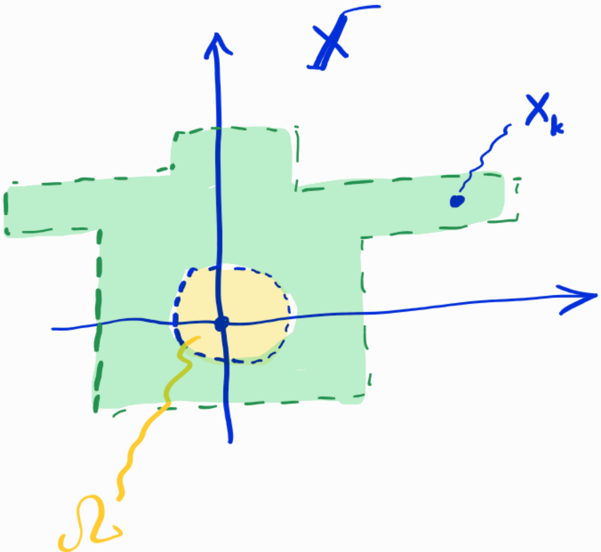
Stabilization via model-predictive control

$x_{k+1} = f(x_k, u_k)$, state constraint $\mathbb{S}_x$, $u \in \mathbb{U}$

MPC: (a simple setup)

1. Get x_k
2. $$u^*_{k:k+N} \leftarrow \arg\min_{u_{k:k+N}} \sum_{i=k}^{k+N-1} c(x_{i+1}, u_i)$$
   $$\text{s.t. } x_{k+1:k+N+1} \in \mathbb{S}_x^N$$
3. Apply u^*_k



A tweak:

Terminal cost

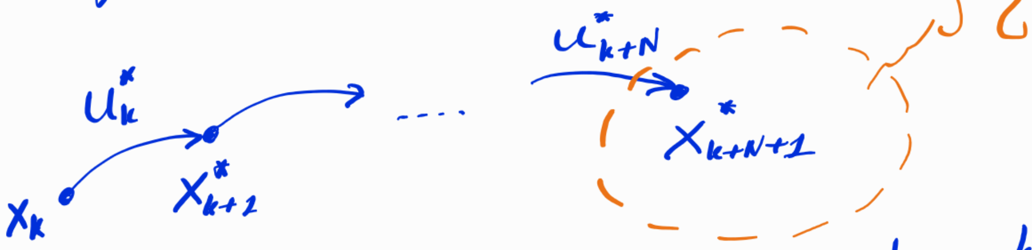
$$\arg\min_{u_{k:k+N+1}} \sum_{i=k}^{k+N-1} c(x_{i+1}, u_i) + L(x_{k+N+1})$$
$$\text{s.t. } x_{k+1:k+N+1} \in \mathbb{S}_x^N$$
$$x_{k+N+1} \in \Omega$$

Terminal constraint

Let's assume: Ω is a level set of some local CLF L with a corresponding stabilizer, i.e., stabilizing controller. η

Feasibility:

let's say, the problem was feasible at step k. Denote the respective solution $u^*_{k:k+N+1}$ and the respective trajectory $x^*_{k+1:k+N+2}$



Let's say, we proceeded to step $k+1$.

Is there a feasible candidate?

Suggestion:

(k)	$(k+1)$	
u^*_k	u'_{k+1}	$\leftarrow u^*_{k+1}$
u^*_{k+1}	u'_{k+2}	$\leftarrow u^*_{k+2}$
$\vdots$	$\vdots$	
u^*_{k+N-1}	u'_{k+N}	$\leftarrow u^*_{k+N}$
u^*_{k+N}	u'_{k+N+1}	$\leftarrow \eta(x^*_{k+N+1})$

This provides an action sequence that is feasible at step $k+1$

Stabilization:

Denote
$$J^*_k := \sum_{i=k}^{k+N-1} c(x^*_{i+1}, u^*_i) + L(x^*_{k+N+1})$$

meaning the minimum of the MPC cost at time step k with the respective minimizer $u^*_{k:k+N+1}$.

Idea: demonstrate that J^*_k is an LF.

$$J^*_{k+1} - J^*_k \overset{!}{<} 0$$

Sketch: take the feasible cand-te at step $k+1$ that we derived above, notice $J^*_{k+1} \le \underbrace{\langle \text{MPC cost at step } k+1 \text{ under feas. cand-te} \rangle}_{=: J'_{k+1}}$,

and analyze:

$$J'_{k+1} - J^*_k = \sum + \overbrace{L - \sum}^{\Delta L} - L$$

cyclic shift

$$= -C_k + \underbrace{\Delta L + C_{last}}_{\text{assume}} \le 0$$

$\forall x \in \Omega$

$$L(f(x, \eta(x))) - L(x) \le -C(x, \eta(x))$$