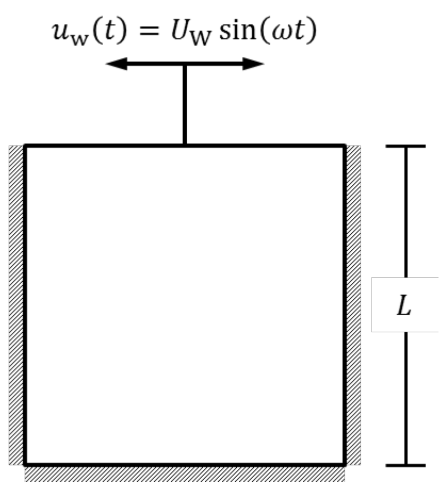
MAE/CSE 557
M.E. Mueller
Fall 2025

MAE/CSE 557: Simulation and Modeling of Fluid Flows

Mini-Project Two
Due: October 27, 2025

Directions: The compute portion of all assignments must be completed in a compiled language (C, C++, or Fortran) or another performant language (Julia). However, scripting languages can be used to manage a program as long the compute components are in a performant language, and any package including MATLAB may be used for making plots. Obviously, good programming practice (comments, etc.) are highly encouraged, and your source code must be submitted electronically with your write-up along with a readme with directions for compiling the code and running the program(s), preferably in a Linux environment (e.g., Nobel or Adroit). Your write-up should be typed and address all aspects of the problem in the instructions below, and you will need plots to support your discussion. The mini-project should be submitted by email to muellerm@princeton.edu and Israel.bonilla@princeton.edu by 9:34am on the due date as a single tarball or zip archive. The filename should have the format "lastname_firstname.zip" (or .tar, .tar.gz, .tgz, etc.) and should contain a report in PDF format entitled "report.pdf", the readme file entitled "readme.txt", and the source code.



Consider the flow inside a square with sides of length L in which the left, bottom, and right walls are stationary and the top wall moves perpendicular to its normal with a sinusoidal velocity profile in time. Initially, the flow is stationary inside the box. The reference parameters for your flow are provided below:

$$L = 1\ [\mathrm{m}]$$
$$L\sqrt{\omega/2\nu} = 1$$
$$\mathrm{Re} = U_W L/\nu = 100$$
$$\gamma = 1.4$$
$$\mathrm{Ma} = U_W/a_0 = 0.025$$
$$Pr = \rho c_p \nu/\lambda = 0.7$$

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Assume that the specific heats are constant and further assume that the initial pressure and temperature are uniformly $1\ \mathrm{bar}$ and $300\ \mathrm{K}$, respectively, with a specific gas constant of $287\ \mathrm{J/kg \cdot K}$. This initial temperature is used to define the reference speed of sound a_0. The walls are isothermal with a wall temperature of $300\ \mathrm{K}$. Assume that the kinematic viscosity ν and thermal diffusivity $\lambda/\rho c_p$ are constant but that the dynamic viscosity $\mu = \rho\nu$ and the ratio λ/c_p vary with the density ρ.

Your task will be to develop a compressible flow solver utilizing the finite difference method for the flow inside the box by following the steps below. In your write-up, be sure to address all of the questions in the steps below and include any additional information as you see fit.

a. Choose [the form of] your governing equations for compressible flow. Justify your choices.

b. Choose appropriate finite difference operators for the spatial operators in the governing equations. Justify your choices.

c. Choose appropriate temporal schemes and solution methods for your flow solver. Justify your choices.

d. For your choice of temporal schemes and spatial discretization, what are your expected (approximate) CFL limits? (Remember that the flow is compressible!)

e. Write your algorithm in pseudo-code.

f. Implement your approach. Prove that your implementation is correct by demonstrating expected convergence in both time and space.

g. Systematically increase the Mach number of the flow (0.05, 0.1, 0.2, 0.4, 0.8) by increasing the wall velocity magnitude. Keep the Reynolds number constant (i.e., $\mathrm{Re} = U_{\mathrm{W}} L/\nu = 100$) by decreasing the box size accordingly, and increase the frequency of the oscillation to keep the non-dimensional frequency constant (i.e., $L\sqrt{\omega/2\nu} = 1$). Compare the solutions at $\omega t = 10$ and discuss any qualitative differences. Discuss the relative computational efficiency of your approach as the Mach number changes.

h. <u>Bonus</u>: Change your temperature boundary condition such that the walls are adiabatic. Is the solution more strongly affected at smaller or larger Mach numbers?