2-Layer Neural-Network

$x, w^{[1]}, b \rightarrow z^{[1]} = w^{[1]}x + b \rightarrow a^{[1]} = g(z^{[1]}) \rightarrow z^{[2]} = w^{[2]}a^{[1]} + b^{[2]} \rightarrow a^{[2]} = g(z^{[2]}) \rightarrow \mathcal{L}(a^{[2]}, y)$

$$\mathcal{L}(a^{[2]}, y) = -(y \log a^{[2]} + (1-y) \log(1-a^{[2]}))$$

① $$\frac{d\mathcal{L}}{da^{[2]}} = -\frac{y}{a^{[2]}} + \frac{1-y}{1-a^{[2]}} \rightarrow da^{[2]} = -\frac{y}{a^{[2]}} + \frac{1-y}{1-a^{[2]}}$$

② $$\frac{d\mathcal{L}}{dz^{[2]}} = \frac{d\mathcal{L}}{da^{[2]}} \cdot \frac{da^{[2]}}{dz^{[2]}}$$

$$\frac{da^{[2]}}{dz^{[2]}} = \frac{d}{dz}\left(\frac{1}{1+e^{-z}}\right) = \frac{(1+e^{-z})^{-1}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} * \left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= a * (1-a)$$

(Assuming Sigmoid)

$$\frac{d\mathcal{L}}{dz^{[2]}} = \left(-\frac{y}{a^{[2]}} + \frac{1-y}{1-a^{[2]}}\right) * (a^{[2]} * (1-a^{[2]}))$$

$$= -y(1-a^{[2]}) + (1-y)a^{[2]}$$

$$\frac{d\mathcal{L}}{dz^{[2]}} = a^{[2]} - y \rightarrow dz^{[2]} = a^{[2]} - y$$

③ $\frac{dL}{dw^{[2]}} = \frac{dL}{dz^{[2]}} \times \frac{dz^{[2]}}{dw^{[2]}}$

$\frac{dz^{[2]}}{dw^{[2]}} = \frac{d}{dw^{[2]}}\left(w^{[2]} a^{[1]} + b^{[2]}\right) = a^{[1]}$

$\frac{dL}{dw^{[2]}} = \left(a^{[2]} - y\right) \times a^{[1]} \rightarrow$ $\boxed{dw^{[2]} = \frac{1}{m} dz^{[2]} \cdot A^{[1]T}}$

* $\frac{1}{m}$ indicates Average.

④ $\frac{dL}{db^{[2]}} = \frac{dL}{dz^{[2]}} * \frac{dz^{[2]}}{db^{[2]}}$

$\frac{dz^{[2]}}{db^{[2]}} = \frac{d}{db^{[2]}}\left(w^{[2]} a^{[1]} + b^{[2]}\right) = 1.$

$\frac{dL}{db^{[2]}} = \left(a^{[2]} - y\right) * 1 \rightarrow$ $\boxed{db^{[2]} = \frac{1}{m} dz^{[2]}}$

* Sum of all the elements

⑤ Similarly, $\frac{dL}{dz^{[1]}} = \frac{dL}{dz^{[2]}} \frac{dz^{[2]}}{da^{[1]}} \frac{da^{[1]}}{dz^{[1]}}$

$\frac{dz^{[2]}}{da^{[1]}} = \frac{d}{da^{[1]}} * \left(w^{[2]} a^{[1]} + b^{[2]}\right) = w^{[2]}$

$\frac{da^{[1]}}{dz^{[1]}} = \frac{d}{dz^{[1]}} \left(\frac{1}{1+e^{-z}}\right) = a^{[1]} * (1 - a^{[1]}) \Rightarrow g^{[1]'}(z^{[1]})$

Element-wise Multiplication

$\frac{dL}{dz^{[1]}} = dz^{[2]} * w^{[2]} * g^{[1]'}(z^{[1]}) \Rightarrow$ $\boxed{dz^{[1]} = w^{[2]T} dz^{[2]} .* g^{[1]'}(z^{[1]})}$

~~db[1]~~ ⑥ $\frac{dL}{dw^{[1]}} = \frac{dL}{dz^{[1]}} \frac{dz^{[1]}}{dw^{[1]}} = dz^{[1]} x$ $\boxed{dw^{[1]} = \frac{1}{m} dz^{[1]} x^T}$

⑦ $\boxed{\frac{dL}{db^{[1]}} = \frac{1}{m} * dz^{[1]}}$

Dimensional Analysis!- Forward Propogation

Input $x \rightarrow n^{[0]}$

Hidden layer $\rightarrow n^{[1]}$

Output layer $\rightarrow n^{[2]} = 1$

} No. of features.

$$W^{[1]} = (n^{[1]}, n^{[0]})$$

$$Z^{[1]} = W^{[1]} x + b^{[1]}$$

$(n^{[1]}, 1) \quad (n^{[1]}, n^{[0]}) \quad (n^{[0]}, 1)$

$b^{[1]} \rightarrow$ single element Broadcasts into $(n^{[1]}, 1)$

$$A^{[1]} = g(Z^{[1]})$$

$(n^{[1]}, 1) \quad (n^{[1]}, 1)$

$$W^{[2]} = (n^{[2]}, n^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$(n^{[2]}, 1) \quad (n^{[2]}, n^{[1]}) \quad (n^{[1]}, 1) \quad (n^{[2]}, 1)$

$$A^{[2]} = g(Z^{[2]})$$

$(n^{[2]}, 1) \quad (n^{[2]}, 1)$

Dimensional Analysis :-

Backward Propagation :-

$$dZ^{[2]} = A^{[2]} - Y$$
$$(n^{[2]}, 1) \quad (n^{[2]}, 1) \quad (n^{[2]}, 1)$$

$$dw^{[2]} = \frac{1}{m} dZ^{[2]} \cdot A^{[1]T}$$
$$(n^{[2]}, 1) \quad (n^{[2]}, m) \quad (1, n^{[1]}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]})$$
$$(1,1) \quad (1,1)$$

$$dZ^{[1]} = W^{[2]T} * dZ^{[2]} * g^{[1]'}(Z^{[1]})$$
$$(n^{[1]}, 1) \quad (n^{[1]}, n^{[2]}) \quad (n^{[2]}, 1) \quad (n^{[1]}, 1)$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]})$$
$$(1,1) \quad (1,1)$$