Applications Of Linear Algebra in Game Development.

Mikhailov V.V.

February 27, 2017

Overview

Vector, dot and cross product

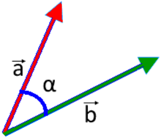


Figure 1: For dot product

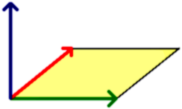


Figure 2: For cross product

Vector is ...

An element of a vector space

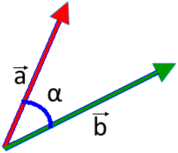
Basics (assume $\|a\| = \|b\| = 1$, $\vec{a}, \vec{b} \in R^3$)

Dot product: $\cos\alpha = \vec{a} \cdot \vec{b} = a^i b_i$,
Cross product: $|\sin(\alpha)| = \|\vec{a} \times \vec{b}\|$,
where $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$
Wedge product: $\omega = \vec{a} \wedge \vec{b}$, $\omega \in \Lambda^2$

"Types" of vectors

- *Polar*
- *Axial (aka pseudovector).*

Angle between two vectors



Problem

Find angle α between two vectors $\vec{a}$ and $\vec{b}$

$\cos\alpha = \vec{a}\cdot\vec{b}$, but cosine function is an *even* $(\cos(-\alpha) = \cos\alpha)$.
sgn α - ?

Solution

In general, we need a reference axis $\vec{z}_{ref}$.
Let $\vec{z} = \vec{a}\times\vec{b}$.
If $\vec{z}\cdot\vec{z}_{ref} \geq 0$, then $\alpha = \arccos\vec{a}\cdot\vec{b}$, else $\alpha = -\arccos\vec{a}\cdot\vec{b}$.
2D case: Check the sign of $\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$

Geometric features

- $\|\vec{a} \times \vec{b}\| = S_{ABCD}$
- $\frac{1}{2}\|\vec{a} \times \vec{b}\| = S_{ABC}$ determinant of |a b c|? scalar triple product (or mixed product) proper and improper rotation