SADA 8 -MILAN WIKARSKI-

PRÍKLAD a

$$\int \sin^3(x)\,dx = \int \sin^2(x)\sin(x)\,dx =$$

POUŽIJEME SUBSTITÚCIU

$$u = \cos x$$
$$du = -\sin x\,dx$$

DOSTÁVAME

$$= -\int \sin^2(x)\,du = -\int \left(1-\cos^2(x)\right)du =$$

$$= -\int 1\,du + \int \cos^2(x)\,du = -\int 1\,du + \int u^2\,du =$$

$$= -u + \frac{u^3}{3} = \underline{\underline{-\cos(x) + \frac{\cos^3(x)}{3} + C}}, \quad C \in \mathbb{R}$$

SADA 8 -MILAN WIKARSKI-

PRÍKLAD b

$$\int x^3 \ln^2(x)\, dx$$

POUŽIJEME METÓDU PER PARTES

$$u = \ln^2(x) \qquad v' = x^3$$

$$u' = 2(\ln(x)) \cdot \frac{1}{x} \qquad v = \frac{x^4}{4}$$

DOSTÁVAME

$$= \ln^2(x) \cdot \frac{x^4}{4} - \int 2(\ln(x)) \frac{1}{x} \cdot \frac{x^4}{4}\, dx =$$

$$= \frac{\ln^2(x) \cdot x^4}{4} - \frac{1}{2} \int \ln(x)\, x^3\, dx =$$

OPÄŤ POUŽIJEME METÓDU PER PARTES

$$u = \ln(x) \qquad v' = x^3$$

$$u' = \frac{1}{x} \qquad v = \frac{x^4}{4}$$

DOSTÁVAME

$$= \frac{x^4 \ln^2(x)}{4} - \frac{1}{2}\left(\frac{\ln(x)\, x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{x}\, dx \right) =$$

$$= \frac{x^4 \ln^2(x)}{4} - \frac{\ln(x)\ x^4}{8} - \frac{1}{2}\int \frac{x^3}{4}\,dx =$$

$$= \frac{x^4 \ln^2(x)}{4} - \frac{\ln(x)\ x^4}{8} - \frac{1}{8}\int x^3 dx =$$

$$= \frac{x^4 \ln^2(x)}{4} - \frac{x^4 \ln(x)}{8} - \frac{x^4}{32} + c, \quad c \in \mathbb{R}$$

$$= x^4\left(\frac{\log^2(x)}{4} - \frac{\log(x)}{8} - \frac{1}{32}\right) + c \quad c \in \mathbb{R}$$

SADA 8 -MILAN WIKARSKI-

PRÍKLAD C

$$\int \frac{x}{x^2-x+2}\,dx = \int\left(\frac{2x-1}{2(x^2-x+2)} + \frac{1}{2(x^2-x+2)}\right)dx =$$

$$= \int \frac{2x-1}{2(x^2-x+2)}\,dx + \int \frac{1}{2(x^2-x+2)}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-1}{x^2-x+2}\,dx + \frac{1}{2}\int \frac{1}{x^2-x+2}\,dx$$

I. NAJPR SPOČÍTAME

$$\int \frac{2x-1}{x^2-x+2}\,dx$$

POUŽIJEME SUBSTITÚCIU

$$u = x^2 - x + 2$$

$$du = 2x-1\ dx$$

DOSTANEME

$$\int \frac{2x-1}{x^2-x+2}\,dx = \int \frac{1}{u}\,du = \ln(|u|) =$$

$$= \ln\left(\left|x^2 - x + 2\right|\right)$$

II. TERAZ SPOČÍTAME

$$\int \frac{1}{x^2-x+2}\, dx =$$

DOPLNÍME NA ŠTVOREC

$$= \int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \frac{7}{4}}\, dx$$

POUŽIJEME SUBSTITÚCIU

$$t = x - \frac{1}{2}$$

$$dt = 1\, dx$$

DOSTANEME

$$= \int \frac{1}{t^2 + \frac{7}{4}}\, dt = \int \frac{1}{t^2 + \left(\sqrt{\frac{7}{4}}\right)^2}\, dt =$$

$$= \frac{1}{\sqrt{\frac{7}{4}}} \cdot \arctan\left(\frac{t}{\sqrt{\frac{7}{4}}}\right) + c \qquad c \in \mathbb{R}$$

$$= \frac{1}{\sqrt{\frac{7}{4}}} \cdot \arctan\left(\frac{x-\frac{1}{2}}{\sqrt{\frac{7}{4}}}\right) + c$$

$$= \frac{1}{\sqrt{\frac{7}{4}}} \cdot \operatorname{arctan}\left(\frac{\frac{2x-1}{2}}{\sqrt{\frac{7}{4}}}\right) + c$$

$$= \frac{1}{\frac{\sqrt{7}}{2}} \cdot \operatorname{arctan}\left(\frac{\frac{2x-1}{2}}{\frac{\sqrt{7}}{2}}\right) + c$$

$$= \frac{2}{\sqrt{7}} \cdot \operatorname{arctan}\left(\frac{2x-1}{\sqrt{7}}\right) + c$$

VRÁTIME SA K PÔVODNÉMU PRÍKLADU

$$= \frac{1}{2} \ln\left(|x^2 - x + 2|\right) + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{7}} \cdot \operatorname{arctan}\left(\frac{2x-1}{\sqrt{7}}\right) + c$$

MÁME

$\forall x \in \mathbb{R} \quad x^2 - x + 2 \geq 0 \quad$ LEBO $\quad x^2 - x + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4}$

DOSTÁVAME

$$= \frac{1}{2} \ln\left(x^2 - x + 2\right) + \frac{1}{\sqrt{7}} \cdot \operatorname{arctan}\left(\frac{2x-1}{\sqrt{7}}\right) + c$$

A EŠTE SA VIEME ZBAVIŤ ODMOCNÍN V MENOVATEĽOCH

$$= \frac{1}{2}\ln\left(x^2 - x + 2\right) + \frac{\sqrt{7}}{7} \cdot \arctan\left(\frac{\sqrt{7}\,(2x-1)}{7}\right) + C$$