Philosophical Boundaries IV: Sheaf-Theoretic Co-Constitution

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Abstract

This paper contains a few revisions on the boundary relatare avan-propos and formalism attempts, derived from new insights related to identity-as-process, replacing hierarchical axioms with sheaf-theoretic coherence and structured cospans. We then re-attempt to address some paradoxes treated in the previous papers "On Boundaries" by treating boundaries as *contextual sections* rather than absolute separators, formalizing how entities co-constitute one another through asymmetric interaction patterns. Empirical validations experiments are re-designed to include temporal attractors and relational scaling.

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1 Introduction

In earlier papers concerning boundaries, I explored if they could be rendered contradiction-free by enforcing hierarchical distinctions: composition versus membership, part versus whole, each neatly separated by a rank axiom. The cell's membrane, used as an example of a static interface, a necessary wall between self and other, with hierarchy as the ultimate safeguard against paradox. While question on the nature of identity [1], it occurred to me, that my some of the previous notes on boundaries had limitations that contradicted some of the insights gathered in the process of formalise identity. Previously I was looking at boundaries as interfaces, and attempted to formalise without fully capturing their dynamism. Perhaps framing them even as absolute partitions, without full acknowledgement of the the cases where they appear *fuzzy*, such as clouds, where boundaries become contextual, dynamic, and co-constitutively embedded in their niches, that may not be

[1] "On Identity: Ideas On Being & Becoming"

reflected in the past approaches grounded in rank-theoretic axioms, and symmetric adjunctions.

By treating boundaries as static interfaces between pre-existing entities rather than as **processual co-constitutive dynamics** as explored in the Identity paper, the hierarchical axiom

$$\text{rank}(x) < \text{rank}(y) \implies x \notin y$$

presupposes that entities possess determinate ranks independently of their relational contexts, when in fact both entities and their organizational levels may be given through boundary interactions themselves. This was of course mentioned throughout the Boundary series papers, however the formulation need a revision. The identity paper's insight that existence is **relational becoming** rather than substantial being exposes this fundamental category error.

Mathematically, my reliance on ZF set theory and rank functions reflects what sheaf theorists recognize as the inadequacy of classical foundations for capturing contextual relationships (Chen and Harris 2025). The identity paper's sheaf formulation

$$\mathcal{I}_S(U) = \{\text{Properties of } S \text{ observable in context } U\}$$

reveals that boundaries are **sections of sheaves**, locally defined but globally coherent structures, rather than absolute separators between ranked sets.

My mistake was fruit of ignorance, I came across the realisation that set-theoretic approaches assume that membership relations ($x \in y$) are context-independent, but biological and social systems demonstrate that part-whole relationships are fundamentally **contextual**(**boundaries-2?**). A mitochondrion "belongs to" a cell in metabolic contexts but "composes" tissue in developmental contexts, relationships that sheaf theory captures through its gluing conditions but that rank hierarchies cannot accommodate.

The experimental validation in Boundaries III sought universal convergence thresholds ($\lambda \approx 0.9957$) across biological systems, and in the bioelectric experiments I conducted, measuring scaling laws in Xenopus tissue and slime mold networks, actually demonstrate contextual boundary formation rather than universal constants (Levin 2019; Chao, Fakhreddin, and Shimerov 2014).

The identity paper's core insight, that identity emerges through **structured cospans** $S \to I \leftarrow E$ where intrinsic invariants (I) maintain stability amid extrinsic flux (E), seems to somewhat challenges the boundary hypothesis symmetric assumptions. The retract condition $r : E \to I$ may better capture how environmental pressures asymmetrically constitute boundaries while preserving internal coherence, a dynamic that my previous symmetric adjunctions $B_S \dashv R_S$ completely missed.

Critically, the identity paper's temporal formulation

$$\mathrm{Id}(X) = \lim_{\to \text{interactions}} X_t$$

may better express boundaries as **temporal attractors** rather than spatial demarcations (Nimbalkar 2024).

More tangibly, *Boundaries I* explores the interface axiom but failed to recognize that interfaces are co-constituted rather than mediating. It is expressed in verbatim, however in *Boundaries II* the developed hierarchical rank functions impose artificial separations on what may be naturally continuous processes. Then, in *Boundaries III*, it is provided experimental validation of a shared transaction via scaling factor for bioelectric-patterns, that require a careful consideration to test this contextual variability require in case of sheaf-theoretic interpretation. In each paper it is proposed that fomalising boundaries could solve paradoxes, achieved by separating conflicting domains, and the operators for such separation did not express the tension, or asymmetry through which interactions occur, which in case of a cell's membrane, the asymmetry in voltage potentials seems to be what confers its identity as a cell. In this light, self-referential paradoxes may dissolves not through rank separation, which is incomplete, but through contextual restriction of self-membership domains, which a sheaf-theoretic approach expresses better than the hierarchical solution.

Here, we'll attempt to reconstruct the boundary theory through three revisions:
- First: replacing hierarchical axioms with **sheaf gluing conditions** to capture contextual coherence,
- Second: incorporating **asymmetric structured cospans** as an attempt to re-formalize the dynamics in boundary formation
- Third: redesigning experiments to incorporate the **temporal attractors** and **relational scaling**

1.1 Revisions

1.1.1 Revision 1: From Hierarchical Ranks to Sheaf Gluing

The original hierarchical axiom $\text{rank}(x) < \text{rank}(y) \implies x \notin y$ fails to capture part-whole relationships which are context-dependent. Model boundaries as sections of a sheaf $\mathcal{I}_S(U)$, where U denotes observational context:

$$\mathcal{I}_S(U) = \{\text{Properties of } S \text{ observable in } U\}$$

Cell:

$$\mathcal{I}_{\text{cell}}(\text{metabolism}) = \{\text{ATP production}\}$$

$$\mathcal{I}_{\text{cell}}(\text{immune}) = \{\text{MHC presentation}\}$$

.

Todo: Attempt to prove that Russell's set dissolves when U restricts self-membership contexts.

1.1.2 Revision 2: Structured Cospans -> Asymmetry in Boundary Formation

Replacing symmetric adjunctions $B_S \dashv R_S$ with **structured cospans** encoding intrinsic/extrinsic asymmetry:

$$S \to I \leftarrow E$$

- I: Intrinsic invariants (e.g., ion channels maintaining membrane potential).
- E: Extrinsic interactions (e.g., neurotransmitter release).
- $r : E \to I$ ensures homeostasis (e.g., membrane repair post-injury).

- Neuron: Synaptic inputs (E) rewire connectomes, but ion channel configurations (I) constrain plasticity, formalizing neurocentric asymmetry.
- Todo: Need to find experimental data -> test

1.1.3 Revision 3: Temporal Attractors In Boundaries States

Replacing static spatial demarcations with:

$$\text{Id}(X) = \lim_{\rightarrow \text{interactions}} X_t$$

- River boundary: Persists as a terrain-flow feedback loop, not a fixed bank.

- Todo: Review literature + find experimenatal data on "social identity", mentioning recursive discourse patterns, not just demographic categories.

1.2 Discussion

The reconceptualization of boundaries as *enabling constraints* overturns the classical view of boundaries as passive mediators between pre-existing entities. Where Boundaries I–III sought to resolve paradoxes through hierarchical containment

$$\text{rank}(x) < \text{rank}(y) \implies x \notin y$$

here we contemplate that things may exist through relational tension at boundaries, which seems to somewhat aligned with Brentano-Chisholm theory's of mutual ontological dependence between boundaries and continua (todo: cit) evolves into a dynamic asymmetry, where boundaries materialise not as static interfaces but as *processual stratifications* where intrinsic invariants (I) and extrinsic interactions (E) negotiate via structured cospans $S \to I \leftarrow E$ (todo: cit).

In Boundaries I, I argued that boundaries resolve paradoxes by mediating between contradictory properties (e.g., a cell as both whole and part). Boundaries II extended this to hierarchical stratification, positing that scale separation dissolves logical tensions (e.g., quantum-classical transitions via decoherence). But this framework failed to explain how boundaries themselves arise from the very systems they mediate, a circularity only resolved by the identity paper's insight: boundaries materialize through relational tension, not prior mediation.

The experimental validation in Boundaries III, which measured universal λ thresholds, is reinterpreted through sheaf gluing conditions. A boundary's sharpness becomes context-dependent:

$$\lambda(U) \geq 0.98 \quad \text{locally, not globally}$$

as demonstrated in slime mold networks under viscosity gradients [Results 4.2]. This aligns with Gieryn's boundary-work theory, where scientific demarcations adapt to contextual power dynamics [11]. Empirical rigor now demands tracking *sectional coherence* across observational contexts U, akin to software testing's boundary value analysis [12][20]. For example, a geopolitical border's permeability varies when analyzed as $\mathcal{I}_{\text{border}}(\text{migration})$ versus $\mathcal{I}_{\text{border}}(\text{trade})$ [8].

The structured cospan formalism exposes inherent asymmetries in boundary formation. Where Boundaries II treated adjoint functors $B_S \dashv R_S$ symmetrically, Boundaries IV's retract $r : E \to I$ codifies power imbalances:
- **Bioelectric systems**: Ion channels (I) constrain synaptic plasticity (E) [5.1.1]
- **Social hierarchies**: Legal borders asymmetrically privilege resident populations over migrants (**lamont2002boundaries?**)

The hierarchical scale separation in Boundaries I–II (cellular < tissue < organism) is replaced by:

$$\text{Scale}(S) = \int_U \mathcal{I}_S(U) \otimes \mathcal{I}_E(U)\, dU$$

where scales co-constitute through boundary interactions, or for instance, when integrin signaling between cells and ECM [5.1] generates tissue-scale coherence not by containment but through sheaf-section overlaps. This formalizes Latour's "flat ontology," where molecular and social scales interoperate via boundary functors.

1.3 Conclusion: Existence as Co-Constitutive Negotiation

Boundaries II treated scales (molecular, cellular, social) as pre-existing layers. The new framework reveals scales as **emergent negotiations**:

$$\text{Scale}(S) = \int_U \mathcal{I}_S(U) \otimes \mathcal{I}_E(U)\, dU$$

- **todo biological example**: A mitochondrion's scale shifts depending on whether we observe metabolic flux ($U =$ energy) or evolutionary history ($U =$ phylogeny).

- **todo social example**: A "family" scales as a nuclear unit in legal contexts ($U =$ custody law) but as an extended network in cultural ones ($U =$ genealogy).

  This replaces hierarchical ranks with *processual trajectories*, boundaries stratify not by containment, but through iterative reconfiguration.

1.3.1 References

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