Elements of robustness and fault-tolerant control

$$\dot{x} = f(x,u) + q$$ (additive disturbance, system noise here)

$$q : \underset{\mathbb{R}_{\geq 0}}{\overset{\shortparallel}{\mathbb{T}}} \longrightarrow \mathbb{R}^n$$

$$\|q\|_\infty \leq \bar{q}, \quad \|\dot{q}\|_\infty \leq \bar{q}_d, \quad \mathrm{Lip}(q) < \infty$$

$$dX_t = \underbrace{f(X_t, U_t)}_{\text{drift}}\, dt + \sigma(X_t, U_t)\, dB_t$$

$$\mathbb{T} = \mathbb{Z}_{\geq 0} : \quad X_{t+1} = f(X_t, U_t) + Q_t$$

Actuator fault

$$\dot{x} = f(x, u + d)$$

Measurement error

$$\dot{x} = f(x, \rho(x + e))$$

— Robustness —

ρ — designed, L — CLF

$$\dot L \le -\nu(\|x\|), \quad \nu \in \mathcal{K}$$

1. $\dot x = f(x,u) + q$

what happens here?

$$\dot L = \underbrace{\langle \nabla L, f(x,\rho(x)) \rangle}_{\le -\nu(\|x\|)} + \mathcal{L}_q L$$

Let's assume $\|q\|_\infty \le \bar q$, then

$$\dot L \le -\nu(\|x\|) + \|\nabla L\| \bar q$$

Conclusion: ...

2. $\dot x = f(x, u+d)$

$\mathrm{Lip}_{\mathcal{X}_0}(f)$

$\Rightarrow$

$\dot x = f(x,u) + q'$, where

$$\|q'\|_\infty \le \mathrm{Lip}_{\mathcal{X}_0}(f) \cdot \|d\|_\infty$$

(i) Lipschitz-continuity:

$f: \mathbb{R}^n \to \mathbb{R}^n$

locally Lip. cont. if

$\forall$ compact $\mathcal{X} \subset \mathbb{R}^n$ $\exists$ $\mathrm{Lip}_{\mathcal{X}}(f) > 0$ s.t.

$\forall x, x' \in \mathcal{X}$ $\quad \|f(x) - f(x')\| \le \mathrm{Lip}_{\mathcal{X}}(f)\|x - x'\|$

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3. $\dot{x} = f(x, \rho(x+e))$

Say, ρ is loc. Lip., and f also, then the control system can (locally) be cast into:

$$\dot{x} = f(x, \rho(x)) + q'',$$

where $\|q''\|_\infty \leq Lip(f) \cdot Lip(\rho) \cdot \|e\|_\infty$

$$\dot{x} = f(x) + g(x)u$$

(L, ν, ρ) : CLF, decay rate, policy

Now:

$$\dot{x} = f(x) + g(x)u + q$$

$\uparrow$ Unknown, $\|\dot{q}\|_\infty \leq Lip(q)$

$$\tilde{q} := \hat{q} - q$$

$L_c := L + \frac{1}{2\alpha}\tilde{q}^T\tilde{q}$, α – learning rate

$$\dot{L}_c = \mathcal{L}_f L + \mathcal{L}_g L u + \mathcal{L}_q L + \frac{1}{\alpha}\left(\tilde{q}^T\dot{\hat{q}} - \tilde{q}^T\dot{q}\right)$$

$$\leq \mathcal{L}_f L + \mathcal{L}_g L u + \mathcal{L}_{\hat{q}} L - \underbrace{\mathcal{L}_{\tilde{q}} L}_{= \tilde{q}^T \nabla L} + \frac{1}{\alpha}\left(\tilde{q}^T\dot{\hat{q}} + \|\tilde{q}\| \cdot Lip(q)\right)$$

$$\boxed{\begin{aligned} u &\leftarrow \rho(x) - g^{\dagger}\hat{q} \\ \dot{\hat{q}} &\leftarrow \alpha \nabla L \end{aligned}}$$

$$\nabla L^T (g u + \hat{q})$$

Say, g has lin-ly indep. rows, then we may use right inverse $g^{\dagger}$:

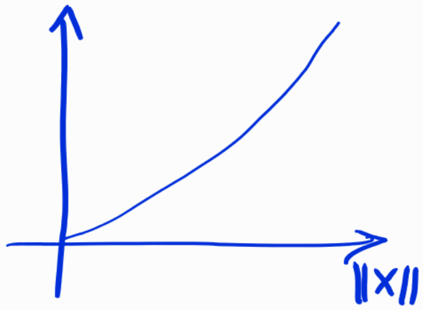
$g g^{+} = I$, $g^{+} = g^{T}(g g^{T})^{-1}$

Then, $-g(g^{+}\hat{q}) = -\hat{q}$

Resulting derivative

$\leq -\nu(\|x\|) + \frac{1}{\alpha} Lip(q) \|\tilde{q}\|$

Say, $\nu \in \mathcal{K}_{\infty}$



$L_c = L + \frac{1}{\alpha} \|\tilde{q}\|^2$

x outside a vicinity of the origin $\Rightarrow$ $\dot{L}_c \leq 0$ and always $\dot{L} < 0$, $\|\tilde{q}\|$ may not grow.

So, the vicinity depends on the initial guess $\|\tilde{q}(0)\|$