Multilevel Mediation

Within-subject mediation analysis for experimental data in cognitive psychology and neuroscience

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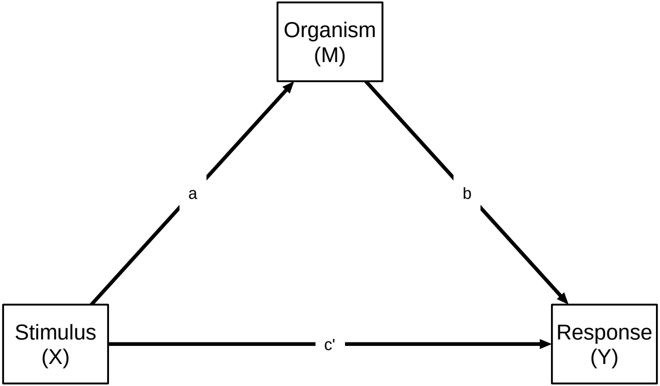
Mediation

What is mediation?

- Mediation is a hypothesized causal model, whereby effect of an IV to a DV is transmitted through an intermediary variable M



Assessing mediation

Experimental approach

- Experiment 1: manipulate X and measure M
- Experiment 2: manipulate M and measure Y
- *Establishing a causal chain: Why experiments are often more effective than mediational analyses in examining psychological processes* (Spencer, Zanna, and Fong 2005)

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Statistical modeling approach

- Experiment: manipulate X, measure M and Y
- Regress M on X; Y on X and M
- Assume that
    - Y does not affect M
    - No 3rd variable on M to Y relationship
    - M is measured without error
    - Y and M residuals are not correlated

Between- vs. within-subject causal models

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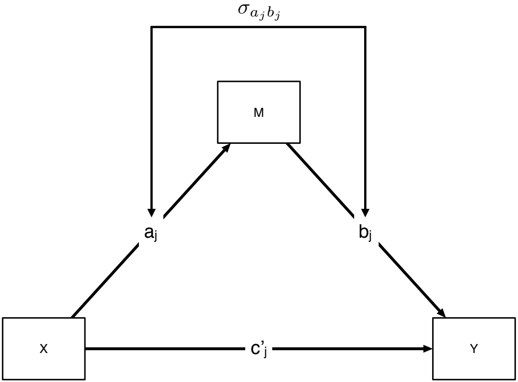
- Multilevel model for trial-level data
  - Average person's within-person causal process ("fixed" effects)
  - Causal effects' heterogeneity ("random" effects)
  - Hierarchical Bayes estimates for individuals in current sample

Multilevel Mediation

Multilevel mediation



- Subject-specific parameters (e.g. a_1)
- Parameters' prior distribution is estimated from data
- $\sigma_{a_j b_j}$ can indicate an omitted moderator (Tofighi, West, and MacKinnon 2013)

Multilevel mediation: Practical implementation

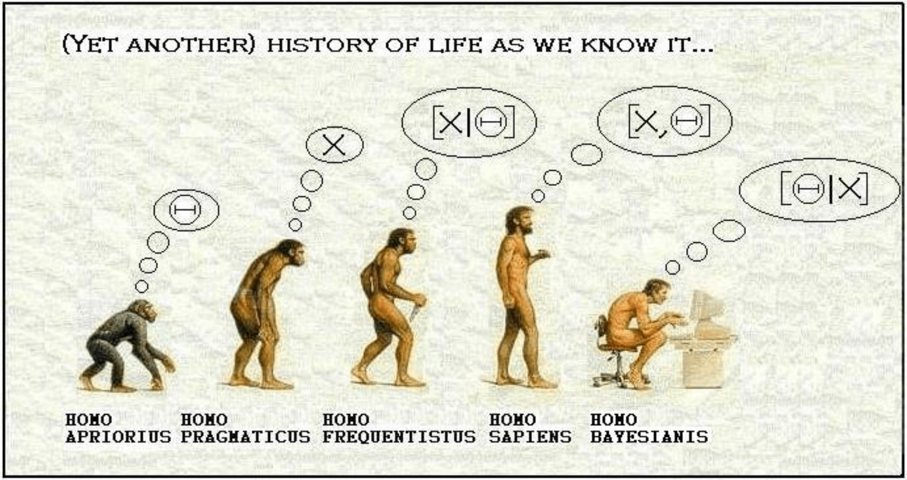
We developed software for Bayesian estimation of multilevel mediation models (Vuorre and Bolger 2017; Vuorre 2017)

bmlm: Bayesian Multi-Level Mediation

- R package
- Bayesian inference
- Data preprocessing, model estimation, summarizing, and visualization
- Continuous and binary Y
- https://mvuorre.github.io/bmlm/

```r
install.packages("bmlm")
```

Bayesian data analysis and inference



Example Multilevel Mediation Analysis

Tip-of-the-tongue, ERPs, learning (Bloom et al., in prep)

- Tip-of-the-tongue state (ToT) predicts increased curiosity and answer seeking (Metcalfe, Schwartz, and Bloom 2017), and possibly learning

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    - 30 participants presented with general info questions
    - "What's the capital of Australia?"
    - After 3 seconds, asked if they are in a ToT state
    - After 1 second, correct feedback presented
    - ERPs timelocked to feedback
    - After 150 items, a surprise recall test on all items

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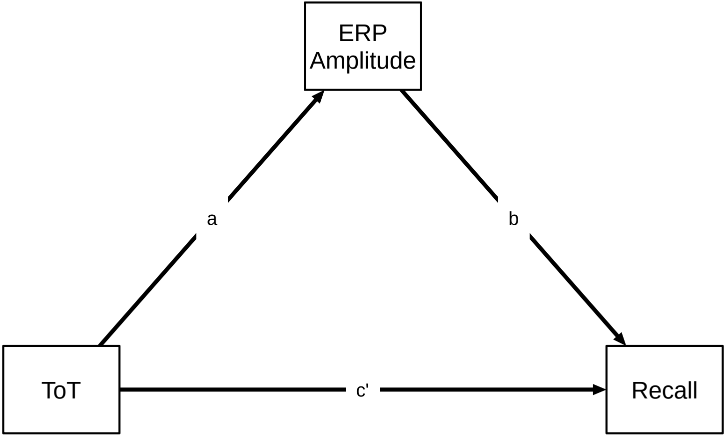
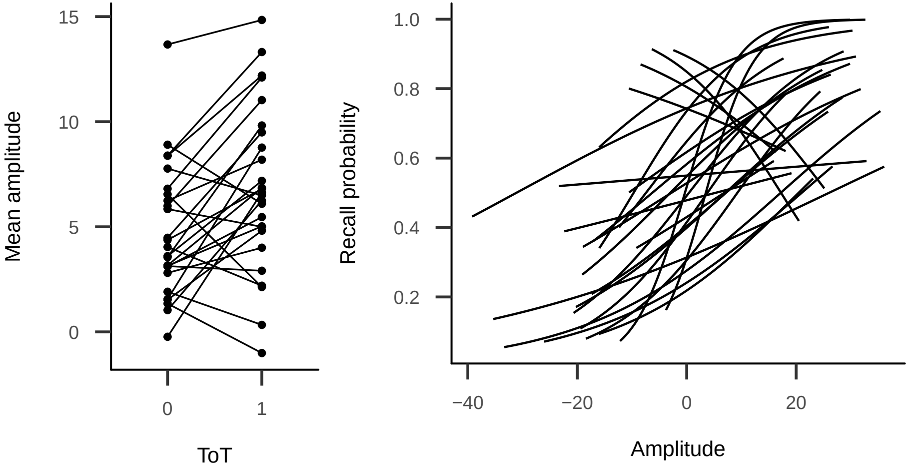
  - ERPs timelocked to feedback
  - After 150 items, a surprise recall test on all items
- We examined to what extent
  - ToT state during learning predicts correct recall
  - Late positive (centro-parietal) ERP amplitude mediates ToT -> recall effect

Hypothesized causal model



ToT Data



15
10
5
0
Mean amplitude
0
1
ToT
1.0
0.8
0.6
0.4
0.2
Recall probability
−40
−20
0
20
Amplitude

ToT Data

id	trial	tot	amplitude	recall
1	1	0	5.53	1
1	2	1	-2.45	1
1	3	0	8.19	0

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Remove between-subject variability from mediator:

```
tot <- isolate(tot, by = "id", value = "amplitude")
```

id	trial	tot	amplitude	recall	amplitude_cw
1	1	0	5.53	1	-1.757
1	2	1	-2.45	1	-9.739
1	3	0	8.19	0	0.907

Model estimation

```
fit <- mlm(tot, id = "id", x = "tot",
           m = "amplitude_cw", y = "recall",
           binary_y = TRUE, cores = 4)
```

This function returns the model's posterior distribution. Users specify data and variables within. Additional options include prior distributions, binary outcomes and multiple CPUs.

Model estimation

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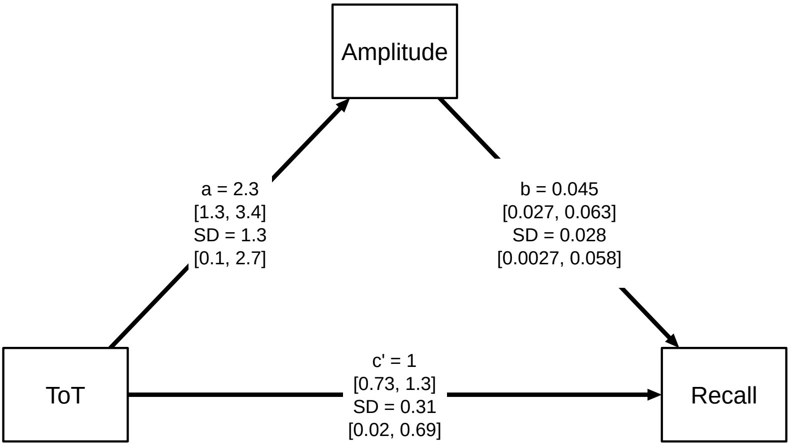
bmlm estimates the posterior distribution using MCMC sampling (HMC; Stan Development Team (2016)).

Model summary: Path diagram

```
?mlm_path_plot
```



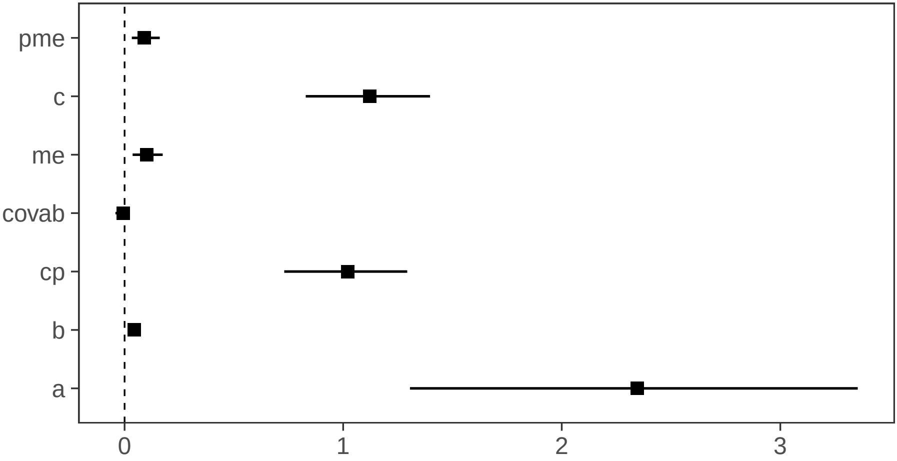
Model summary: Numerical

```
?mlm_summary
```

Parameter	Mean	SE	Median	2.5%	97.5%	n_eff	Rhat
a	2.35	0.52	2.35	1.31	3.35	6294	1
b	0.05	0.01	0.05	0.03	0.06	5716	1
cp	1.02	0.14	1.02	0.73	1.29	5149	1
me	0.10	0.03	0.10	0.04	0.17	6292	1
c	1.12	0.14	1.12	0.83	1.40	5241	1
pme	0.09	0.03	0.09	0.03	0.16	5634	1

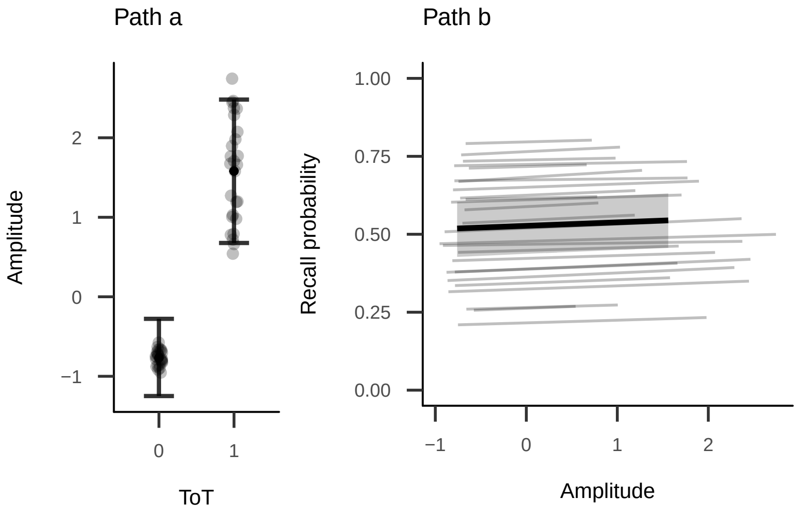
Model summary: Graphical

```
?mlm_pars_plot
```



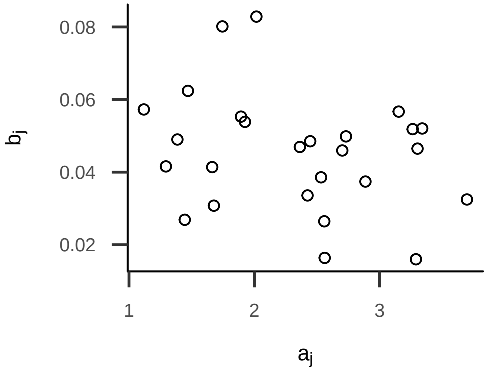
Fitted values

```
?mlm_spaghetti_plot
```



Between-subject (co)variance

Parameter	Mean	SE	Median	2.5%	97.5%	n_eff	Rhat
tau_a	1.34	0.69	1.32	0.10	2.74	2463	1
tau_b	0.03	0.01	0.03	0.00	0.06	2006	1
corrab	-0.12	0.36	-0.14	-0.77	0.62	3868	1



Conclusion

- Late positivity mediated ($\sim$10% of) ToT's positive effect on recall
  - Late positivity may index enhanced processing of feedback

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- Evidence of heterogeneity in causal paths
  - Estimate of $a_j - b_j$ correlation negative but very uncertain

Conclusion

- Late positivity mediated (~10% of) ToT's positive effect on recall
  - Late positivity may index enhanced processing of feedback
- Evidence of heterogeneity in causal paths
  - Estimate of $a_j - b_j$ correlation negative but very uncertain
- Formal assessment of within-subject mediation with **bmlm**
  - Relatively easy, free, accessible
  - Probabilistic modeling
    - Intuitive probability statements about parameters
    - Flexible framework for investigating questions about within-person psychological and causal processes

Acknowledgements

Thank you

- Niall Bolger
- Janet Metcalfe
- David Friedman
- Paul A. Bloom
- Judy Xu

Appendix: Mediation equations

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Between-subject mediation

$$
\begin{aligned}
Y_i &\sim N(d_Y + c'X_i + bM_i, \sigma^2_Y) && \text{[Y model]} \\
M_i &\sim N(d_M + aX_i, \sigma^2_M) && \text{[M model]}
\end{aligned}
$$

$$
\begin{aligned}
me &= a \times b && \text{[mediated effect]} \\
c &= c' + me && \text{[total effect]}
\end{aligned}
$$

Appendix: Mediation equations

bmlm's within-subject mediation model, continuous outcome

$$Y_{ij} \sim N(d_{Yj} + c'_j X_{ij} + b_j M_{ij}, \sigma^2_Y) \qquad \text{[Y model]}$$

$$M_{ij} \sim N(d_{Mj} + a_j X_{ij}, \sigma^2_M) \qquad \text{[M model]}$$

$$\begin{pmatrix} d_{Mj} \\ d_{Yj} \\ a_j \\ b_j \\ c'_j \end{pmatrix} \sim N \left[\begin{pmatrix} d_M \\ d_Y \\ a \\ b \\ c' \end{pmatrix}, \begin{pmatrix} \sigma^2_{d_{Mj}} & & & & \\ \sigma_{d_{Mj} d_{Yj}} & \sigma^2_{d_{Y_j}} & & & \\ \sigma_{d_{Mj} a_j} & \sigma_{d_{Yj} a_j} & \sigma^2_{a_j} & & \\ \sigma_{d_{Mj} b_j} & \sigma_{d_{Yj} b_j} & \sigma_{a_j b_j} & \sigma^2_{b_j} & \\ \sigma_{d_{Mj} c'_j} & \sigma_{d_{Yj} c'_j} & \sigma_{a_j c'_j} & \sigma_{b_j c'_j} & \sigma^2_{c'_j} \end{pmatrix} \right]$$

$$me = a \times b + \sigma_{a_j b_j} \qquad \text{[mediated effect]}$$

$$c = c' + me \qquad \text{[total effect]}$$

Appendix: Mediation equations

bmlm's within-subject mediation model, binary outcome

$$Y_{ij} \sim Bernoulli(\text{logit}(d_{yj} + c'_j X_{ij} + b_j M_{ij})) \quad \text{[Y model]}$$
$$M_{ij} \sim N(d_{mj} + a_j X_{ij}, \sigma^2_M) \quad \text{[M model]}$$

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