Ex 1.

$$\dfrac{\dfrac{\dfrac{c \models \neg a}{a \models \neg c} \qquad \dfrac{c \models \neg b}{b \models \neg c}}{a \wedge b \models \neg c}\,\wedge}{c \models \neg(a \wedge b)} \qquad \dfrac{c \models \neg a \qquad c \models \neg b}{c \models \neg a \vee \neg b}\,\vee$$

$$\therefore \dfrac{c \models \neg(a \wedge b)}{c \models \neg a \vee \neg b}$$

$$\rightarrow \dfrac{\dfrac{}{\neg a \vee \neg b \models \neg a \vee \neg b}}{\neg a \vee \neg b \models \neg(a \wedge b)} \qquad \dfrac{\neg(a \wedge b) \models \neg(a \wedge b)}{\neg(a \wedge b) \models \neg a \vee \neg b}$$

$$\therefore \neg a \vee \neg b \subseteq \neg(a \wedge b)$$

$$\neg(a \wedge b) \subseteq \neg a \vee \neg b$$

$$\therefore \neg(a \wedge b) = \neg a \vee \neg b$$

Ex 2.

$$\dfrac{(x \wedge y) \vee (x \wedge z) \models x \wedge (y \vee z)}{(x \wedge y) \vee (x \wedge z) \models x \qquad (x \wedge y) \vee (x \wedge z) \models (y \vee z)}\,\wedge R$$

$$\dfrac{(x \wedge y) \vee (x \wedge z) \models x \qquad (x \wedge y) \vee (x \wedge z) \models (y \vee z)}{(x \wedge y) \models x \quad (x \wedge z) \models x \quad (x \wedge y) \models (y \vee z) \quad (x \wedge z) \models (y \vee z)}\,\vee L$$

$$\dfrac{(x \wedge y) \models x \quad (x \wedge z) \models x \quad (x \wedge y) \models (y \vee z) \quad (x \wedge z) \models (y \vee z)}{x, y \models x \quad x, z \models x \quad x, y \models y, z \quad x, z \models y, z}\,\vee R, \wedge L$$

$$\therefore \dfrac{(x \wedge y) \vee (x \wedge z) \models x \wedge (y \vee z)}{x, y \models x \quad x, z \models x \quad x, y \models y, z \quad x, z \models y, z}$$

Yes, it is universally valid.
($a \wedge b$ will always be included in a;
$a, b \models b, c$ is immediate.)

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Ex 3. $\vDash (x \wedge \neg y) \vee (\neg(x \vee z) \vee (y \vee z))$

VR

$\vDash (x \wedge \neg y), (\neg(x \vee z) \vee (y \vee z))$

VR

$\vDash (x \wedge \neg y), (\neg(x \vee z)), (y \vee z)$

VR

$\vDash (x \wedge \neg y), (\neg(x \vee z)), y, z$

De Morgan's

$\vDash (x \wedge \neg y), (\neg x \wedge \neg z), y, z$

$\wedge$R

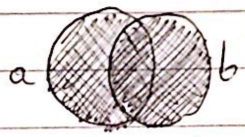
$\vDash x, (\neg x \wedge \neg z), y, z \qquad \vDash \neg y, (\neg x \wedge \neg z), y, z$

$\wedge$R

$\vDash x, \neg x, z, y \qquad \vDash \neg y, \neg x, z, y \qquad \vDash x, \neg z, z, y \qquad \vDash \neg y, \neg x, y, z$

This is a tautology.
This would not be a contradiction if it were in the format $\varphi \vDash$

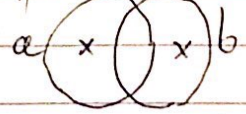
Ex 4. $\neg a \wedge \neg b \vDash \neg(a \wedge b) \Longleftrightarrow \neg a \wedge \neg b \vDash \neg a \vee \neg b$
$\Longleftrightarrow \neg a, \neg b \vDash \neg a, \neg b.$



This is univesally valid.

$\neg(a \wedge b) \vDash \neg a \wedge \neg b \Longleftrightarrow \neg a \vee \neg b \vDash \neg a \wedge \neg b$
$\Longleftrightarrow \neg a \vDash \neg a \wedge \neg b$
and $\neg b \vDash \neg a \wedge \neg b$
$\Longleftrightarrow \neg a \vDash \neg a$
and $\neg a \vDash \neg b$
and $\neg b \vDash \neg a$
and $\neg b \vDash \neg b$

- This is <u>not</u> universally valid.
- A counter example is any universe where region $\neg a \wedge \neg b$ is occupied.



Excercise 5 is on Things_quickcheck.hs.