On Identity: Ideas On Being & Becoming

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Abstract

This article explores the possible formalism for understanding how boundaries operate across scales.

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> I celebrate myself, and sing myself, And what I assume you shall assume, For every atom belonging to me as good belongs to you. Song of Myself (1892 version) By Walt Whitman poetry foundation

1 Introduction: On Identity

Traditionally, identity was seen as an intrinsic property, somewhat of a "self" defined by an immutable essence (e.g., Aristotle's *ousia*), and while Hegel and Mead argued that identity arises dialectically through interaction with the "other", I wonder if this holds true outside the human-specific context. I find quite a satisfaction "playing" with these notion of selves, identities and

relationships mathematically, due to the implicit clarity it requires, while remaining delightfully challenging to be precise towards it. The dialect in question reflect a position when someone is at stage of comparison of someone else in a context. Which is congruent with A in relation to B. In mathematics, identity is generally relational as well, where objects (e.g., groups, spaces) are defined *up to isomorphism* [1], by their morphisms (structure-preserving maps), and said group G's identity lies in how it maps to other groups (e.g., via homomorphisms), not just its internal elements, or entities derive meaning from their place in a hierarchy [2] (e.g., natural numbers in Peano arithmetic).

But what does it mean for a "thing" to be "itself"? Whichever name we chose to represent this, or any other "thing", is a reference to it, thus it would quite a reduction to say *A thing is a thing as a ball is a ball*. However, Aristotle's *ousia* finds numerous attempts to specify identity through distinction, and conversely, via tautological expressions of identity. The relation a thing bears with itself seem to be partially useful, as the relations it holds to what is not "itself", may reveal a sort of proprietary and informing uniqueness. On *various ways of questioning about the thing* some make yet another clear distinction between objects and things (Heidegger 1967) where "things" are human concepts where "thingness" could not be reducible to its material composition or its role as a bearer of properties. While I disagre with these statements, his proposal of "things" constituted through its relations, functions, seems of insightful value.

Transcending the paradigm of human-object relation, that draws from Heidegger and critical theory, into the origin and nature of identity, to think about how something bears a relation to itself as "Everything is identical to itself" $(x = x)$, which is one of the foundational axioms of classical logic and metaphysics (Lawvere 2003), where the Law of Identity $(\forall x(x = x))$ gives way to relational predicates: $x =_F y$ (x is the same F as y) ("Identity and Contextual Semantics in Complex Systems" 2019).

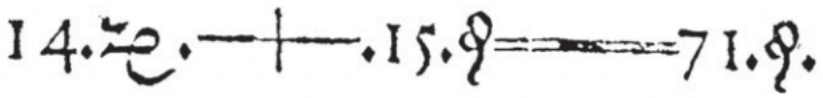

Figure 1: first use of =

The equality sign '=' (as well as the plus-sign '+', the minus-sign '–' and the word alge-

[1] cat: Category Theory
[2] type: Type Theory

> bra) saw the light of day in the treatise The Whetstone of Witte, whiche is the seconde parte of Arithmeteke: containing thextraction of Rootes; the Cossike practise, with the rule of Equation; and the workes of Surde Nombers (London, 1557), written by Robert Recorde (1510–1558), in the days before Shakespeare: And to avoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, of Gemowe (or twin) lines of one lengthe, thus: ======, bicause noe 2 thynges can be moare equalle. (Müller 2023)

While language certainly plays a role in how we individuate and talk about things, the axioms of identity are not merely linguistic conventions.

In the world of things, nothing can be a part of itself, or so it is said. To think about this is rather interesting. Regardless of human symbolic attribution interprets and generates observations, materialising them in structured categorisation and classification of things such as a cell, a cell is a cell via its membrane. No membrane, no cell. There is no contradiction, as the cell cannot be and not be simultaneously. But if we call it "kgkds" instead of a cell, and attribute it to the same object, it would not change the fact that the object in question requires an identity to persist, and that part of that identity is its own morphological description. This, partially agrees with Aristotelians and partially disagrees with positivists who argue that contradictions in nature result from linguistic or cognitive errors given by our perceptual limitations. A proposal is that scale and perspective at scale is where the issue resides. Moreover, paradoxes such as *Who shaves the barber*, and *Does infinity contain itself or not?* are familiar and I'm curious as to which methods we could use to test a hypothesis where in both cases, for the barber and the infinity, they are a boundary, a formal interface to something else much like the cell or the reference object "kgkds". If we make a clear distinction between belonging (cell belonging to tissue) and composing (morphisms of atoms and molecules into cells), and allow specification and regularity (set/subset - blocking self membership Zermelo-Fraenkel) atoms compose and belong to cells simultaneously and there is no contradiction because they refer (belonging / composition) to different levels of abstraction (hierarchical description composition < belonging).

They are formalized in logic and mathematics as equivalence relations (reflexivity, symmetry, transitivity). Some have argued that identity predicates are "ontologically generative," meaning that the very act of identifying or individuating things helps constitute the structure of reality itself. What would happen if we merged aristotle distinction, and had a numerical identity (being one

and the same) and qualitative identity (sharing properties) both formalised? Would it be sound to frame these properties as relational domain and range? Carrying the proposal to a bio-physical realm, a cell's identity, for instance, is potentially given by what allows for metabolic interfaces with its environment, a membrane. Without it, a cell would not be a cell, but loose environmental material. Perhaps identity is better put as requiring the "allowing" of self, which could be obfuscating the "allowing" for relation.

Further tension seems to point in the case of cross-domain applicability, as boundary ambiguity requires having an answer to how "sharp" must a boundary be? And how *stable*? Do ranges the ranges of variation in identity alike the established "tree-ness" throughout the seasons impose a clear constraint on the identitarian boundaries of a tree as an object, that according to a substance-ontology, where change is part of persistence.

> Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same river. (Plato Cratylus 402a = A6)

The central ideas of Heraclitus's saw a shared trait in the *opposites* and this was *change*. His insights on the perceived world as *constantly in flux, always "becoming" but never "being"" [@] and expressed as "No man ever steps in the same river twice" contrasting the ancient philosopher Parmenides, who is said to belie in a form of "being" that was immutably static. (Curd 2022)

In the river's aphorism the river persists as "river" despite complete molecular turnover, while the man's identity shifts through the very act of stepping. We we entertain the idea and explore it trough formalism, where can it takes us?

There is an implicit suggestion that identity is neither static substance nor mere flux, but a processual coherence that emerges through relation, river maintaining form despite molecular turnover via the terrain where if flows (Whitehead 1929) alike a human body replaces most cells from time to time, yet is still structurally recognizable through DNA-protein feedback loops. Similarly, online communities persist through member turnover via shared discourse patterns. Here, the relation is an state-enabled happening that occurs both in space and time, regardless of embodiment. Can the boundaries of self, and the embodied interface of *things* be some form of adjoint functors, and persist not despite change, but *through* it?

$$\text{Man}_t@ >>> \text{River}_t@VVV@VVV\text{Man}_{t+1}@ >>> \text{River}_{t+1}$$

Here, both man and river are transformed through their interface, the ability to interact, and what it interacts with. Hegel's *becoming* identity is a fixed point in the dialectic of boundary maintenance and relational adaptation, and via reflexivity of **higher-order adjunctions**, boundaries become objects of interaction, suggesting that *to exist is to be a morphism in the category of becoming* (Simondon 1958; DeLanda 2006).

The river's identity depends on both its internal flow *and* its interaction with the man, and pointed to as a reference by the man. The river's identity is co-constitutive. The man's identity is also altered by the river (e.g., wet feet, shifted perspective), suggesting a relational asymmetry.

> As the same thing in us are living and dead, waking and sleeping, young and old. For these things having changed around are those, and those in turn having changed around are these.(B88)

In question is the dual nature of identity, one one hand we have the relation to itself, what "allows" persistence, but persistence seems to be then connected to whatever is not part ot that identity almost as a constructive-coupling. This echoes notions of intrinsic and extrinsic *information*. Absolute identity is unique to each object; relative identity allows for sameness "in some respect" (e.g., two things might be identical in structure but not in context). which is analogous to identity being differential, in the sense that difference is what makes a pattern persist amid noise (Spencer-Brown 1969), but both the asymmetry and the opposite cannot be seen as static when the essence is transience.

Both Mead and Hagel posed that this relationship with a *"generalised-other"* allows for the development of self-awareness [3] which imply clear boundary of *self—allowing* for relation, but also demarcating what is not-self.

If we entertain the notion of *"self-maintenance and environmental coupling"* rather than a static essence of *"things"*, where identity requires both "allowing" self and relational aspect, then identity may procedural as well, the implication is *processual and transient*. Gilbert Simondon's *On the Mode of Existence of Technical Objects* (Simondon 1958) challenges the traditional cultural divide between hu-

[3] human: social-dynamics and self-awareness

mans and technology, arguing for a deeper understanding of technical objects as dynamic, evolving entities. While he beautifully critiques the reduction of technology to mere utility or alienating forces, proposing instead a framework that examines their genesis, evolution, and integration into human culture, from it a insight lies when he emphasizes the open-ended potential *nature* of objects.

Other have found this insight pertinent as it intersects with Alfred North Whitehead's process philosophy and Manuel DeLanda's assemblage theory, contributing to redefine *objects*, *identity*, and relational ontologies, destabilizing classical notions of identity, repositioning objects as dynamic, relational, and *multi-scale* processes (Whitehead 1929; DeLanda 2006).

So if there are no fixed "things", identity is given by intrinsic organization (self) and extrinsic interactions (relations), which are not fixed either, and what containing a *"thing"* into *"becoming"* are gradient-like transient states (e.g., cell membranes, social identities) that permit selective exchange *"allowing"* relation, how can we make sense of what is a *thing* and how to generalise identity?

If we juxtapose this river analogy and Heraclitus' aphorism "No man ever steps in the same river twice, …" [4] then persistence is both enabled and destabilised by interaction, and identity persistence is both bounded and allow by the asymmetry of being and relating.

This sounds rather familiar with Hoffmeyer's and Emmeche's notion of Code-duality (the semiotics of nature) where something encondes through its morphological composition its identity, and at the sime time, because of its morphology, there's an embedded description of it, which can be translated as a sign, pointer to it through abstraction. Now that I thing about it, when they (Hoffmeyer and Emmeche) state that "Biological information is not a substance", when referring to morphology, from the Latin *morph*, which puts a great smile on my face due to the an unforeseen and elegant overload (potentially a coincidence and said beauty requires rigorous scrutiny as I'm not a subscriber of elegance as a guide), [5] because information can be seen as a **morphism**, a structure-preserving map that encodes identity through relational patterns.

We must also contemplate the mereological contributions of part-whole relationships, however static they may be in classic mereology there's something of interest to said regarding levels of abstraction, composition and belonging. Identity can be mereologically defined:$x = y$ if and only if x and y have the same parts, and this approach makes mereology a simpler choice comapred to those

[4] river: No man ever steps in the same river twice, for it's not the same river and he's not the same man.
[5] informare: to bring something into form, which is the root of the now fashionable word information.

requiring separate identity predicates. However, in processual ontologies, this definition becomes problematic since parts are constantly changing while identity persists (Simons 1987). Comparing a relational perspective (relational ontology), which treats concrete particulars as "blobs" without internal ontological structure, with constituent ontologies, that analyze internal composition, a processual approaches offers us here a middle path, one where entities have dynamic structure given from ongoing part-whole relationships rather than fixed composition alone.

So a process-mereological coumpond could be proposed in principle by:
- Gradient boundaries create selective permeability rather than absolute separation
- Part-whole relationships are dynamically constituted through ongoing interactions
- Identity is given by organizational patterns rather than fixed composition
- Multiple scales of parthood operate simultaneously (molecular, cellular, organismic, social)

I previously emphasize how identity seems to have a duality, that must reflected in pre-axiomatic principles, which we could re-write them considering that we require: - Diachronic identity: Same organism across developmental stages
- Compositional flexibility: Parts can be replaced without losing systemic identity
- Contextual membership: Part-whole relationships vary across observational scales
- Adaptive boundaries: Membrane selectivity adjusts to environmental pressures + Parts can belong to multiple wholes simultaneously
- Beyond Composition: Wholes exhibit capacities absent in isolated parts

And we could simplify it by saying that the identity of a *thing* is: * Neither substance nor mere relation * It's a process which is transient * Has local properties * May have global properties / reflections

Furthermore we could propose that a thing is:
* a sign
* a thing (composition, boundary)
* relational
* a state

Elaborating on the thing we have:
* a sign implies information gradients that enable boundary formation
* a thing with categorical structures formalize the consistency conditions

* relational with a thing with categorical structures formalize the relational potential (heaves model how local sign-relations glue into global identity)
* a state allowing for entropy measures quantifying the information exchange required for state-persistent distinction

Which in turn seems to (if we play along) suggest that to answer the question "what makes a thing a thing?" we need:
* Its identity
* Its relational potential
* Its context (local - global)
* Its bundles of properties mapping (local - global)

The temporal and phenomenological dimensions of identity formation, in the process of "becoming and persistence" suggests that things are co-constituted and co-actors by the contextual embeddings at scale (local - global) wich seems to then implicate a spacio-temporal significante in the sign-making process wich seems in turn aligned with everything we know in developmental and evolutionary disciplines.

First things first, the intrinsic and extrinsic properties of things, and the tension allowing persistente we may draft that:

$$Identity = lim \rightarrow (InternalPhaseSeparation) \times lim \leftarrow (ExternalInteraction)$$

Summing up the idea of a *dynamic tension* between internal *differentiation* and *external coupling*, which at first seems phenomenological grounding and explains persistence amid change, but faces challenges in operationalising "process" boundaries. In the example of a cell, no mebrane no cell argument is not valid for a cloud, or what the sign of a cloud references to.

So instead of:

$$Identity = lim \rightarrow (InternalPhaseSeparation) \times lim \leftarrow (ExternalInteraction)$$

and

$$\text{Hom}(\text{Sign}(X), \text{Object}(Y)) \cong \text{Hom}(X, \text{Interpretant}(Y))$$

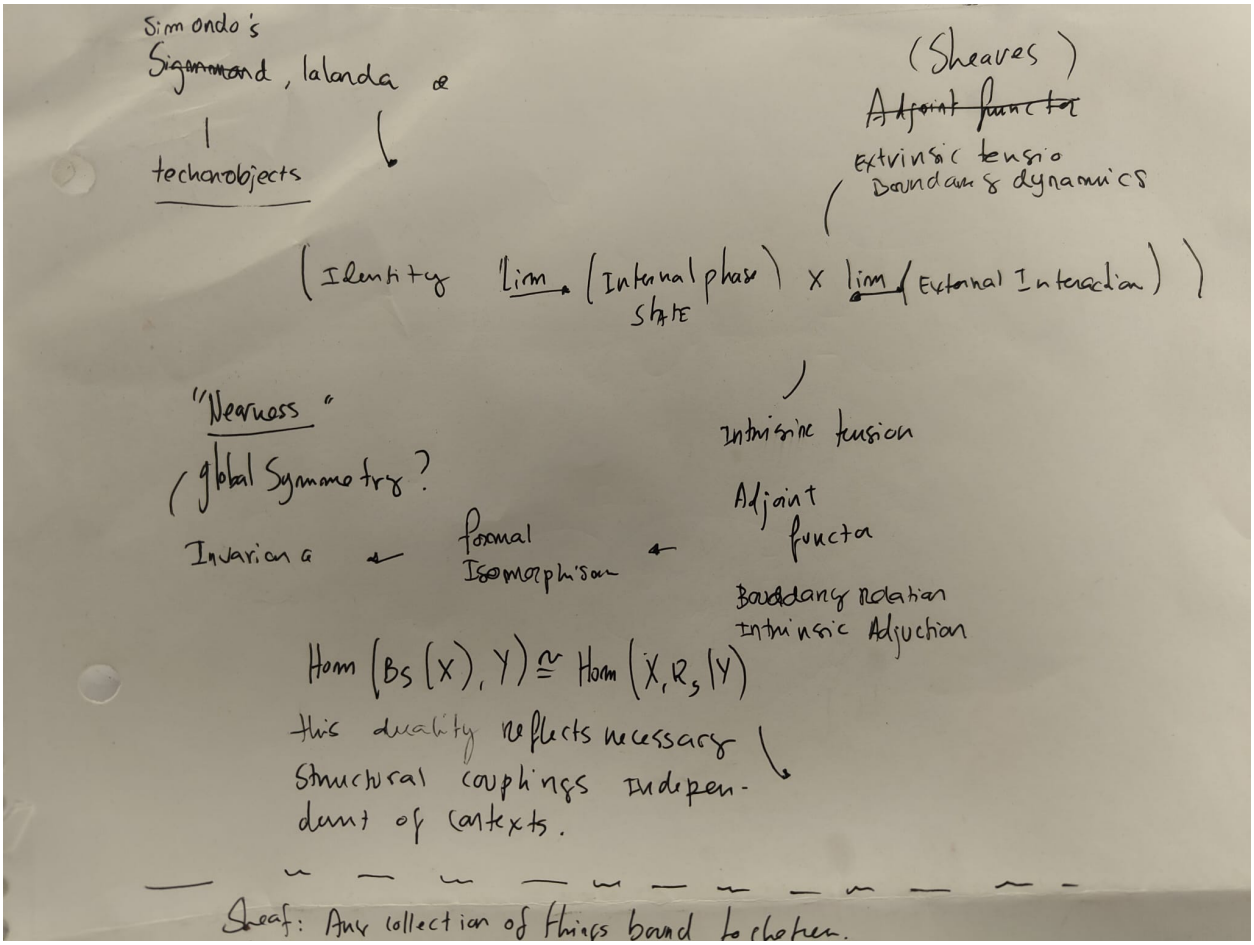

Figure 2: notebook-drawing

We may consider:

$$\text{Identity} = \varinjlim(\text{Internal Phase Separation}) \times \varprojlim(\text{External Interaction})$$

$$\text{Hom}_{\mathcal{D}}(B_S(X,t), Y) \cong \text{Hom}_{\mathcal{C}}(X, R_S(Y,t))$$

Where intrinsic tension expressed as:

$$B_S \dashv R_S : \mathcal{C} \to \mathcal{D}$$

$$\mathrm{Hom}_{\mathcal{D}}(B_S(X,t),Y) \cong \mathrm{Hom}_{\mathcal{C}}(X,R_S(Y,t))$$

and fuzzy boundaries, our dynamims of said boundaries is expressed as:

$$\partial S = \int_{\mathcal{E}} \phi(S,E)\, dE$$

$$\mathcal{I}_S(U) = \{\text{Properties of } S \text{ observable in context } U\}$$

To handle fuzzy boundaries and contextual variation, we model identity as a sheaf over a site of interactions:

$$I_S(U) = \{\text{Properties of } S \text{ observable in context } U\}$$

Identity persists when local sections glue coherently across overlapping contexts. This resolves the boundary ambiguity problem by treating sharpness as contextual rather than absolute.

With temporal properties as:

$$\mathrm{Id}(X) = \lim_{\to \text{interactions}} X_t$$

With identity criteria as:

$$\overline{S} = \{f \in \mathrm{Hom}(S,S) \mid B_S(f,t) = \mathrm{id}_S\}$$

$$\mathrm{Id}(S,\mathcal{K}) = \{R \mid \forall f \in \mathcal{K} : S \to T,\ f \circ R = R \circ f\}$$

Mutual parthood is possible during **phase transitions** when structured cospans exhibit non-trivial automorphisms, formalizing developmental plasticity. We may represent dynamic systems as structured cospans:

$$S \to I \leftarrow E$$

Where:

I: Intrinsic invariants (core organizational patterns)

E: Extrinsic interactions (environmental coupling)

S: The system mediating between internal and external domains

Persistence Condition: A system maintains identity if there exists a retract $r : E \to I$ such that

$$r \circ S = \mathrm{id}_I$$

This ensures intrinsic stability amid extrinsic flux.

And given than identity may recur:

$$\text{Boundary} \xrightarrow{\text{enables}} \text{Interaction} \xrightarrow{\text{reinforces}} \text{Boundary}$$

This creates a *state like point* where being and becoming coincide:

$$\mathcal{B}(\mathcal{I}(S)) = S \quad \text{and} \quad \mathcal{I}(\mathcal{B}(S)) = S$$

where $\mathcal{B}$ is the boundary operator and $\mathcal{I}$ is the interaction operator.

> Things stand in differenttruths, Heidegger says, meaning that the way a thing is understood depends on the conceptual and practical attitudes brought to it. (Gendlin 1967)

The differential aspect of what is it and what is not it may be expressed mathematically to have something like

$$\int iF(x) = \int iG(x), but \int eF(x) \neq \int eG(x)$$

in an attempt of capturing how entities can change externally while maintaining internal coherence.

Where:

- i represents the intrinsic domain (invariant structure)
- e represents the extrinsic domain (context-dependent interactions)
- $F(x)$ and $G(x)$ represent different temporal states of the same entity

which says that $F(x)$ and $G(x)$ are identical over their intrinsic domains (i) but differ over their extrinsic domains (e).

Through differential patterns that maintain intrinsic invariants while allowing extrinsic variation:

$$\int_i F(x) = \int_i G(x), \quad \int_e F(x) \neq \int_e G(x)$$

::: {.column-page} ## Postulating Identity Axioms

Positioning identity as a transient yet coherent *pattern* sustained by differential relation:

Classical Identity Foundations - **Law of Identity**:

$$\forall x(x = x)$$

, asserting that every entity is identical to itself[1].

- **Equivalence Relations**:

- *Reflexivity*:

$$x = x$$

- *Symmetry*:

$$x = y \implies y = x$$

- *Transitivity*:

$$x = y \land y = z \implies x = z$$

Domain-Relative Identity Entities exhibit distinct identities across intrinsic (internal structure) and extrinsic (contextual interactions) domains:

$$\int_i F(x) = \int_i G(x), \quad \int_e F(x) \neq \int_e G(x)$$

Here, $ F(x) $ and $ G(x) $ represent temporal states of an entity, with intrinsic invariants (i) enabling persistence amid extrinsic variation (e).

Processual Identity Identity is a dynamic tension between **Internal Phase Separation**, where self-maintenance via organizational patterns, and **External Interaction**, where elective coupling with the environment is allowed and enabled.

Formally:

$$\text{Identity} = \varinjlim(\text{Internal Phase Separation}) \times \varprojlim(\text{External Interaction})$$

This captures persistence through change, as seen in biological systems (e.g., cell membranes).

Identity is modeled as a **sheaf** over interaction contexts:

$$\mathcal{I}_S(U) = \{\text{Properties of } S \text{ observable in context } U\}$$

Sharp boundaries may exist by contextually, allowing for an array of boundaries contextually, with identity persisting when local properties cohere across overlapping contexts.

Diachronic Stability, where entities maintain identity across time via **Retract Condition**, where a mapping

$$r : E \to I$$

ensuring intrinsic invariants (I) stabilize extrinsic interactions (E):

$$r \circ S = \text{id}_I$$

and **Phase Transitions** structured cospans

$$S \to I \leftarrow E$$

formalize plasticity during developmental changes.

2 Discussion

Exploring this notion of boundaries via contextual sheaves seems to allow a few interesting configurations. Another approach as a boundary-centric model treating identity as a fixed container, like a cell membrane statically filtering molecules, would conflict with real-world systems such as immune cells that self from non-self, or social identities adapting to cultural contexts. Boundaries must be transient, adaptive permeable states. Via sheaves, or better put, by framing identity as a sheaf

$$\mathcal{J}_S(U)$$

, where boundarie *sharpness* is *locally defined* but *globally coherent*.

A tree, finds its identity across seasons, not inspite of having leaves or not, but by budding spring leaves (extrinsic domain

$$\int_e F(x)$$

) which vary wildly, but with vascular invariants (intrinsic

$$\int_i F(x)$$

) persist. In winter, leafless branches still satisfy the sheaf's gluing condition, overlapping contexts which maintain coherence. This oposes substance ontology's "tree-as-object" in favor of **processual glueing**, where identity exhibits local properties (e.g., photosynthesis in summer, dormancy in winter) yet agree on overlaps (e.g., nutrient storage in autumn), unlike static boundaries, sheaves allow identity to be *renegotiate* as its edges change contextually.

The implicit code-duality which I originally expressed via morphisms to signs (

$$\text{Info} : \text{Morph}(\mathcal{C}) \to \text{Sign}(\mathcal{S})$$

), struggled with ambiguous encodings (a gene encoding multiple protein isoforms), however now,

information can actually be expressed as a **sheaf morphisms**

$$\text{Info} : \text{Hom}(A, B) \to \Gamma(\mathcal{J}_S, \mathcal{J}_T)$$

where for instance a DNA sequence (A) maps to a *section* of possible protein folds (

$$\Gamma(\mathcal{J}_S)$$

), constrained by cellular context (e.g., chaperone availability).

More spceifically, thegene *FOXP2* has a role in speech evolution which isn't just about nucleotide sequence (intrinsic domain) but its interaction with neural crest cell environments (extrinsic sheaf

$$\mathcal{J}_S(U)$$

). Code-duality thus becomes a dialogue between invariant genetic "text" and context-dependent "meaning."

The poetics rivers, retracts, and phase-transitions may go beyond metaphor.

$$\begin{array}{ccc} \text{Man}_t & \longrightarrow & I_{\text{metabolism}} \\ \downarrow & & \downarrow \\ \text{Man}_{t+1} & \longrightarrow & \text{River}_{t+1} \end{array}$$

The map r ensures metabolic invariants (I) stabilize the system against external perturbations (e.g., the man drinking river water, the river reshaping its banks). And phase-transitions may be tracked when perturbations exceed $\$ \square < 0.3\$$ (pollution altering river pH?), the cospan $S \to I \leftarrow E$ bifurcates, forcing identity reconfiguration (need better expamples than the man developing immunity, the river evolving microbial communities).

In Heraclitus' paradox, the river's identity is not *despite* molecular turnover but *because* its sheaf

$$\mathcal{J}_{\text{river}}(U)$$

gluing local flow states (rapids, pools) into a global process, strenghening the argument of a

relational identity, which beyond adjoint functors,

$$B_S \dashv R_S$$

(boundaries □ relations) implied symmetry, however beautiful, conflics observations. Observations concerning identity suggest that this is morelikely to be *asymmetric*. A neuron's identity, for example, seems more influenced by synaptic inputs ($ R_S $)$thanitsownfiringpatterns$(B_S $).Ibeleivethatvia * *structuredcospans * *and * *domainrelativity * *the * *intrinsicdomain * *maybewellexpressedthroughionchannelconfigurations$(∫_i F(x) $)whichmaintainhomeostasis, and**extrinsicdomain**byneurotransmitterinteractions$(∫_e F(x) $) which then rewire the connectome.

Almost as if identity becomes a **limit** reconciling these domains (intrinsic - extrinsic)

$$\text{Id}(\text{neuron}) = \lim_{\rightarrow \text{network activity}} \left(\int_i F(x) \times \int_e F(x) \right)$$

This seems to have minor implications in the inquiry domains of Philosophy and Biology, as the anti-assentialism is revisited and the dissolve the essence/accident dichotomy. A lymphocyte's identity isn't defined by surface receptors (old Axiom 1) but by its sheaf

$$\mathcal{J}_{\text{lymphocyte}}(U)$$

, which might include:
- **Infection Context**: Presenting antigens (extrinsic).
- **Developmental History**: Bone marrow training (intrinsic).

2.0.1 Symbiogenesis

Lichens—fungus-algae composites—challenge the old boundary model. The sheaf framework treats them as **nested identities**:

$$\mathcal{J}_{\text{lichen}}(U) = \mathcal{J}_{\text{fungus}}(U) \oplus \mathcal{J}_{\text{alga}}(U)$$

where

$$\oplus$$

represents resource exchange (e.g., carbon sharing) gluing overlapping contexts.

3 Synthesis: Identity as Convergent Process

The river-man system epitomizes the new axioms:

1. **Sheaf-Theoretic Boundaries**: The river's "edge" is defined differently by a fish (habitat), a farmer (irrigation), and a poet (metaphor).

2. **Processual Stability**: The river retains its identity not through fixed banks but through *erosional feedback*—the very act of flowing reshapes the terrain that guides it.

3. **Diachronic Limit**: Over geologic time, the river converges to a graded profile (

$$S^*$$

), a fixed point balancing uplift and erosion.

In this view, identity is neither static nor fragmented but a **convergent narrative**—a story that coheres across scales and contexts, much like a novel whose chapters are written by different authors yet resolve into a plot. The new axioms provide the grammar for this narrative, unifying Heraclitus' flux with Hegel's synthesis in a mathematical framework ripe for testing in systems biology and AI ethics.

3.1 Conclusion: To Step Again is to Redefine

Identity is neither a static essence nor an ephemeral illusion but a **contextually coherent process** sustained by differential relations and sheaf-theoretic gluing. It emerges as the convergent limit where intrinsic invariants (

$$\int_i F(x)$$

) stabilize extrinsic interactions (

$$\int_e F(x)$$

), much like a river's flow is simultaneously shaped by and shaping its terrain. To "step into the river twice" is to navigate the **sheaf of becoming**, where each interaction reveals new local sections (contextual properties) while preserving global coherence through retract mappings (Axiom 5).

Hegel's dialectic and Heraclitus' flux converge in the **processual identity** framework: identity is a directed, relational rhythm, harmonizing phase separation (internal cohesion) and environmental coupling (external adaptation). Just as a lichen's symbiotic identity arises from nested sheaves (fungal-algal

$$\mathcal{J}_{\text{lichen}}(U)$$

), human selves persist through recursive adjunctions—psychological boundaries (

$$B_S$$

) filtering social interactions (

$$R_S$$

) that, in turn, reshape those very boundaries. This mirrors mathematical **sheaf morphisms**, where compatibility conditions ensure local-to-global consistency without rigid invariance.

The "self" is not merely the limit

$$\text{Id}(X) = \lim_{\rightarrow \text{interactions}} X_t$$

but the **structured cospan** reconciling intrinsic and extrinsic domains:

$$X_t @ >>> I @ >> I @ <<< E_{t+1}$$

Here, the retract

$$r$$

ensures metabolic, cognitive, or social invariants (

$$I$$

) anchor identity amid environmental flux (

$$E_t$$

). Persistence is thus a dynamic equilibrium, akin to immune cells adapting to pathogens while maintaining self-tolerance through contextual sheaves.

Non-commutativity persists—the man's footstep alters the river's sediment, while the river's chill reshapes the man's gait—but **asymmetric adjunctions** (

$$B_{\text{river}} \dashv R_{\text{man}}$$

) formalize this interplay. Boundaries are not passive containers but active participants, as seen in neuronal identity: ion channels (intrinsic

$$\int_i$$

) stabilize synaptic input patterns (extrinsic

$$\int_e$$

), each interaction a phase transition in the connectome's sheaf.

Hegel's terrain-river feedback becomes a **natural transformation** in the category of process:

$$\mathcal{H} : \text{River} \Rightarrow \text{Terrain}, \quad \mathcal{H}(t+1) = r \circ \mathcal{H}(t) \circ \mathcal{H}_{\text{erosion}}$$

where erosion (external interaction) and retraction (internal invariance) co-constitute the river's diachronic identity. Similarly, a cell's membrane (

$$\mathcal{I}_{\text{cell}}(U)$$

) is both a boundary and an interface, its lipid sheaf gluing metabolic states across time.

The recursive loop of selfhood—observing the river while being transformed by it—echoes Gödelian self-reference but finds resolution in **sheaf cohomology**: identity's consistency is a

higher-order section (

$$H^1(\mathcal{I}_S)$$

), locally verifiable but globally emergent. To exist is to be a **section of becoming**, where every interaction updates the sheaf while preserving gluing conditions.

Thus, identity is the **commuting diagram of existence**, a symphony of structured cospans and contextual sheaves. Each step into the river is a morphism rewriting both man and river, yet the composition converges to a melody of persistence—not despite change, but *through* it. In this light, Heraclitus' paradox dissolves: we *are* the river, not because our molecules persist, but because our sheaves cohere.

3.2 Steps Forward

4 Future Work: Sheaves, Symmetries, and Post-Structuralist Syntheses

The integration of sheaf theory with identity dynamics and post-structuralist philosophy opens transformative research avenues. Below, we outline key directions for advancing this synthesis, grounded in mathematical rigor and empirical applicability.

4.1 1. Sheaves and Symmetries: Quantifying Identity Dynamics

Global vs. Local Symmetries in Identity Mapping

Sheaf theory's capacity to model **local-to-global coherence** (via

$$H^0(\mathcal{F})$$

) and **obstructions** (via

$$H^1(\mathcal{F}) \neq 0$$

) provides a framework to formalize identity persistence and transformation:

- **Local Symmetries**: Automorphisms within stalks (e.g., metabolic homeostasis in cells) represent *invariant subprocesses* under perturbation.

- **Global Symmetries**: Natural transformations between sheaves (e.g., developmental phase transitions) encode *identity-preserving morphisms* across scales.

Example: In immune cell differentiation,

$$H^1$$

obstructions could quantify failures in maintaining lineage identity under cytokine noise, while

$$H^0$$

sections model successful differentiation trajectories.

Future Direction: Develop **sheaf-based invariants** for identity persistence:
- For biological systems: Compute

$$H^1(\mathcal{J}_{\text{cell}})$$

to predict carcinogenic transitions (loss of cellular identity).
- For social systems: Model institutional identities as sheaves over discourse networks, with

$$H^1$$

detecting ideological fractures.

4.2 2. Experimental Validation: Protocols and Metrics

Proposed Empirical Frameworks

- **Biological Identity**:
- **Protocol**: Track *Planaria* regeneration under RNA interference, mapping gene expression sheaves

$$\mathcal{J}_{\text{planaria}}(U)$$

across wound sites.
- **Metric**: Obstruction degree

$$\dim H^1$$

correlates with failed regeneration (loss of body-plan identity).

- **Social Identity**:

– **Protocol**: Analyze online community dynamics (Reddit, Twitter) as sheaves over interaction graphs.

– **Metric**: Sheaf Laplacians (from [2][6]) quantify polarization (

$$\lambda > 0.3$$

) vs. consensus (

$$\lambda < 0.1$$

).

Challenge: Bridging qualitative post-structuralist critiques (e.g., Foucault's *biopower*) with quantitative sheaf obstructions.

4.3 3. Post-Structuralism and Higher-Categorical Structures

Beyond Sheaves: ∞-Categories and Homotopy

- **∞-Sheaves**: Model identity as a *homotopy-coherent process*, where higher morphisms (2-cells, 3-cells) encode meta-stable transitions (e.g., trauma recovery in psychology).
- **Example**: Deleuze's *body without organs* as an ∞-sheaf resisting stratification into fixed stalks.

Ethical Implications:

- **Normative Shift**: Replacing essentialist legal/political categories (race, gender) with **processual adjunctions** (boundary □ relation functors).
- **Risk**: Sheaf obstructions (

$$H^1$$

) could inadvertently naturalize systemic inequities as "inevitable coherence failures."

4.4 4. Computational Implementation

Sheaf Neural Networks for Identity Modeling

- **Architecture**: Adapt *sheaf diffusion networks* ([2][6]) to simulate identity dynamics:

“‘python class SheafIdentityLayer(nn.Module): def **init**(self, stalk_dim, restriction_maps): super().__init__() self.stalks = nn.ParameterDict({U: nn.Linear(stalk_dim, stalk_dim) for U in open_sets}) self.restrictions = restriction_maps # Morphisms between stalks

```
def forward(self, x, context_U):
    # Restrict input to context U and transform
    x_local = self.stalks[context_U](x)
    # Glue with adjacent contexts via restrictions
    x_global = sheaf_gluing(x_local, self.restrictions)
    return x_global
```

“‘ - **Application**: Train on longitudinal biodata (e.g., microbiome shifts) to predict healthspan identity transitions.

4.5 5. Post-Structuralist Synthesis: Rigorizing Metaphors

From Différance to Differential Sheaves

- **Formalizing Derrida**: Model *différance* as a **connection on a sheaf**, where meaning deferral arises from non-integrable sections (

$$\nabla^2 \neq 0$$

).

- **Foucault's Discourses**: Represent *epistemic regimes* as sheaves over power-topological spaces, with restriction maps enforcing ideological compatibility.

Critique: Current humanities applications often use sheaf theory metaphorically. Future work must:

- Ground claims in explicit adjunctions (e.g., knowledge □ power as

$$B_S \dashv R_S$$

).

- Validate against historical/textual datasets (e.g., sheaf cohomology of archival networks).

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5 Conclusion: Toward a Processual Science of Identity

The fusion of sheaf theory, post-structuralism, and complex systems science promises to redefine identity as a **convergent sheaf of becoming**. Key milestones include:

1. **Experimental Protocols**: Quantifying

$$H^1$$

obstructions in biological/social systems.

2. **Higher-Categorical Models**: Encoding identity transitions via ∞-sheaves.

3. **Ethical Frameworks**: Replacing essentialism with processual adjunctions in policy.

This program demands interdisciplinary collaboration—mathematicians formalizing philosophers' intuitions, empiricists testing sheaf-theoretic predictions, and ethicists navigating the normative shifts such models entail. The result could be a unified science of identity, as fluid as Heraclitus' river yet as rigorous as Grothendieck's topos.

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