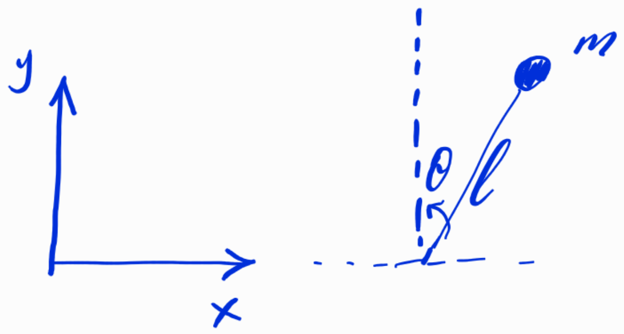
Adaptive control basics

Example: inv. pendulum with friction



$$x = \begin{pmatrix} \theta \\ \dot\theta \end{pmatrix}$$

$$\dot x_1 = x_2$$

$$\dot x_2 = \frac{g}{l} \sin x_1 + \frac{u}{ml^2} - c\, x_2^2 \cdot \mathrm{sign}(x_2)$$

$$E_{tot} = \frac{ml^2 x_2^2}{2} + mgl(\cos x_1 - 1)$$

LFC:

$$L \leftarrow \frac{E_{tot}^2}{2} \Rightarrow$$

$$\dot L = (ml^2 x_2 \dot x_2 - mgl \sin x_1\, x_2)\, E_{tot}$$

$$= E_{tot}\left(ml^2 x_2 \left(\frac{g}{l}\sin x_1 + \frac{u}{ml^2} - c x_2^2\, \mathrm{sign}(x_2)\right) - mgl \sin x_1\, x_2\right)$$

$$= E_{tot}\left(u x_2 - cml^2 |x_2|^3\right) = -K x_2^2 E_{tot}^2$$

$$u \leftarrow cml^2 x_2 |x_2| - K x_2 \cdot E_{tot}$$

Alternative: $-K\, \mathrm{sign}(x_2 E_{tot})$

Certainty-equivalence adaptive controller

Assumption: c is not known.

Instead of c, the controller will now use an estimate $\hat{c}$.

$\dot{\hat{c}}$ – ?

α : learning rate

$$L_c \leftarrow L + \frac{1}{2\alpha}(\hat{c} - c)^2$$

$$\dot{L}_c = E_{tot}\left(u x_2 - c\, m l^2 |x_2|^3\right) + \frac{1}{\alpha}\underbrace{(\hat{c} - c)}_{=:\tilde{c}}\dot{\hat{c}}$$

$$= E_{tot}\left(u x_2 - \hat{c}\, m l^2 |x_2|^3\right) + E_{tot}\,\tilde{c}\, m l^2 |x_2|^3 + \frac{1}{\alpha}\tilde{c}\,\dot{\hat{c}}$$

$$u \leftarrow -K\,\mathrm{sign}(E_{tot}\, x_2) + \hat{c}\, m l^2 |x_2| x_2$$

$$\dot{\hat{c}} = -\alpha \cdot E_{tot}\, m l^2 |x_2|^3$$

$$\tilde{c}\left(E_{tot}\, m l^2 |x_2|^3 + \frac{1}{\alpha}\dot{\hat{c}}\right)$$

$$\dot{L}_c = E_{tot}\left(-Kx_2 \,\mathrm{sign}\left(E_{tot}x_2\right)\right) =$$

$$= -K\left|E_{tot}x_2\right|$$