ÚLOHA 9

-MILAN WIKARSKI-

PRÍKLAD a

$$\int_1^e \frac{\ln^2(x)}{x}\, dx$$

POUŽIJEME SUBSTITÚCIU

$$u = \ln(x)$$

$$du = 1/x \; dx$$

DOSTANEME NOVÉ HRANICE INTEGRÁLU

$$e \longrightarrow \ln(e) = 1$$

$$1 \longrightarrow \ln(1) = 0$$

A VYPOČÍTAME

$$\int_0^1 u^2\, du = \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3} - \frac{0}{3} = \underline{\underline{\frac{1}{3}}}$$

ÚLOHA 9 -MILAN KIKARSKI-

PRÍKLAD b

$$\int_0^{\pi/4} \tan^3(x)\, dx = \int_0^{\pi/4} \tan^2(x) \cdot \tan(x)\, dx$$

POUŽIJEME IDENTITU $\sec^2(x) - \tan^2(x) = 1$

$$\int_0^{\pi/4} \left(-1 + \sec^2(x)\right) \tan(x)\, dx$$

POUŽIJEME SUBSTITÚCIU

$$u = \sec(x)$$
$$du = \tan(x)\sec(x)\, dx$$

DOSTANEME NOVÉ HRANICE INTEGRÁLU

$$\pi/4 \longrightarrow \sec(\pi/4) = \sqrt{2}$$
$$0 \longrightarrow \sec(0) = 1$$

VYPOČÍTAME

$$\int_1^{\sqrt{2}} \frac{-1+u^2}{u}\, du = \int_1^{\sqrt{2}} \frac{-1}{u}\, du + \int_1^{\sqrt{2}} u\, du =$$

$$= -\int_1^{\sqrt{2}} \frac{1}{u}\, du + \int_1^{\sqrt{2}} u\, du = -\Big[\ln(u)\Big]_1^{\sqrt{2}} + \left[\frac{u^2}{2}\right]_1^{\sqrt{2}} =$$

$$= -\frac{1}{2}\ln(2) + 0 + 1 - \frac{1}{2} = -\frac{1}{2}\ln(2) + \frac{1}{2} = \frac{1}{2}\left(1 - \ln(2)\right)$$