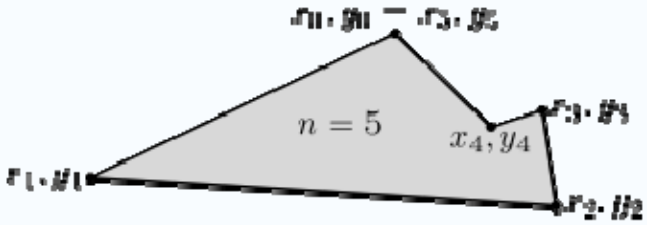
Area and centroid

http://en.wikipedia.org/wiki/Polygon_area#Area_and_centroid



Nomenclature of a 2D polygon.

The area of a polygon is the measurement of the 2-dimensional region enclosed by the polygon. For a non-self-intersecting (simple) polygon with *n* vertices, the area and centroid are given by[1]:

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$$

$$C_x = \frac{1}{6A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$C_y = \frac{1}{6A}\sum_{i=0}^{n-1}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

To close the polygon, the first and last vertices are the same, i.e., $x_n, y_n = x_0, y_0$. The vertices must be ordered clockwise or counterclockwise; if they are ordered clockwise, the area will be negative but correct in absolute value. This is commonly called the Surveyor's Formula.[

Moment of Inertia of any cross section defined as polygon

http://en.wikipedia.org/wiki/Second_moment_of_area#Any_cross_section_defined_as_polygon

The second moments of area for any cross section defined as a simple polygon on XY plane can be computed in a generic way by summing contributions from each segment of a polygon.

For each segment defined by two consecutive points of the polygon, consider a triangle with two corners at these points and third corner at the origin of the coordinates. Integration by the area of that triangle and summing by the polygon segments yields:

$$I_x = \frac{1}{12}\sum_{i=1}^{n}(y_i^2 + y_i y_{i+1} + y_{i+1}^2)a_i$$

$$I_y = \frac{1}{12}\sum_{i=1}^{n}(x_i^2 + x_i x_{i+1} + x_{i+1}^2)a_i$$

from Wikipedia

$$I_{xy} = \frac{1}{24}\sum_{i=1}^{n}(x_i y_{i+1} + 2x_i y_i + 2x_{i+1}y_{i+1} + x_{i+1}y_i)a_i$$

- $a_i = x_i y_{i+1} - x_{i+1} y_i$ is twice the (signed) area of the elementary triangle,
- index *i* passes over all *n* points in the polygon, which is considered closed, i.e. point *n+1* is point *1*

These formulae imply that points defining the polygon are ordered in anticlockwise manner; for clockwisely defined polygons it will give negative values.