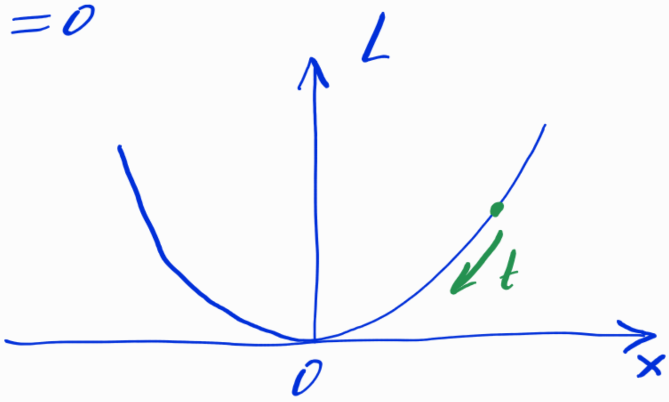
Basics of Lyapunov-based control

$\dot{x} = f(x), \; x(0) = x_0$ $\qquad$ $x_{k+1} = f(x_k), \; k \in \mathbb{Z}_{\geq 0}$

Equilibrium state: $\quad x_e \Rightarrow f(x_e) = 0$

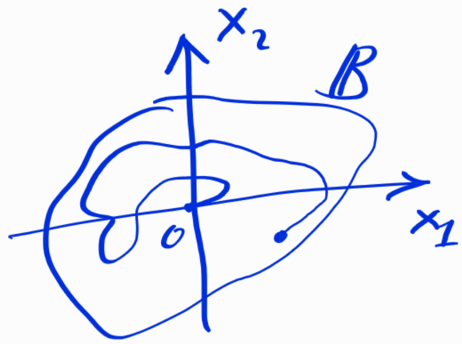
Assume $x_e = 0$



Positive-def.

Asympt. stability (local) $\quad \exists \mathbb{B} \ni 0$ s.t.

$x_0 \in \mathbb{B} \Rightarrow \lim_{t \to \infty} x(t) = 0$



Global: $\mathbb{B} = \mathbb{X}$

$L : \mathbb{X} \longrightarrow \mathbb{R}_{\geq 0}$

(Lyapunov) decay property: $\quad \dot{L} < 0$

$L_{k+1} - L_k < 0$

Decay rate ν : $\quad \dot{L}(x) \leq -\nu(x)$

p.-d.

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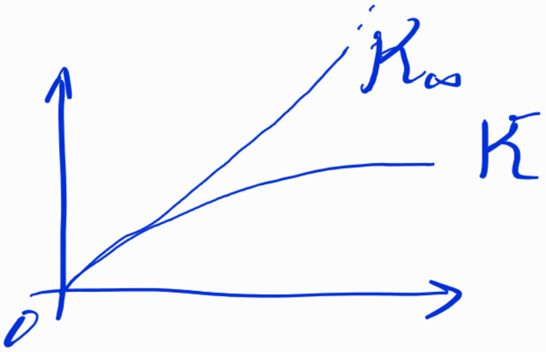
$\mathcal{K}$, $\mathcal{K}_\infty$

↳ Functions $\mathbb{R} \to \mathbb{R}$, monoton. increasing ✓ pos.-def., limit of the function as the arg. tends to ∞ is in turn ∞



$\exists\ K_{low}, K_{up} \in \mathcal{K}_\infty$, $\nu \in \mathcal{K}$ s.t.

$$\forall x \quad K_{low}(\|x\|) \le L(x) \le K_{up}(\|x\|)$$

$$\dot{L}(x) \le -\nu(\|x\|)$$

$$\dot{x} = f(x,u), \quad x \in \mathbb{R}^n$$

Control Lyapunov function (CLF):

$$\forall x \quad K_{low}(\|x\|) \le L(x) \le K_{up}(\|x\|)$$

$$\min_u \langle \nabla L, f(x,u) \rangle \le -\nu(\|x\|)$$