{p_n}{1-p_n} x_n\right]$

(Assuming $\theta = p_1$)

$f(E[x]) < p_1 f(x_1) + (1-p_1) f\left[\frac{p_2}{1-p_1} x_2 + \frac{p_3}{1-p_1} x_3 \cdots \frac{p_n}{1-p_n} x_n\right]$ — (1)

Let us consider the term,

$= f\left[\frac{p_2}{1-p_1} x_2 + \frac{p_3}{1-p_1} x_3 \cdots \frac{p_n}{1-p_n} x_n\right]$

$= f\left(\frac{p_2 x_2}{1-p_1} + \frac{1-p_1-p_2}{1-p_1}\left[\frac{p_3 x_3}{1-p_1-p_2} + \frac{p_4 x_4}{1-p_1-p_2} \cdots + \frac{p_n x_n}{1-p_1-p_2}\right]\right.$

$$< \frac{p_2}{1-p_1} f(x_2) + \frac{1-p_1-p_2}{1-p_1} f\left[\frac{p_3 x_3}{1-p_1-p_2} + \frac{p_4 x_4}{1-p_1-p_2} \cdots + \frac{p_n x_n}{1-p_1-p_2}\right]$$

$$\theta = \frac{p_2}{1-p_1} \qquad 1-\theta = 1 - \frac{p_2}{1-p_1} = \frac{1-p_1-p_2}{1-p_1}$$

$$\Rightarrow (1-p_1) f\left(\frac{p_2 x_2}{1-p_1} + \frac{p_3 x_3}{1-p_1} \cdots \frac{p_n x_n}{1-p_1}\right)$$

$$< p_2 f(x_2) + (1-p_1-p_2) f\left(\frac{p_3 x_3}{1-p_1-p_2} \cdots + \frac{p_n x_n}{1-p_1-p_2}\right) \quad — (2)$$

From (1) & (2), we get,

$$f(E[x]) < p_1 f(x_1) + p_2 f(x_2) + (1-p_1-p_2) f\left(\frac{p_3 x_3}{1-p_1-p_2} \cdots + \frac{p_n x_n}{1-p_1-p_2}\right)$$

If we generalize the above term, we get

$$f(E[x]) < p_1 f(x_1) + p_2 f(x_2) + p_3 f(x_3) \cdots + p_{n-1} f(x_{n-1}) + (1-p_1-p_2 \cdots p_{n-1}) f\left(\frac{p_n x_n}{1-p_1-p_2 \cdots -p_{n-1}}\right)$$

$\Rightarrow p_1 + p_2 + p_3 \dots p_n = 1$

$\Rightarrow 1 - p_1 - p_2 - p_3 \dots - p_{n-1} = p_n$

$f(E[x]) \leq p_1 f(x_1) + p_2 f(x_2) \dots$

$+ p_{n-1} f(x_{n-1}) + p_n f(x_n)$

$$\boxed{\therefore f(E[x]) \leq E[f(x)]}$$

Q4) $P_\lambda(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{else} \end{cases}$

+0.5

(a) Likelihood (L),

$$L(\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$$

$$= \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i}$$

$$\log(L(\lambda)) = n \log \lambda - \lambda \sum_{i=1}^{n} x_i$$

For maximum, set derivative to '0'.

$$\frac{dL(\lambda)}{d\lambda} = \frac{n}{\lambda} - \Sigma x_i = 0$$

$$\lambda_{MLE} = \frac{n}{\Sigma x_i}$$

$$\lambda_{MLE} = \frac{1}{\bar{\theta}} \qquad \left(\bar{\theta} = \frac{\sum_{i=1}^{n} x_i}{n}\right)$$

b) $E[\hat{\lambda}] = E\left[\frac{n}{\sum_{i=1}^{n} x_i}\right]$

$= n \times E\left[\frac{1}{x_i}\right]$

$$E[x^{-1}] = \int_0^\infty \frac{1}{\lambda} \lambda e^{-\lambda x} dx = \frac{\lambda}{n-1}$$

$$E(\hat{\lambda}) = \frac{n}{n-1} \lambda \neq \lambda$$

→ It is a biased estimator

Ok, but see soln for an alternative proof

+0.25

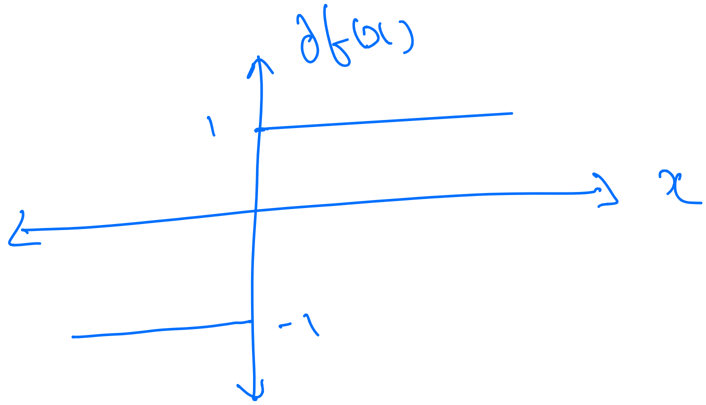
Every gradient is a subgradient
a subgradient = gradient if f is differentiable
convex doesn't really matter

Q2)

(a) If f is convex and differentiable, then gradient at x is a sub gradient.

But a sub-gradient can exist even when f is non-differentiable.

OK




A function f is called sub-differentiable at x if there exists atleast one sub-gradient at x.

this is always true if f is continuous

Consider, $f(x) = |x|$

For, $x < 0 \rightarrow$ sub-~~gradiet~~ differential $\partial f(x) = \{-1\}$

For, $x > 0 \longrightarrow$ sub-~~gradiet~~ differential $\partial f(x) = \{1\}$

At $\boxed{x \quad = 0}$

one sub-gradiet is defined by the

equality, $\boxed{|z| \geq gz}$ $\forall z$ which is satisfied

iff $\boxed{g \in [-1, 1]}$. ok

$\therefore \partial f(0) = [-1, 1]$

$$\therefore \partial f(x) = \begin{cases} \{1\} & \text{if } x > 0 \\ \{-1\} & \text{if } x < 0 \\ [-1, 1] & x = 0 \end{cases}$$

b) A Point $x^*$ is a minimizer of a convex function iff $f$ is sub-differentiable at $x^*$ and $0 \in \partial f(x^*)$,

i.e. $g = 0$ is a sub-gradient of $f$ at $x^*$.

[as $f(x) \geq f(x^*)$]

$\Rightarrow 0 \in \partial f(x^*)$ reduces to $\nabla f(x^*) = 0$

if $f$ is differentiable at $x^*$. ?