Backstepping control of a differential drive robot

Kinematic

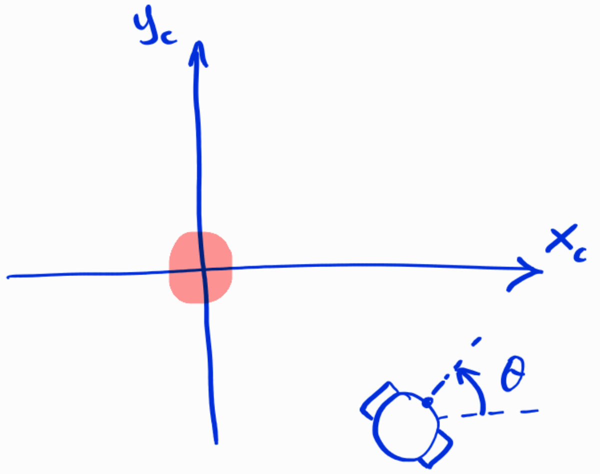
$$\dot{x}_c = v\cos\theta$$
$$\dot{y}_c = v\sin\theta$$
$$\dot{\theta} = \omega$$

$$x = \begin{pmatrix} x_c \\ y_c \\ \theta \end{pmatrix}, \quad u = \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Dynamic

$$\dot{x}_c = v\cos\theta$$
$$\dot{y}_c = v\sin\theta$$
$$\dot{\theta} = \omega$$
$$\dot{v} = \frac{1}{m}F$$
$$\dot{\omega} = \frac{1}{J}M$$

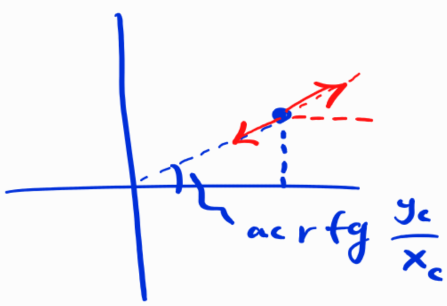
$$z = \begin{pmatrix} x_c \\ y_c \\ \theta \\ v \\ \omega \end{pmatrix}, \quad a = \begin{pmatrix} F \\ M \end{pmatrix}$$



LF candidate:

$$L := \frac{1}{2}\left(x_c^2 + y_c^2 + \left(\theta - \operatorname{arctg}\frac{y_c}{x_c}\right)^2\right)$$

Illustration

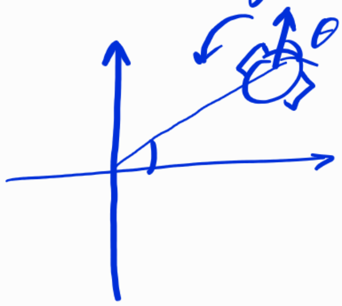


$$\dot{L} = x_c \dot{x}_c + y_c \dot{y}_c + \left(\theta - \operatorname{arctg}\frac{y_c}{x_c}\right)\left(\dot{\theta} - \frac{\dot{y}_c x_c - \dot{x}_c y}{x_c^2 + y_c^2}\right)$$

$$= v x_c \cos\theta + v y_c \sin\theta + \left(\theta - \operatorname{arctg}\frac{y_c}{x_c}\right)\left(\omega - \frac{v x_c \sin\theta - v y_c \cos\theta}{x_c^2 + y_c^2}\right)$$

Hint: case distinction on $\theta = \vee \neq \operatorname{arctg}\frac{y_c}{x_c}$

$\uparrow$ logical or



1. $\theta \neq \operatorname{arctg}\frac{y_c}{x_c}$

$\omega \leftarrow -\operatorname{sign}\left(\theta - \operatorname{arctg}\frac{y_c}{x_c}\right)$

$v \leftarrow 0$

2. $\theta = \operatorname{arctg}\frac{y_c}{x_c}$

$\dot{L} = v x_c \cos\theta + v y_c \sin\theta = v\left(x_c \cos\theta + y_c \sin\theta\right)$

$\omega \leftarrow 0$

$v \leftarrow -\operatorname{sign}\left(x_c \cos\theta + y_c \sin\theta\right)$

$$\cos\left(\operatorname{arctg}\frac{y_c}{x_c}\right) = \frac{1}{\sqrt{1 + y_c^2/x_c^2}}$$

$$\sin\left(\operatorname{arctg}\frac{y_c}{x_c}\right) = \frac{y_c/x_c}{\sqrt{1 + y_c^2/x_c^2}}$$

So, $$\dot{L} = \frac{\upsilon}{\sqrt{1 + y_c^2/x_c^2}}\left(x_c + \frac{y_c^2}{x_c}\right) = \frac{\upsilon}{\sqrt{1 + y_c^2/x_c^2}}\left(\frac{x_c^2 + y_c^2}{x_c}\right)$$

$\Rightarrow$ (another option) $\upsilon \leftarrow -\mathrm{sign}(x_c)$

Actual control law (policy), for instance:

$$u \leftarrow \min_{u \in U} \dot{L} = \min_{u \in U} \langle \nabla L, \dot{x} \rangle, \quad \dot{x} = f(x, u)$$

action space

$$\dot{x} = f(x,u)$$

Let's call this policy $\rho(x)$

Dynamic robot: $a \leftarrow -K(u - \rho(x))$