Dynamical system (plant)

State : x , s

Transition map : „ $x_{now} \mapsto x_{next}$ "

(Controlled) input : u , a

$\Longrightarrow$ trans. map

„ x_{now} , $u \longmapsto x_{next}$ "

Output (observation) : y , o

Transition maps

$$x_{k+1} = f(x_k, u_k)$$

$k \in \mathbb{Z}_{\geq 0}$

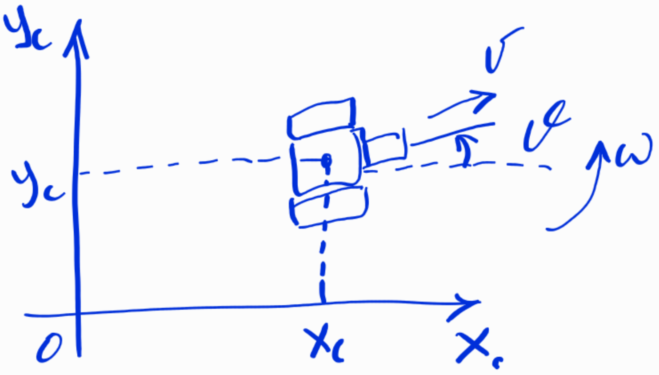
$$\dot{x} = f(x, u)$$

„ $\dot{}$ " $= \frac{d}{dt}$

$$X_{k+1} \sim f(\cdot \,|\, x_k, u_k)$$

$$dX_t = f(X_t, U_t)\,dt + \sigma(X_t, U_t)\,dB_t$$

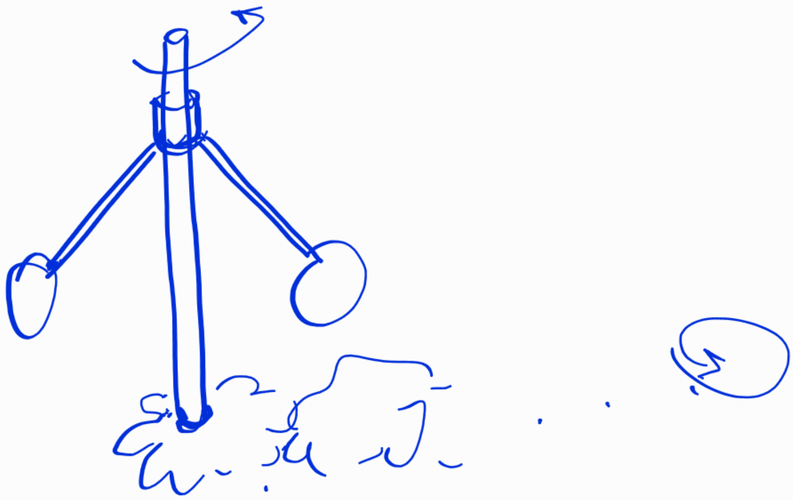
Example: mobile 3-wheel robot



$$X = \begin{pmatrix} x_c \\ y_c \\ \vartheta \end{pmatrix}$$

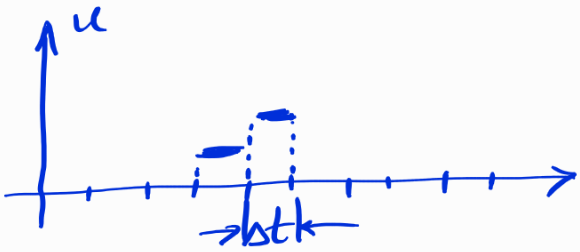
$$\dot{X} = \begin{pmatrix} v \cos\vartheta \\ v \sin\vartheta \\ \omega \end{pmatrix}$$

Centrifugal governor



$$\dot{x} = f(x, u), \quad x \in \underbrace{\mathbb{R}^n}_{\mathbb{X}\ \text{state space}}, \quad u \in \underbrace{\mathbb{R}^m}_{\mathbb{U}\ \text{action space}}$$

$$u \equiv u_k, \quad t \in [k\Delta t, (k+1)\Delta t)$$



Sample-data control
(sample-&-hold control)

$$\dot{x}_c = v\cos\vartheta$$
$$\dot{y}_c = v\sin\vartheta$$
$$\dot{\vartheta} = \omega$$

meas. noise

$$u_k \leftarrow \rho(y_k), \quad y_k = x_k + r_k$$

$r_k \sim$ Unif etc.

Goal: $x \to x^* , \; t \to \infty$

$x^* = 0$

$$\mathbb{X} = \mathbb{R}$$

$$x_{k+1} = a x_k + b u_k \quad (LTI)$$

$$x_{k+1} = a x_k$$

linear feedback control

$$u_k = -K x_k$$

$$\Rightarrow x_{k+1} = \underbrace{(a - Kb)}_{|\cdot| < 1} x_k$$

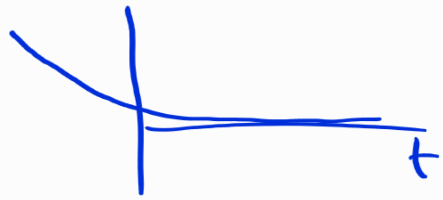
$x \in \mathbb{R}$ $\quad \dot{x} = ax + bu$

$$\dot{x} = ax \, , \quad x(0) = x_0$$

Solution (trajectory)

$$x(t) = x_0 e^{at}$$

$$\dot{x} = \underbrace{x_0 a}\, \underbrace{e^{at}} \quad \checkmark$$



$a < 0$

$$\dot{x} = ax + bu$$

Example: $\quad u \leftarrow -Kx$

$$\Rightarrow \quad \dot{x} = \underbrace{(a - Kb)}_{<0} x$$

$\mathbb{R}^n$: $\quad \underbrace{\dot{x} = Ax + Bu,}_{\text{State-space repr.}} \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m$

$\xrightarrow{\text{want}} \quad \dot{x} = \underbrace{(A - BK)}_{A'} x$

Let's suppose eig(A') are real, distinct

$$\Rightarrow \quad A' = V \underbrace{\Lambda}_{= \mathrm{diag}(\lambda_1, \ldots, \lambda_n)} V^{-1}$$

$$\Longrightarrow \quad \underbrace{\dot{z} = \Lambda z}, \qquad z = V^{-1} x^{(?)}$$

$$z = z_0 e^{\Lambda t}$$