A Summary of SGL Mission Parameters Relevant to Einstein Ring Formation and Deconvolution

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Introduction

Below is a semi-exhaustive list of constraints for an SGL mission which may or may not affect images taken by the telescope, as well as their ability to be deconvolved (to our standards). We discuss the extent to which each constraint may affect measurements and what can be done to alleviate those effects.

Constraints

Here we discuss three types of constraints, corresponding to the three parts of the SGL "system:" the telescope itself, the Sun and the exoplanet being observed.

Telescope constraints

Size and resolution

One of the most obvious constraints on the quality of images the SGL will be capable of producing is the resolution of the telescope itself. [...]

Syzygal sensitivity

An important question we hope to answer with our research is how perfect the image of the Einstein ring has to be, i.e. how far from alignment can the telescope be allowed to stray while still collecting useful data? An SGL in perfect syzygy should obviously yield the best results, but this level of precision may not be something we can guarantee when operating the telescope from a distance of 550 au. **This, along with parametrizing the motion of the focus, are critical to the devonvolution discussion.**

Motion in the focal plane

Motion in the focal plane may not be very complex, but keeping alignment requires attention paid to variables pertaining to the motion of the planet being observed; this puts some constraints on the motion of the telescope itself.

Variables to be considered include:

1. Inclination of the planet's orbital plane,
2. Semimajor and -minor axes of the orbit,
3. Orbital period of the planet.

It is yet unclear whether or not the distance of the planet to the Sun is a relevant variable, so we will consider it last. It is also unclear what effect a focal distance of >550au will have on the final image. For this reason, paired with the notion that the distance of the planet to the Sun is insignificant (specifically at different points in the planet's orbit), we'll consider the motion of the telescope if it is held constant at the minimum opaque focal distance of 550au.

That being said, we how have a constraint on our telescope: *it must be held at a constant radius of 550au from the Sun's surface.*

The next variable to consider is the inclination, i, of the target planet's orbit, which will constrain the motion of the telescope on this 550au shell to a specific oscillatory path. There are two extremes, $i = 0$ and $i = 90$, for which the motion of the telescope is constrained to a straight curve or a small circle, respectively, while all intermediate inclinations constrain the path of the telescope to an ellipse, which complicates the motion somewhat, our orbital shell being a sphere.

This motion is yet to be modeled but, in theory, this adds another constraint to the telescope: *syzygal motion is constrained by the inclination of the exoplanet's orbit.* Of course, the dimensions of this path are not arbitrary, but are *further constrained by the dimensions of the orbit, namely the semiminor and -major axes of the orbit.*

Finally, we should surely consider the speed at which the telescope must move along this path in order to keep up with the moving focus. This depends on the orbital period of the planet. [...]

Solar coronagraph

Gain and magnification

Mission design

Solar constraints

Atmospheric plasma

The effect that plasma in the atmosphere of the Sun might have on light passing through it is a variable that has been considered in initial studies. One paper in particular (Turyshev & Andersson 2003) which investigated this phenomenon found that there exists a critical frequency at which light rays will be deflected, as a function of *impact parameter*. This is the frequency at which, at a given impact parameter, the light begins to be deflected away from the focus of the SGL. It is given by

$$\nu^2_{crit}(b) = (2.161GHz)^2 \times \left[2952\left(\frac{R_\odot}{b}\right)^{15} + 228\left(\frac{R_\odot}{b}\right)^{5} + 1.1\left(\frac{R_\odot}{b}\right)\right]. \tag{1}$$

I've run this calculation myself in the past and found that the solar plasma is transparent to optical light, i.e. there is no deflection in the regime we are most concerned with. This deflection occurs for larger wavelengths, even at $b = R_\odot$. **Thus, this should not be a concern for optical use of the SGL.**

Radiation

Even with a suitable coronograph, some radiation from the Sun's corona will likely cause interference. The effect of this interference will depend on the impact parameter of the Einstein ring produced, and likely the width of the ring.

The issue of impact parameter is simply explained – the further away the ring is from the limb of the Sun, the less coronal radiation will reach the ring to interfere with the image. As for the width of the ring itself, this has to do with the concentration of radiation in the ring, so to speak. The amount of radiation in the ring will decrease radially outward, and so thinner rings will have more solar noise in general.

These two issues in conjunction with one another raise the questions of (a) whether or not there is an impact parameter at which this interference is negligible and (b) how we might use images that have been polluted and what they might look like. **This is a question that is worth being explored more rigorously.**

Target constraints

Orbital parameters

Dimensions of and distance to target

The distance of the target from the Sun and the diameter of the target are variables which should, intuitively, affect the form of the Einstein ring produced. For example, a large planet relatively close to our solar system will likely produce a very thick Einstein ring, while a small planet farther away will produce a much thinner one. There is a tradeoff, then between the size of the planet being observed and its distance from us. **The relationship between these two variables is something to be considered.**

Host star

The brightness and proximity of the host star present yet another tradeoff in image quality. We want a well-illuminated planet for our images, so we can see features on the planet's surface (under the safe assumption there are no city lights to look at when it's dark). Thus, the ideal target ranges from planets very close to dimmer stars or farther away from brighter stars.

However, there may be a critical proximity to the host star at which the star and the planet cannot be resolved separately, or the host star may be seen in the image. This seems unlikely given the FOV we intend to see with the SGL, **but this prospect should be taken into consideration, just in case.**