Philosophical Boundaries

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Abstract

This article explores the concept of boundaries as a foundational principle in logic, mathematics, and systems theory. Here I venture into the possibility that structured boundaries (interfaces) are essential for coherence, complexity, and the avoidance of paradoxes, and since Nature does not seem to contradict itself, this may be informative on the nature of selves, and interaction. Viewing interfaces as non-self-referential connectors that enforce logical consistency, I demonstrate how interfaces dissolve Russell-like contradictions, clarify abstraction layers in biological and computational systems, paving a way to modeling interaction through geometric and algebraic constraints.

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> I celebrate myself, and sing myself, And what I assume you shall assume, For every atom belonging to me as good belongs to you. Song of Myself (1892 version) By Walt Whitman poetry foundation

1 Interface as a Formal Boundary

In the world of things, nothing can be a part of itself, or so it is said. To think about this is rather interesting. Regardless of human symbolic attribution in observations materialising categorisation and the classification of things such as a cell, a cell is cell via its membrane. No membrane, no cell. There is no contradiction, as the cell cannot be and not be simultaneously. But if we call it "kgkds" instead of cell, and attributed it to the same object, it would not change the fact that the object in question requires an identity to persist, and that part of that identity is its own morphological description. This partially agreeing with Aristotelians and partially disagreeing with positivists arguing that contradictions in nature are linguistic or cognitive errors, given by our perceptual limitations. A proposal is that scale and perspective at scale is where the issue resides. Moreover, paradoxes such as *Who shaves the barber*, and *Does infinity contain itself or not?* are familiar and I'm curious to which methods we could use to test a hypothesis where in both cases, for the barber and the infinity, they are a boundary, a formal interface to something else much like the cell or the it's reference object "kgkds". If we make a clear distinction between belonging (cell belonging to tissue) and composing (morphisms of atoms and molecules into cells), and allow specification and regularity (set / subset - blocking self membership Zermelo-Fraenkel) atoms compose and belong to cells simultaneously and there is no contradiction because they refer (belonging / composition) to different levels of abstraction (hierarchical description composition < belonging). There are several aspects of interest here: The first one is self-reference as a reusable pattern in nature, which should be allowed because it is observed, but in either case (allowed / not allow) a sound explanation should exist. The second is, in the case of infinity, what does this pose?

While hierarchical boundaries could in principle resolve logical contradictions by enforcing scale-dependent distinctions between composition (parts forming a whole) and membership (elements belonging to a set), this seems rather profound, touching on questions much greater than what I originally thought of.

1.1 Foundational Definitions & Clarifications

A cell's membrane defines its identity by separating it from what's not a cell. Without this boundary, the cell dissolves into its environment. Aristotle's law of identity ($A = A$) and as some logical positivist state: contradictions arise only when boundaries (physical or conceptual) are ambiguously defined.

At the atomic level, the cell is a dynamic composition of molecules; at the tissue level, it's a functional unit. Contradictions vanish when distinctions between belonging (cell □ tissue) and composing (atoms □ molecules □ organelles □ cell) are maintained across hierarchical scales. Russell's paradox (does the set of all sets that don't contain themselves contain itself?) arises from self-membership. Similarly, the "barber paradox" exposes inconsistencies in self-referential definitions. By banning self-membership (via the axiom of regularity), ZF set theory treats sets as hierarchical containers. A set cannot belong to itself, just as a cell's membrane prevents it from being its own environment.

In the "kgkds" example I used earlier, I attempted a humorous provocation to show that in this context identity depends on structural interfaces (e.g., membranes, mathematical axioms) rather than labels; it does not matter what we call this object, allowing us to introduce a certain secure abstraction and tangibility hereafter.

The implications are that self-reference and infinity are paradoxical inducers only when objects in scope are treated as monoliths rather than nested but well defined interfaces.

Theoretically, by distinguishing belonging (membership) from composing (structure), and embracing scale-dependent perspectives, ww could in principle model nature without logical inconsistency. Self-reference, scale-dependent perspectives, and paradoxes like Russell's, could just be ambiguous descriptions that dissolve under hierarchical structuring.

But can the axioms of regularity, cumulative hierarchies, and category theory be used to show / proof how systems avoid logical inconsistency through stratified organization?

If we let $\mathcal{H}$ denote a hierarchical system with scales indexed by ordinals α, then at each scale, we have lower-scale entities as composition:

$$x_\beta @ > \mathrm{Comp}_\alpha >> X_\alpha @ > \mathrm{Mem}_\alpha >> \mathcal{C}_{\alpha+1}$$
$$@V\partial_\beta VV @V\partial_\alpha VV @V\partial_{\alpha+1} VV$$
$$V_\beta @ > \subseteq >> V_\alpha @ > \subseteq >> V_{\alpha+1}$$

Which commutatively, $\partial_\alpha \circ \mathrm{Comp}_\alpha = \mathrm{Comp}_\alpha \circ \partial_\beta$ ensure boundaries propagate upward, with no self-membership, as the vertical ∂-arrows enforce $X_\alpha \notin X_\alpha$, as $X_\alpha \subseteq V_\alpha$ but $X_\alpha \in V_{\alpha+1}$.

And if we let $R = \{X \mid X \notin X\}$, under hierarchical boundaries, the resolution R is demoted to a **proper class** because $\mathrm{rank}(R) = \sup_{X \in R}(\mathrm{rank}(X) + 1)$, which is not an ordinal in ZF, and as such $R \notin V_\alpha$ for any α, avoiding $R \in R \iff R \notin R$.

For a "cell" C, $C = \mathrm{Comp}(\text{atoms}) \subseteq V_\alpha$, which for both composition, and membership we have $C \in \text{Tissue} \subseteq V_{\alpha+1}$.

Here, the rank inequality, $\mathrm{rank}(\text{atoms}) < \mathrm{rank}(C) < \mathrm{rank}(\text{Tissue})$, eliminates ambiguity.

1.2 Proof-*Kinda*

$$\boxed{\forall S\, \exists \alpha\, \left(\partial_\alpha(S) \neq \emptyset \implies \mathrm{rank}(S) = \alpha \wedge \neg \exists \beta \leq \alpha\, (S \in V_\beta)\right)}$$

- Where in $\boxed{\forall S\, \exists \alpha\ (\partial_\alpha(S) \neq \emptyset)}$: The existence of a boundary $\partial_\alpha(S)$ implies S is constrained to rank α.

- And in $\boxed{\text{rank}(S) = \alpha \wedge \neg\exists\beta \leq \alpha \, (S \in V_\beta)}$: The rank ensures S cannot self-belong or compose itself at any lower scale $\beta \leq \alpha$.

1.3 Conclusion

The question if this is informing us of a fundamental design principle in nature requires humility. It seems sound to say that hierarchical boundaries resolve contradictions by enforcing scale-dependent distinctions. But is objectionable from epistemic and quantum limits. Perhaps the assumption that nature's apparent non-contradiction arises not from human logic, but from boundary-defined strata that approximate reality's complexity. An inconsistency also seems apparent when treating scale separation as intrinsic to nature, because if theories succeed at describing gravity according to to quantum mechanics, the implication that space-time may be discrete at Planck scales blurs hierarchical distinctions. Yet, another point of contentions is the paradox resolve of infinity through proper classes, because for the case of an absolute infinity (canto'r) which is said to transcends all hierarchies, it implies a contradiction at the meta-level.

So, the following

$$Reality = \sum \partial(boundaries)$$

does not hold, because hierarchy may break down.

Maybe just relativity enforces c as a cosmic speed limit, hierarchical boundaries act as logical speed limits, preventing contradictions, and the rank function $rank(S)$ may be seen as analogous to spacetime intervals, invariant across frames (scales). At Planck scales, the hierarchy may break down, but emergent boundaries (e.g., decoherence) restore classicality. Thus, boundaries are effective, not fundamental, and hierarchy can be better seen as a map, and not the territory, although a useful map in understanding.

I care about abstraction-layers because logical levels of abstraction, and clarity in abstraction requires discipline in separation, allowing for higher-order relational reasoning, which in turn allows us to manipulate symbols pointing to large structures of knowledge and things, for sense-making at scale. If we take notions of composition and belonging those are not the same. They express distinct levels of of abstraction, where composition is to a lower-level of abstraction, and belonging a higher-level one. Interfaces as structured boundaries that enforce non-circular interaction, resolving paradoxes and enable scalable complexity without conflating identities. Here by identity I'm expressing a sort of coherence, a persistence, one that does not avoid completely environmental interactions, but one that allows for coherence.

While hierarchical boundaries resolve contradictions by enforcing scale-dependent distinctions as an effective tool. However I must confess to be confused because, quantum coherence expresses phase relationships between waves the ones that, together, describe an object. Then boundaries are coherence-preserving tools via interactions (decoherence), while also phase-relationships between waves, better put stabilized phase relationships?

If so, a workaround for it not to invalidate the boundary problem at the Planck scale is by having decoherence restoring effective boundaries at larger scales by averaging out Planck-scale chaos.

If at a quantum level decoherence means the loss of identity (dissolves quantum superposition), into a new identity (into classical definiteness) then at the collapsing superpositions, decoherence creates stable classical identities (a cell as alive, a set as rank-α).

From a classic point of view decoherence allow for identity then what does coherence do?

Also, if a cell's membrane is no more "real" than a qubit's coherence, given that both are stable patterns, a description of nature's strata that succeeds not by mirroring reality, but by description, what are interactions? Are they more or less real that substances (particles, cells, sets) that we perceive as distinct entities?

If we put as

$$Reality = \sum \partial(interactions)$$

where boundaries are fruit by interactions, then what allows interaction? And if the hierarchical boundaries don't hold between quantum and classical mechanics, but as mere effective negotiations through interaction density and scale, what can be the nature of this negotiation be?

Could it be so that from a relational-ontology perspective, interactions are some kind of fundamental means of reality, while boundaries are the habits of reality? Stable interaction patterns that enable prediction and coherence while decoherence negotiates quantum-classical transitions by filtering noise, much like a cell membrane filters toxins?

Can the hierarchical boundary axiom

$$\boxed{\forall S\,\exists\alpha\,\left(\partial_\alpha(S)\neq\emptyset \implies \text{rank}(S)=\alpha\wedge\neg\exists\beta\leq\alpha\,(S\in V_\beta)\right)}$$

and formalising interaction summation making up reality

$$Reality=\sum\partial(interactions)$$

mean anything at all beyond formal insights?