Advanced Control Methods
Project

Model Predictive Control of the Droplet Generation System

Mikhail Vulf,
Aleksandr Kolomeitcev

Skolkovo
Institute of Science
and Technology

Skoltech

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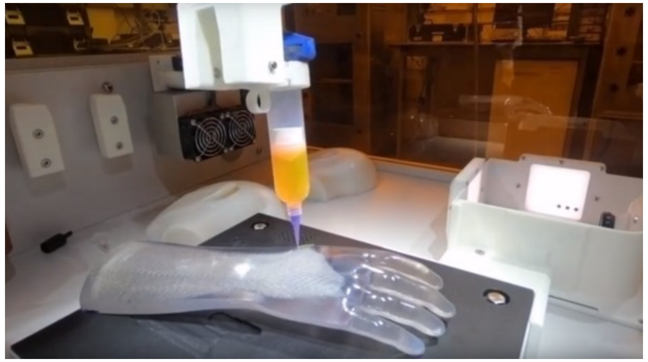
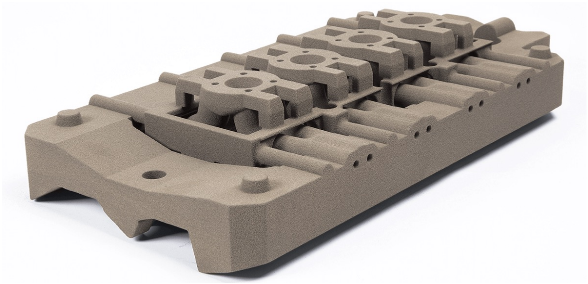
Droplet Generation System. Relevance

Droplet-based 3D-printing and coating

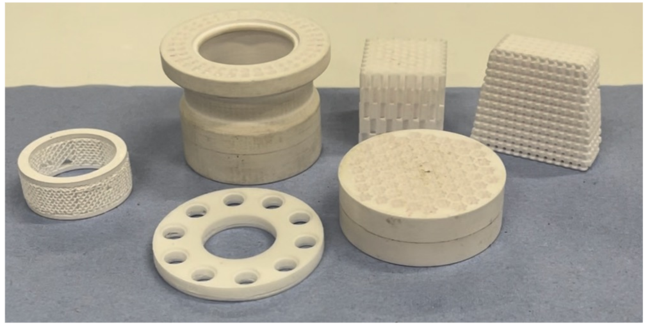
Spheroid Bioprinting *[1-3]*



Sand Molds
[LLC "Globatek", LLC "Robotech"]



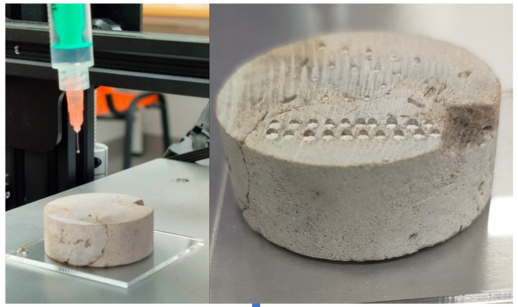
Suspension Ceramic *[4]*

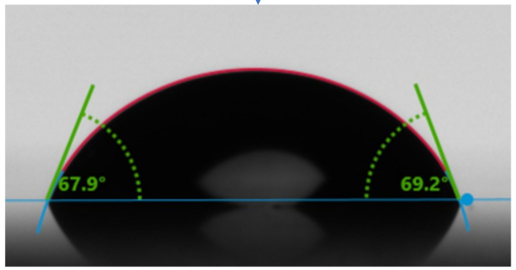


Metal droplet printing *[5]*

Dispersion flows studies

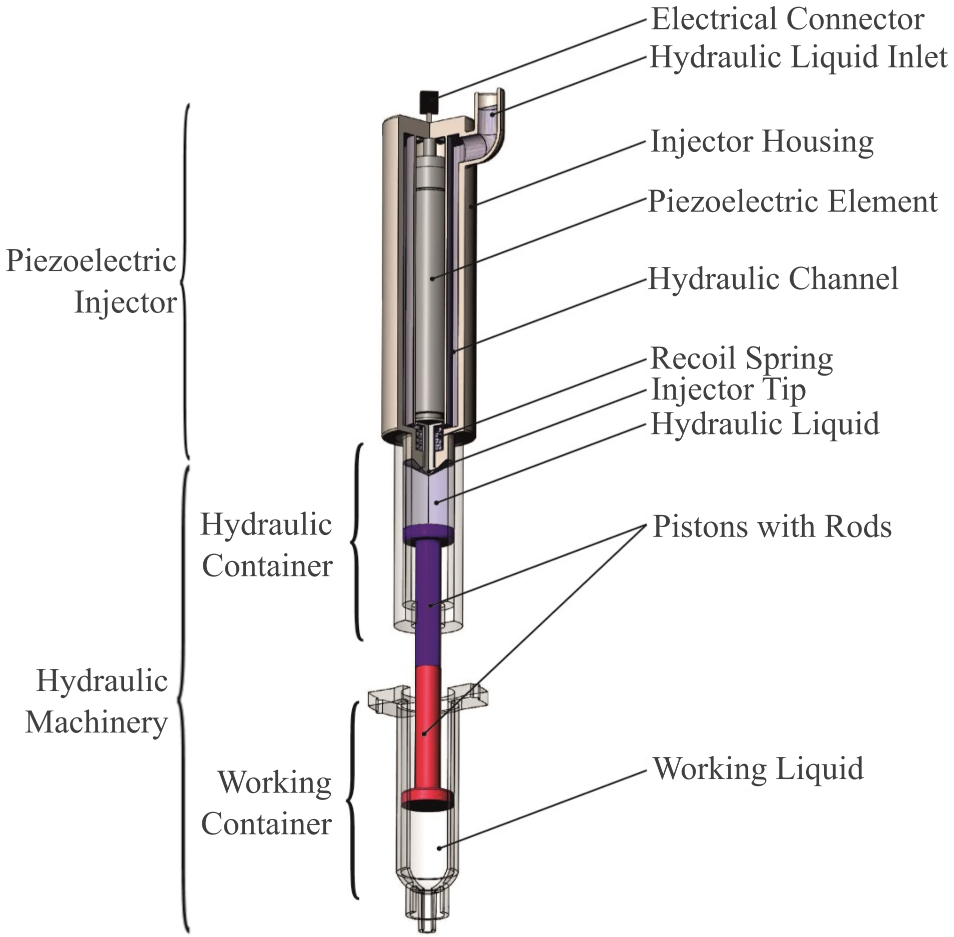
Contact angle measurements
[LLC "Gazprom Neft]





Droplet Generation System

Suspension DoD Droplet Generator



Problem

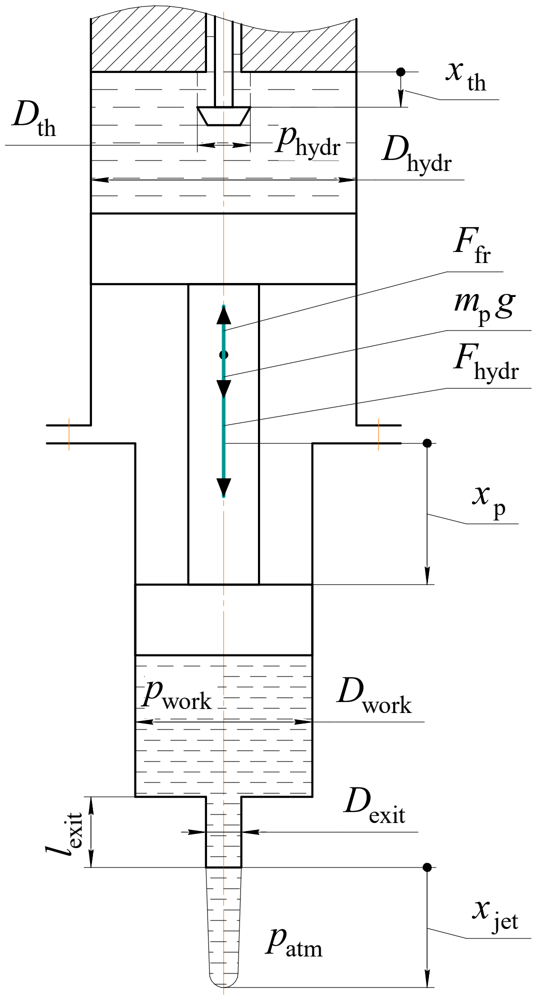
Difficult to adjust the parameters for generating a <u>single drop of the desired size</u>

Single droplet formation [6]

Droplet satellite formation [6]

Problem setup

Calculation Scheme



State

$$\boldsymbol{x} = [\boldsymbol{x}_{\mathbf{p}}, \boldsymbol{v}_{\mathbf{p}}, \boldsymbol{x}_{\mathbf{th}}, \boldsymbol{p}_{\mathbf{hydr}}, \boldsymbol{p}_{\mathbf{work}}]^{\boldsymbol{T}}$$

Action

$$u = \boldsymbol{x}_{\mathbf{th}}^{\mathbf{act}}$$

$$\dot{\boldsymbol{x}}_{\mathbf{th}} = \boldsymbol{f}_{\mathbf{th}}(\boldsymbol{x}_{\mathbf{th}}^{\boldsymbol{act}} - \boldsymbol{x}_{\mathbf{th}})$$

Piston Dynamics

$$\boldsymbol{m}_{\mathbf{p}}\dot{\boldsymbol{v}}_{\mathbf{p}} = \boldsymbol{m}_{\mathbf{p}}\boldsymbol{g} + \boldsymbol{F}_{\mathbf{hydr}} - \boldsymbol{F}_{\mathbf{fr}}$$

$$F_{\mathrm{hydr}} = p_{\mathrm{hydr}}A_{\mathrm{hydr}} - p_{\mathrm{work}}A_{\mathrm{work}}$$

$$F_{\mathrm{fr}} = \max(F_{\mathrm{C}}, (1-\eta)F_{\mathrm{hydr}})$$

where F_{C} - coulomb friction force

Pressures

Liquid compressibility:

$$\beta_V = -\frac{dV}{Vdp} = \mathrm{const}$$

$$\dot{\boldsymbol{p}}_{\boldsymbol{hydr}} = -\frac{\mathbf{1}}{\boldsymbol{V}_{\mathbf{hydr}}\boldsymbol{\beta}_{\boldsymbol{V}_{\mathbf{hydr}}}}(\boldsymbol{A}_{\mathbf{hydr}} \cdot \boldsymbol{v}_{\mathbf{p}} - \boldsymbol{Q}_{\mathbf{th}})$$

$$\dot{\boldsymbol{p}}_{\boldsymbol{work}} = -\frac{\mathbf{1}}{\boldsymbol{V}_{\mathbf{work}}\boldsymbol{\beta}_{\boldsymbol{V}_{\mathbf{work}}}}(\boldsymbol{Q}_{\mathbf{exit}} - \boldsymbol{A}_{\mathbf{work}} \cdot \boldsymbol{v}_{\mathbf{p}})$$

$$Q_{\mathrm{th}}^2 = 2A_{\mathrm{th}}^2(x_{\mathrm{th}})C_{D_{\mathrm{th}}}^2\frac{p_{\mathrm{l}} - p_{\mathrm{hydr}}}{\rho_{\mathrm{hydr}}}$$

$$Q_{\mathrm{exit}}^2 = 2A_{\mathrm{exit}}^2C_{D_{\mathrm{exit}}}^2\frac{p_{\mathrm{work}} - p_{\mathrm{atm}}}{\rho_{\mathrm{work}}}$$

Droplet detaching

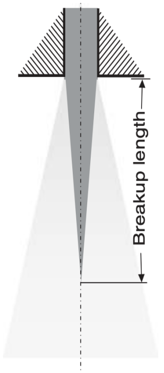
Drop detaching condition [6]

$$l_{\text{jet}} \geq l_{\text{crit}}$$

However, $l_{\text{jet}} \uparrow \rightarrow n_{\text{satellites}} \uparrow$

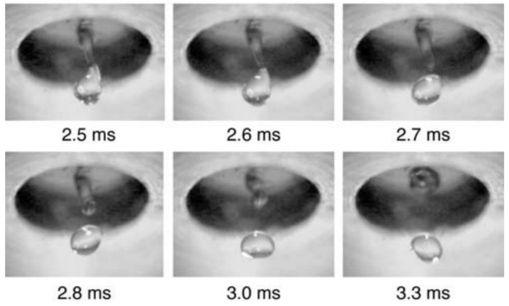
$$\boxed{x_{\text{jet}} \searrow l_{\text{crit}}}$$

Drop formation from jet [8]

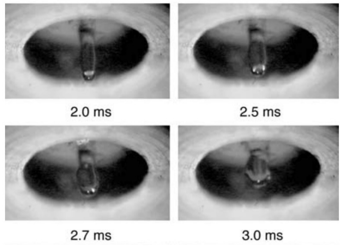


l_{jet}

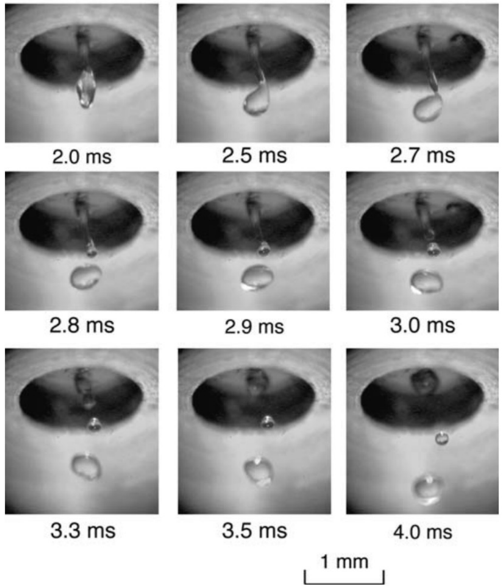
$l_{\text{jet}} = l_{\text{crit}}$



$l_{\text{jet}} < l_{\text{crit}}$

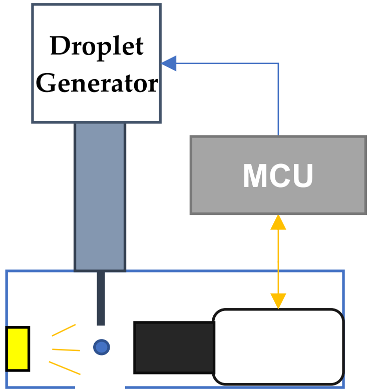


$l_{\text{jet}} > l_{\text{crit}}$



Observations and Running Cost

Observations



$$y = \begin{bmatrix} x_{\text{jet}} \\ v_{\text{jet}} \end{bmatrix}, \qquad \varepsilon_x \sim \mathcal{N}(0, \sigma_x^2), \qquad \varepsilon_v \sim \mathcal{N}(0, \sigma_v^2),$$

$$\hat{x}_{\text{jet}} = 10^{-3} \frac{D_{\text{work}}^2}{D_{\text{exit}}^2}(x_{\text{p}} - x_{\text{p0}}), \qquad \hat{v}_{\text{jet}} = 10^{-6} \frac{D_{\text{work}}^2}{D_{\text{exit}}^2} v_{\text{p}},$$

$$x_{\text{jet}} = \hat{x}_{\text{jet}} + l_{\text{crit}} \varepsilon_x, \qquad v_{\text{jet}} = \hat{v}_{\text{jet}} + \frac{l_{\text{crit}}}{\Delta\tau} \varepsilon_v,$$

Running cost

$$c(\tilde{y}, u) = \tilde{y}^T Q \tilde{y} + u^T R u,$$

$$\tilde{y} = y - \begin{bmatrix} l_{\text{crit}} \\ 0 \end{bmatrix},$$

$Q = \text{diag}(1{,}0),$
$R = \text{diag}(0)$

Relative value function

$$V_{\text{episode}}(\tilde{y}, u) = \sum_{t=1}^{T} \gamma^t \frac{c(\tilde{y}_{\text{t}}, u_{\text{t}})}{l_{\text{crit}}^2} \Delta\tau$$

Averaged value function

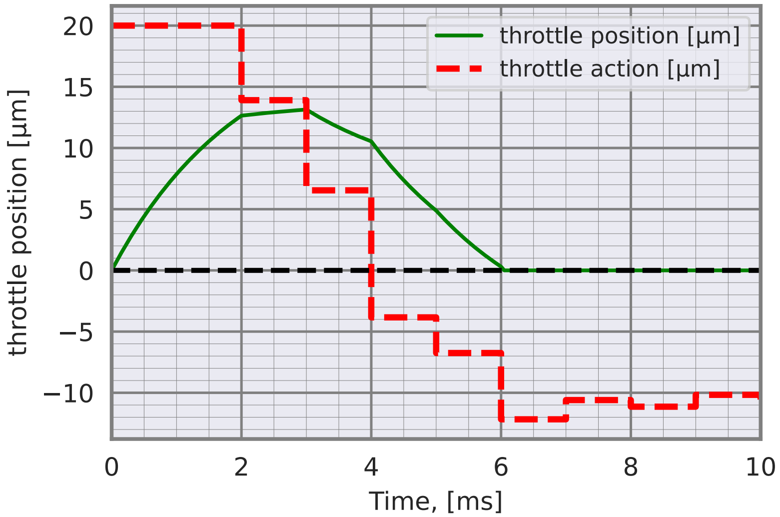
$$V(\tilde{y}, u) = \frac{1}{M} \sum_{j=1}^{M} V_{\text{episode}}(\tilde{y}, u)$$

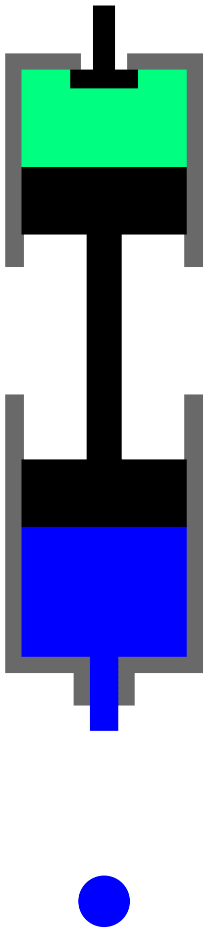
PD-regulator

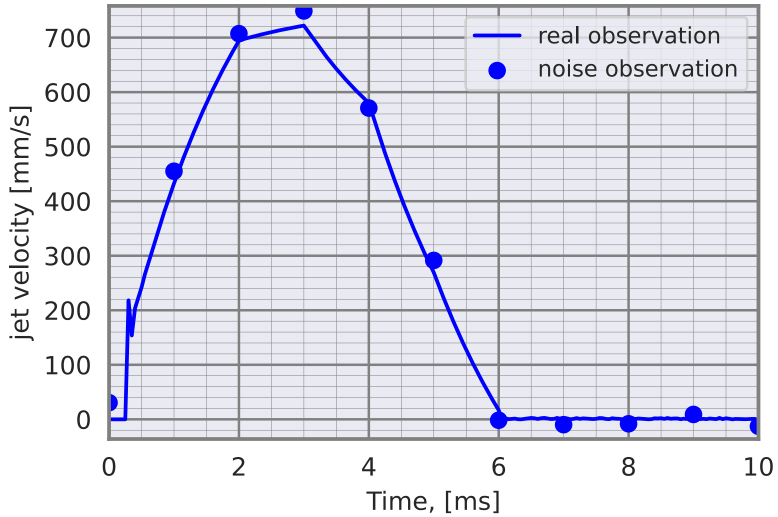
PD-policy

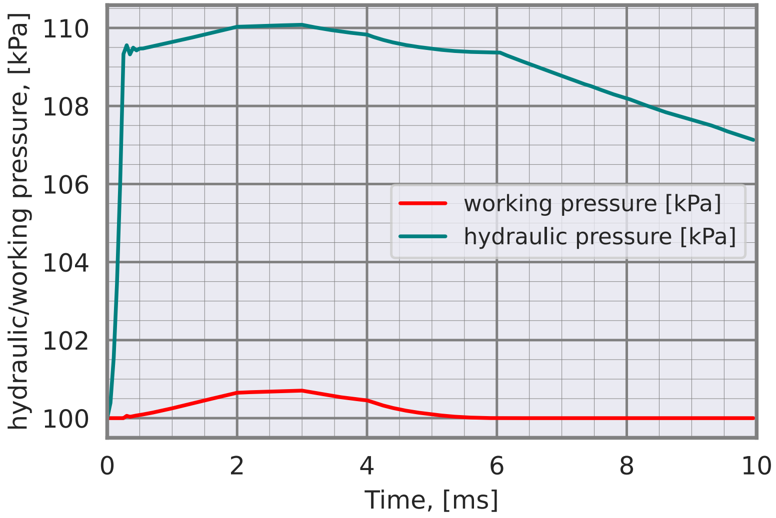
$$u(x_{\mathrm{jet}}, v_{\mathrm{jet}}) := P\left(1 - \frac{x_{\mathrm{jet}}}{l_{\mathrm{crit}}}\right) - D\left(\frac{v_{\mathrm{jet}}\Delta\tau}{l_{\mathrm{crit}}}\right)$$

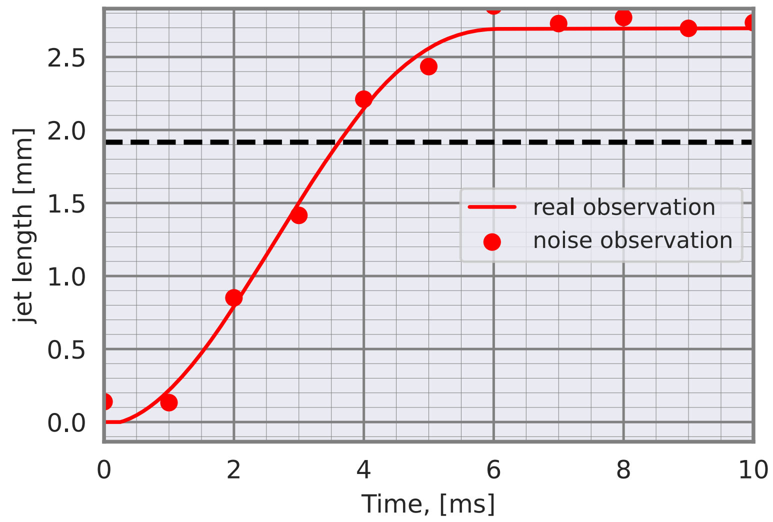
$P = U_{max} = 20,$
$D = 0$





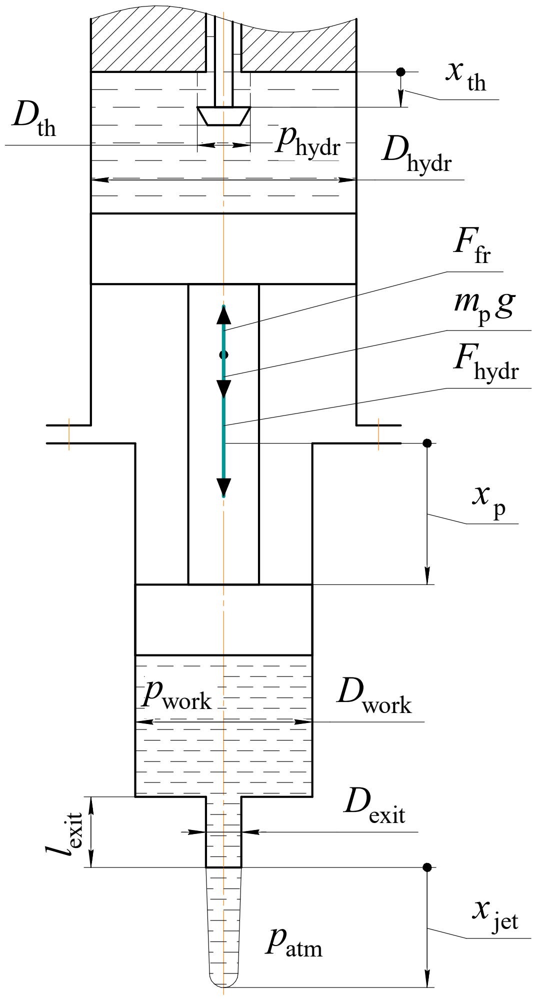






Problem setup

Calculation Scheme

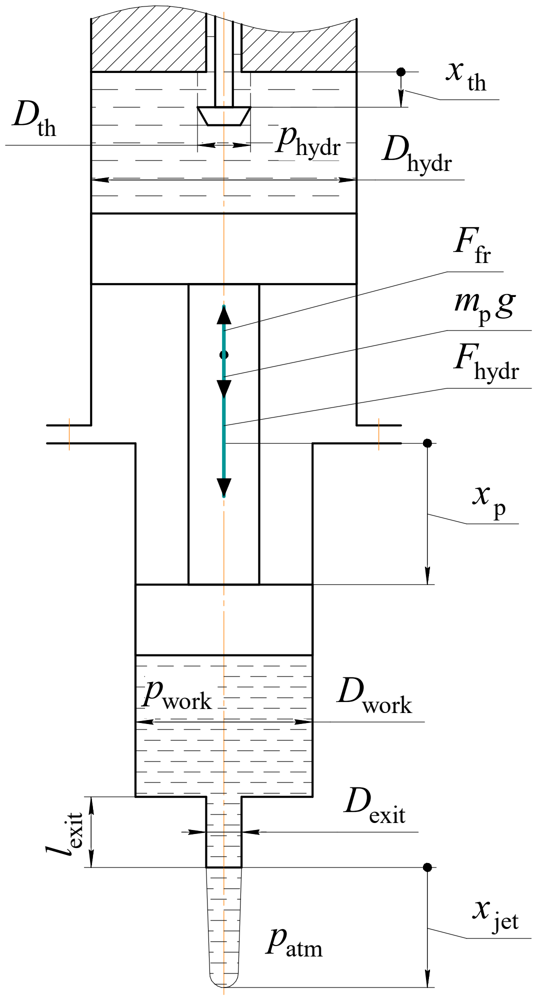


$$\frac{\partial x}{\partial \tau} = \begin{cases} \dot{x}_p = v_p \\ \dot{v}_p = \begin{cases} g + \frac{1}{m_p}(F_{hydr}(p_{hydr}, p_{work}) + F_{fr}(v_p, F_{hydr})), \\ \qquad \text{if } |v_p| > 0 \text{ or } |F_{hydr} + m_p g| > |F_{fr}| \\ 0, \ \text{otherwise} \end{cases} \\ \dot{x}_{th} = f_{th} \cdot (x_{th}^{act} - x_{th}) \\ \dot{p}_{hydr} = K_{hydr} \frac{\text{sign}(p_l - p_{hydr}) B_{th} \hat{x}_{th} \sqrt{|p_l - p_{hydr}|} - v_p}{x_p} \\ \dot{p}_{work} = K_{work} \frac{v_p - \text{sign}(p_{work} - p_{atm}) B_{exit} \sqrt{|p_{work} - p_{atm}|}}{h_{work0} - x_p + x_{p0}} \end{cases},$$

$$\hat{x}_{th} = \text{clip}_0^{x_{th}^{max}}(x_{th}),$$

Quasi-stationary system

Calculation Scheme



$$\dot{\boldsymbol{v}}_{\mathbf{p}} = \mathbf{0}, \qquad \dot{\boldsymbol{p}}_{\mathbf{hydr}} = \mathbf{0}, \qquad \dot{\boldsymbol{p}}_{\mathbf{work}} = \mathbf{0}$$

$$\boxed{\frac{\partial x}{\partial \tau} = \begin{cases} \dot{x}_{\mathrm{p}} = \begin{cases} v_{\mathrm{p}}, & \text{if } x_{\mathrm{th}} > 0 \\ 10^{-10} v_{\mathrm{p}}, & \text{overwise,} \end{cases} \\ \dot{x}_{\mathrm{th}} = f_{\mathrm{th}} \cdot (x_{\mathrm{th}}^{\mathrm{act}} - x_{\mathrm{th}}) \end{cases}}$$

$$v_{\mathrm{p}} = \mathrm{sign}(p_{\mathrm{l}} - \hat{p}_{\mathrm{hydr}}) B_{\mathrm{th}} x_{\mathrm{th}} \sqrt{|p_{\mathrm{l}} - \hat{p}_{\mathrm{hydr}}|}$$

$$\hat{p}_{\mathrm{hydr}} = \frac{\hat{F}_{\mathrm{hydr}} + p_{\mathrm{atm}} A_{\mathrm{work}} + p_{\mathrm{l}} A_{\mathrm{work}} \left(\frac{x_{\mathrm{th}} B_{\mathrm{th}}}{B_{\mathrm{exit}}} \right)^2}{A_{\mathrm{hydr}} + A_{\mathrm{work}} \left(\frac{x_{\mathrm{th}} B_{\mathrm{th}}}{B_{\mathrm{exit}}} \right)^2},$$

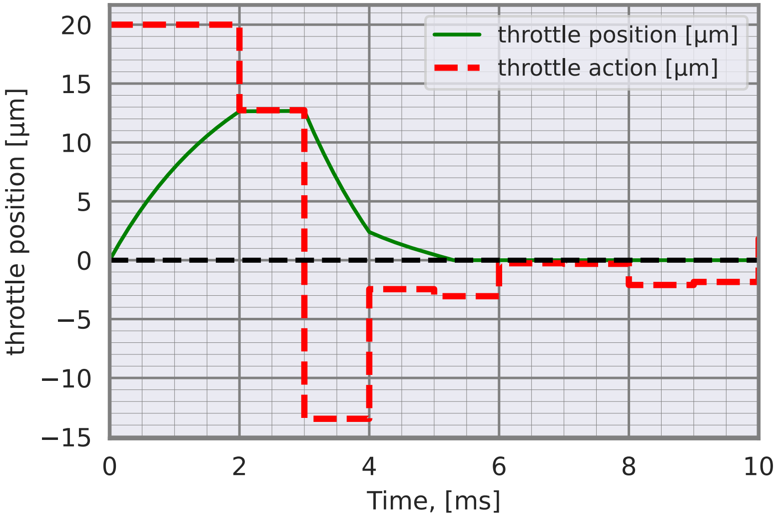
$$\hat{F}_{\mathrm{hydr}} = \begin{cases} \frac{m_{\mathrm{p}} g}{\mathrm{sign}(v_{\mathrm{p}})(1-\eta)-1}, \\ \quad \text{if } (1-\eta) \frac{m_{\mathrm{p}} g}{\mathrm{sign}(v_{\mathrm{p}})(1-\eta)-1} > F_{\mathrm{C}} \\ \mathrm{sign}(v_{\mathrm{p}}) F_{\mathrm{C}} - m_{\mathrm{p}} g, \\ \quad \text{overwise} \end{cases}$$

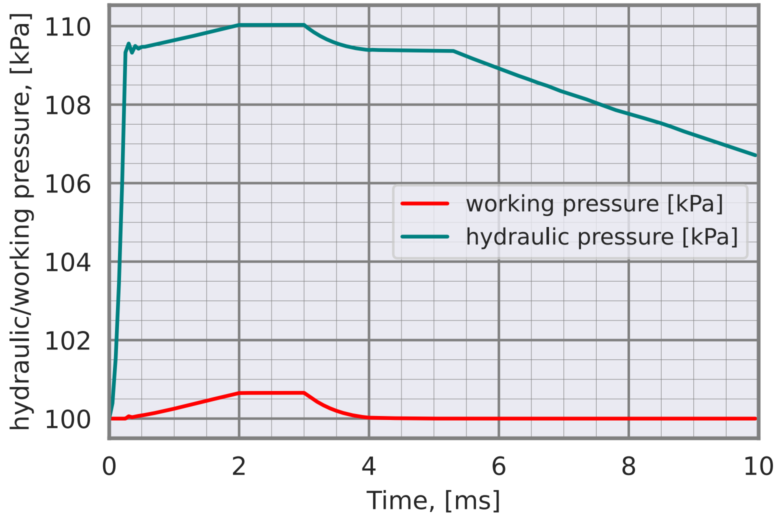
MPC

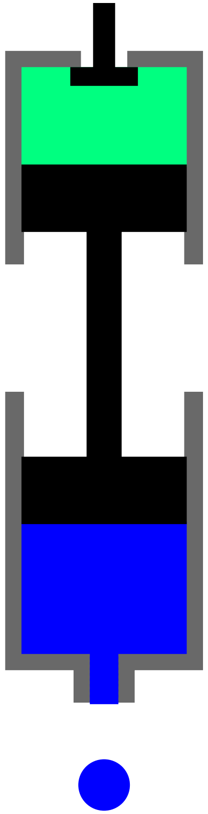
MPC-policy

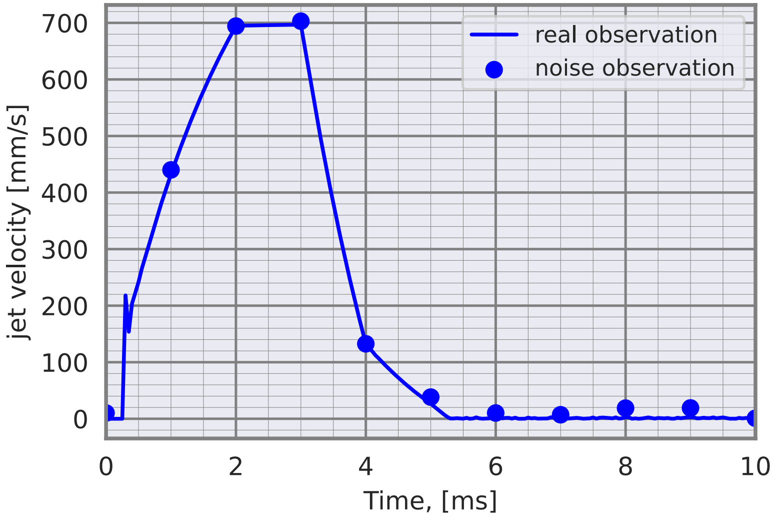
$$\rho(y_t) = \arg\min_{u_t \in \mathbb{U}} \left(\min_{u_{t+i}{}_{i=1}^{N-1} \in \mathbb{U}^{N-1}} \sum_{k=0}^{N-1} c(\tilde{y}_{t+k}, u_{t+k}) \right)$$

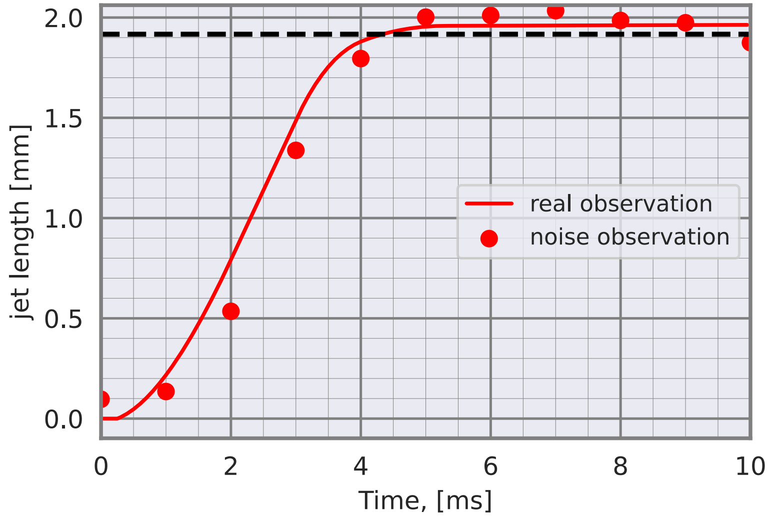
$N = 5$











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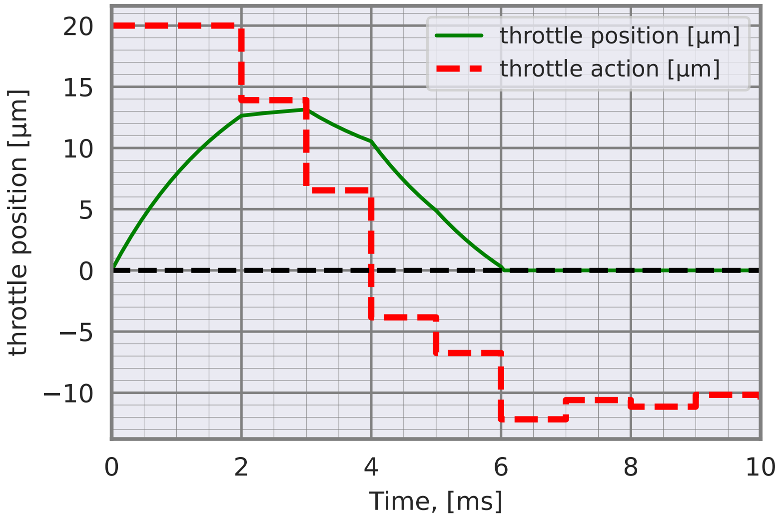

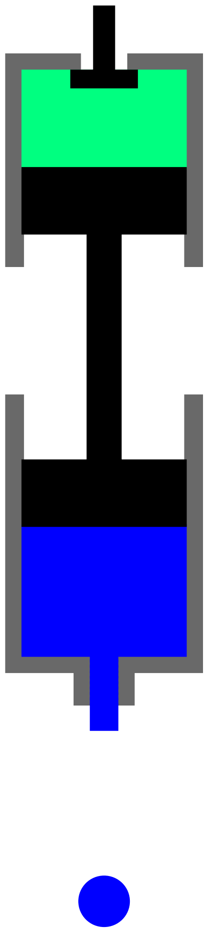

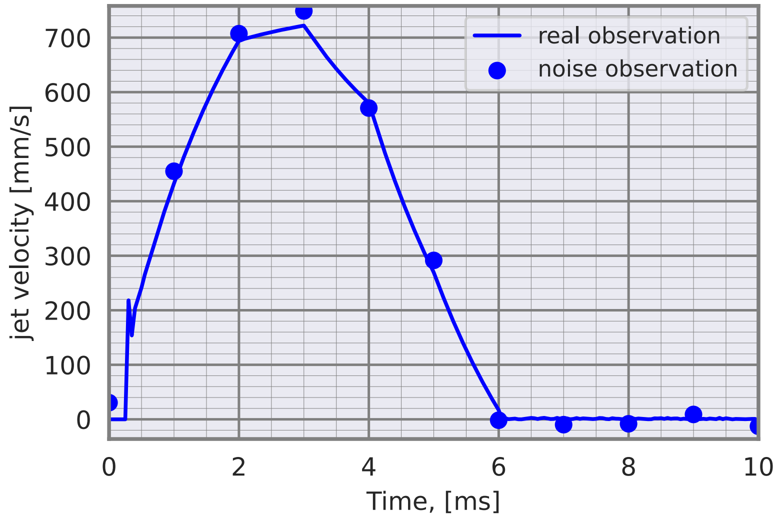

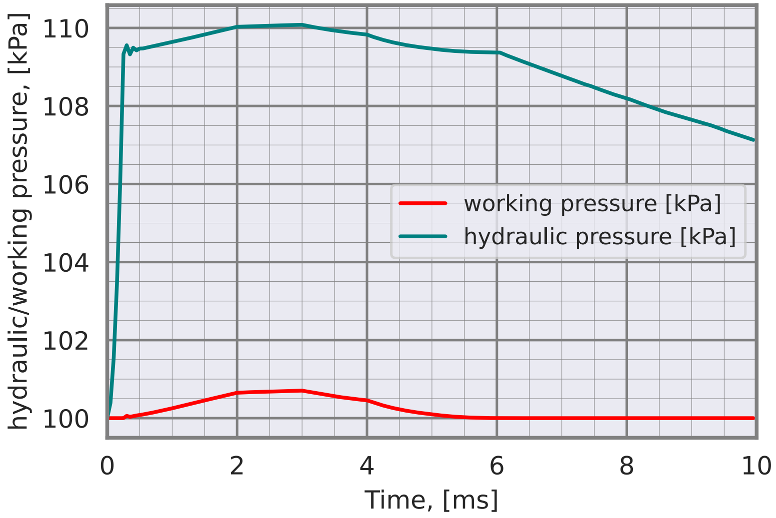

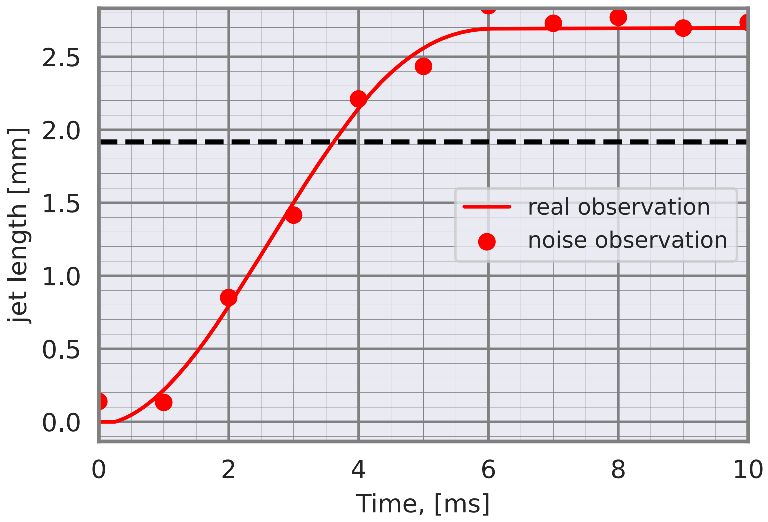

Results

"No noise" experiment

Control	Last running cost	Episodic value
PD	$1.13 \cdot 10^{-1}$	2.86
MPC	$0.1 \cdot 10^{-5}$	2.26

Value was reduced by **21.1%**

Experiments with noise

Control	Last running cost mean	Last running cost std	Episodic value mean	Episodic value std
PD	$1.15 \cdot 10^{-1}$	$2.88 \cdot 10^{-2}$	2.83	0.17
MPC	$3.06 \cdot 10^{-3}$	$4.10 \cdot 10^{-3}$	2.89	1.20

Number of episodes:
$M = 7$

$$\boldsymbol{\sigma_{x_{jet}} = 0.1}$$
$$\boldsymbol{\sigma_{v_{jet}} = 10.0}$$

Further research steps

- Clip negative jet length!
- Add running cost evaluation on "real" observation
- Add droplet-detaching terminal cost
- Add Kalman-filter on observations

References

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