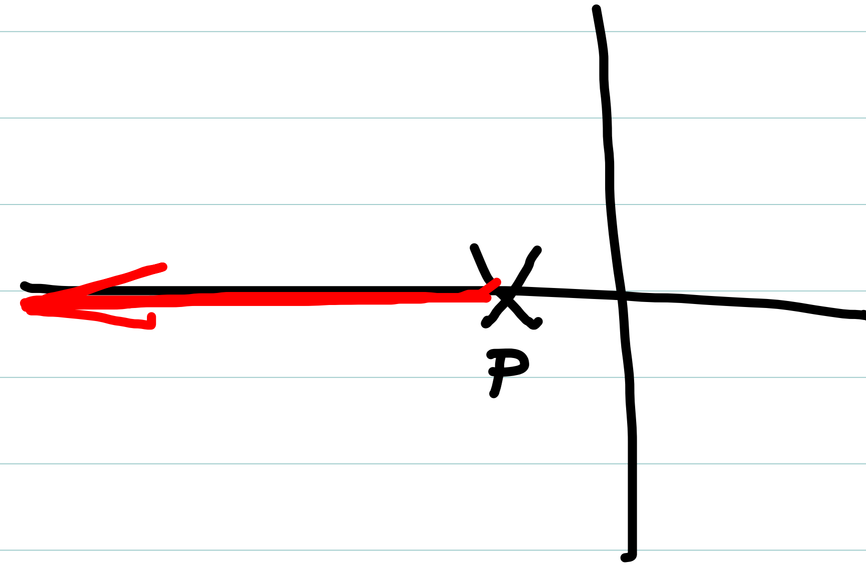
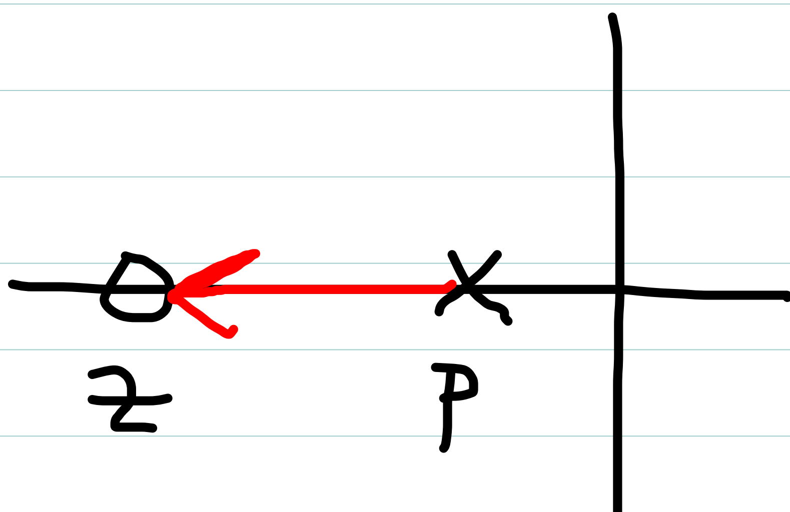
# Simple Examples

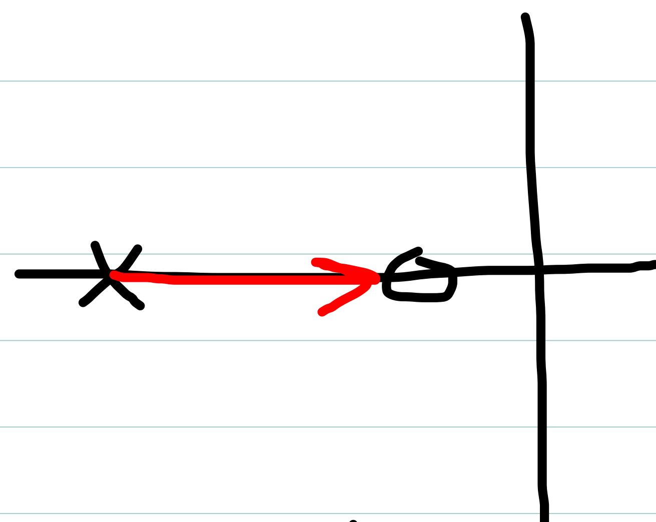
#1) $L(s) = \frac{K}{s-p}$




#2) $L(s) = K\left[\frac{(s-z)}{(s-p)}\right]$

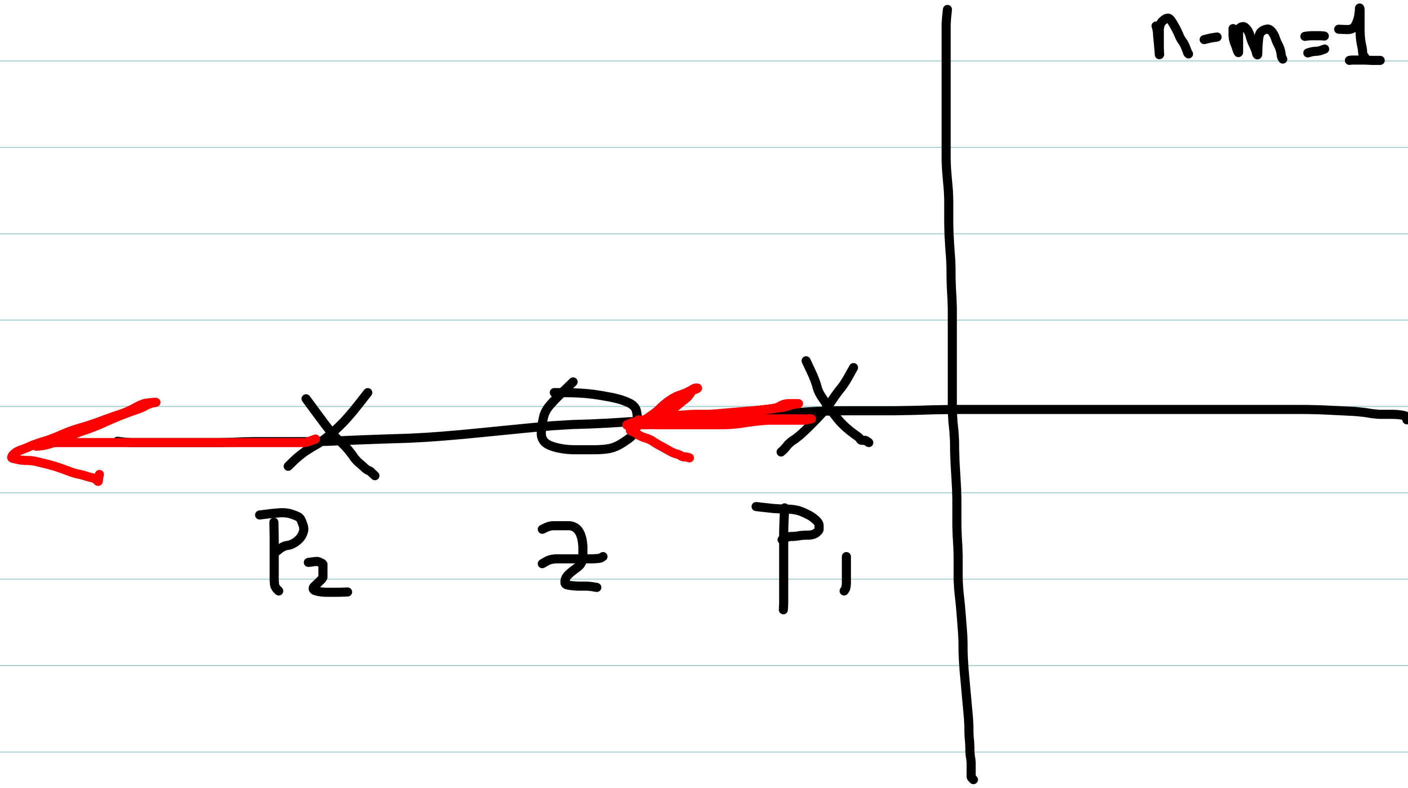



$|z| > |p|$



$|z| < |p|$

#3] $L(s) = K\left[\frac{(s-z)}{(s-p_1)(s-p_2)}\right]$




$n-m=1$ : asymptote along neg real Axis

$|P_2| > |z| > |p_1|$

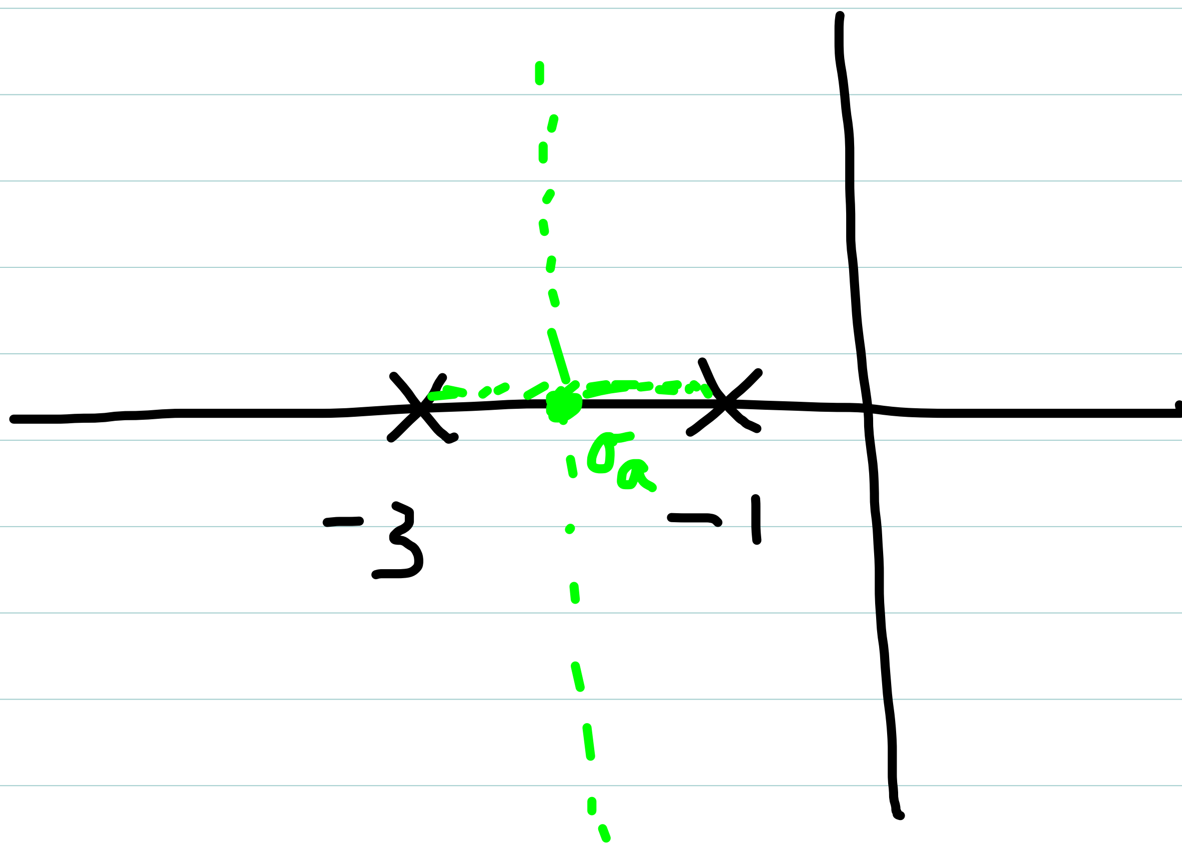
Case where $|z| < |p_1| \leq |p_2|$ is more complicated -- see below.

#4] $L(s) = \frac{K}{(s+1)(s+3)}$

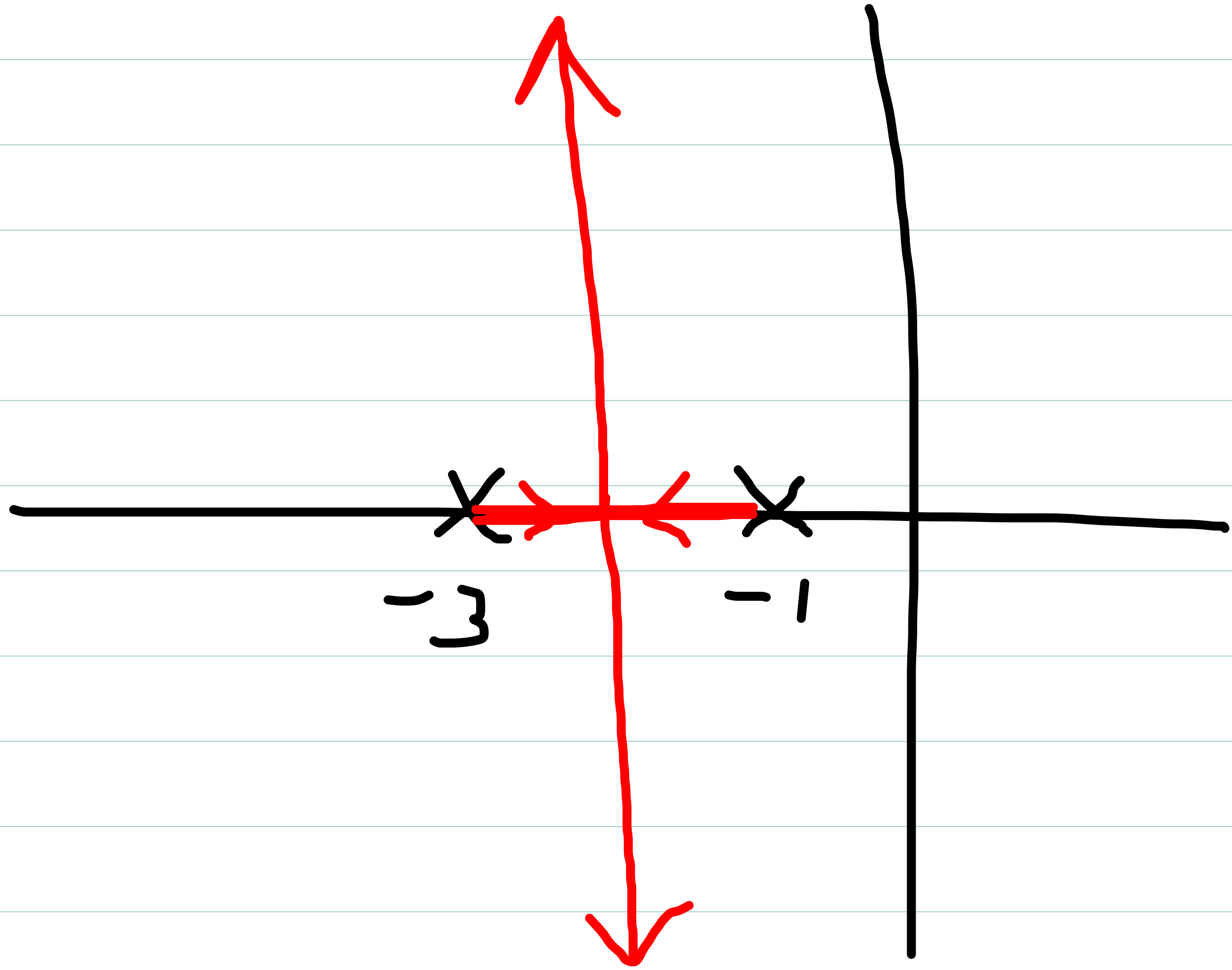
$n - m = 2 \Rightarrow \alpha_l = \pm 90°$

real Axis locus from $-3 \rightarrow -1$

$\sigma_a = \frac{(-1)+(-3)}{2} = -2$



Actual locus:




Compare w/ exact sol'n for CL poles:

$$s^2+4s+(3+K)=0 \implies s=-2\pm\frac{\sqrt{16-(3+K)^2}}{2}$$

# Example #5

$$L(s) = \frac{K}{s(s+1)(s+2)}$$

$n=3$, $m=\emptyset$, $n-m=3$ => 3 branches go to $\infty$ along

asymptotes: $\alpha_e = \pm 60°, 180°$

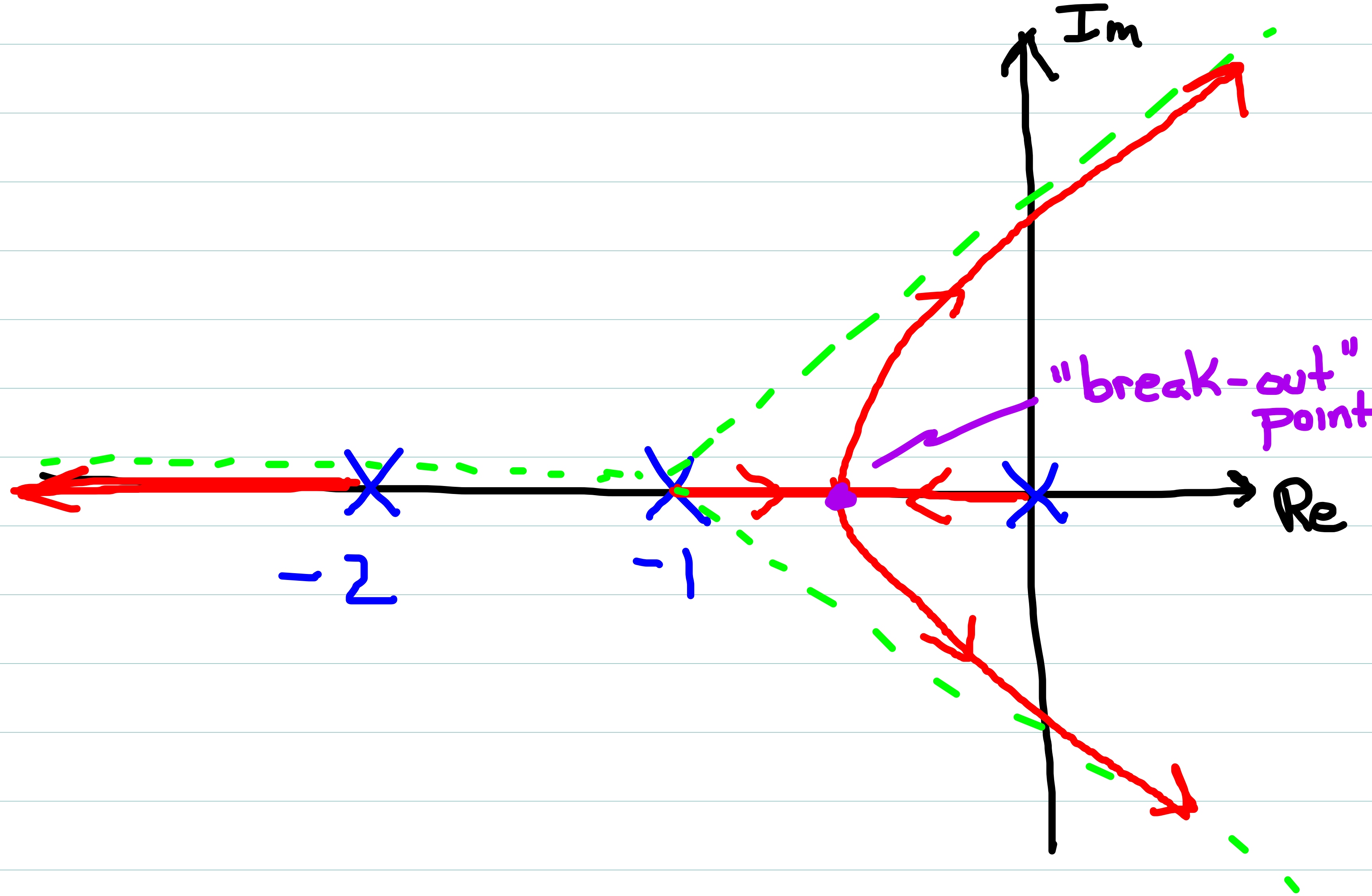
with intercept: $\sigma_a = \frac{0+(-1)+(-2)}{3} = -1$

Real Axis branch locations.

=> between -1 and $\emptyset$

=> left of -2

# Example #5, cont



# Break-out Points

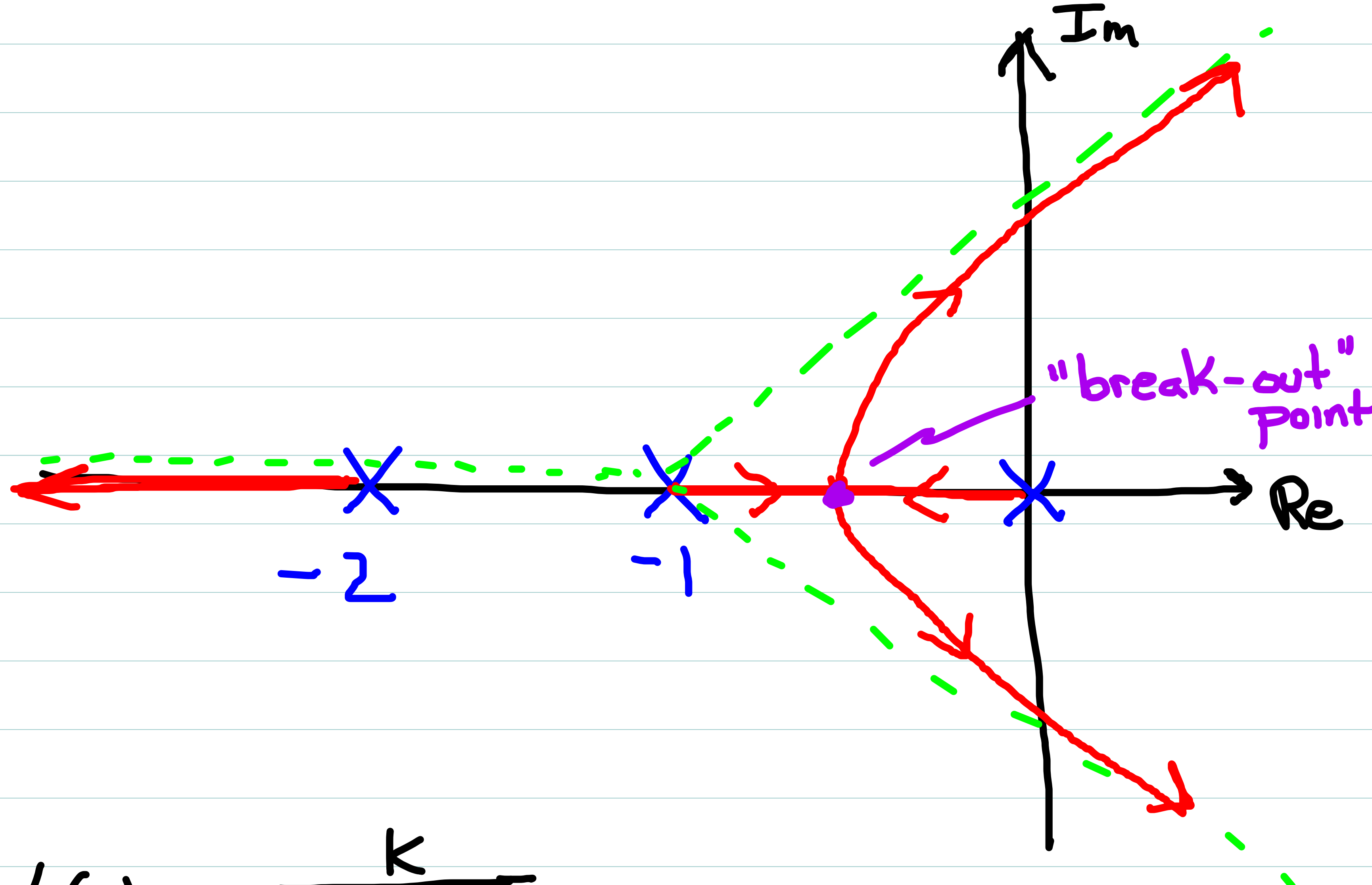
Break-out points occur for values of s satisfying

$$\frac{d}{ds}\left(L(s)\right) = 0$$

Since this (usually) leads to another high-order polynomial to factor, we often just approximate a break-out as occuring half-way along the branch

Use Matlab ("rlocus" command) to nail exact details when needed).

# Example #5, cont




$$L(s) = \frac{K}{s(s+1)(s+2)}$$

# High Gain Instability

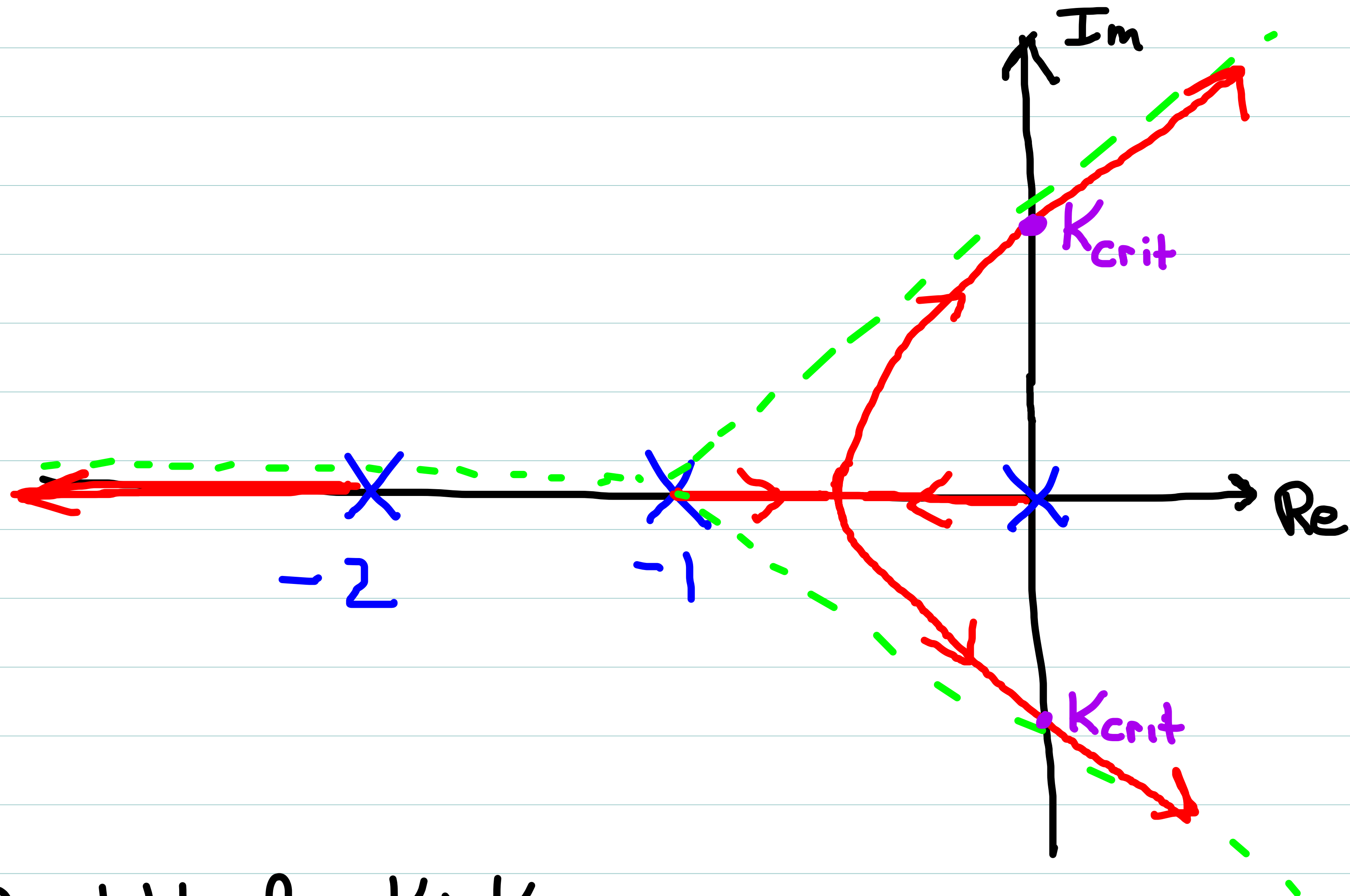
Since this example has Asymptotes in RHP, we can see the CL system <u>will</u> be unstable for sufficiently high gains K.

Whether this is a problem or not depends on the gain we want/need to get the desired CL poles

T(s) is <u>Not</u> automatically unstable b/c the root locus branches in RHP!

Such a locus only tells us T(s) will be unstable for <u>some</u> values of K.

# Example #5, cont




Unstable for $K > K_{crit}$

# Example #6

$$L(s) = \frac{K(s+3)}{(s+1)(s+2)}$$

$\Rightarrow$ $n=2$, $m=1$, $n-m=1$

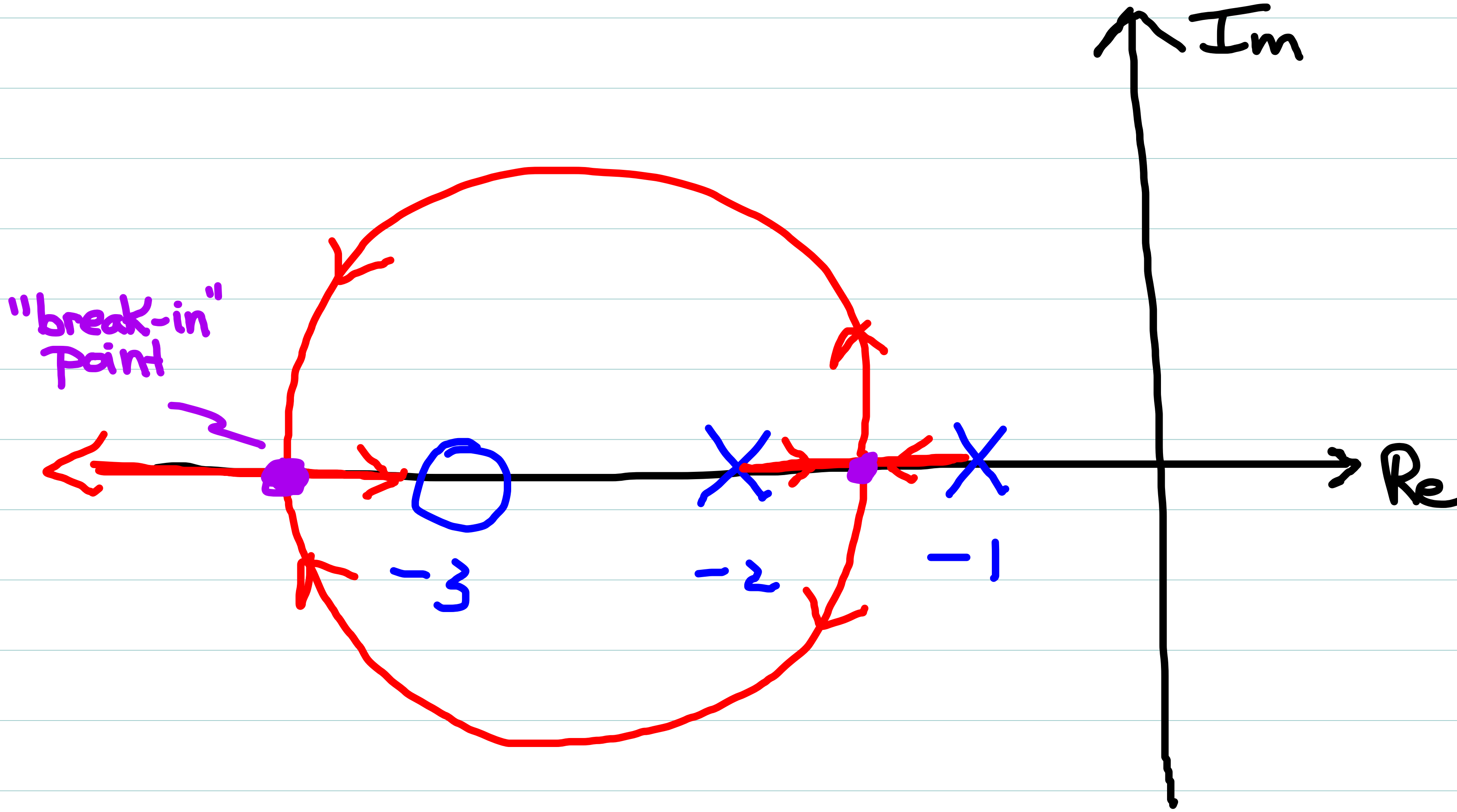
$\Rightarrow$ One branch ends at $-3$ (OL zero). One branch goes to $\infty$ along asymptote $\alpha = 180°$ (negative real axis)

$\Rightarrow$ Segments of branches lie on real axis:

$\Rightarrow$ Between $-2$ and $-1$

$\Rightarrow$ left of $-3$

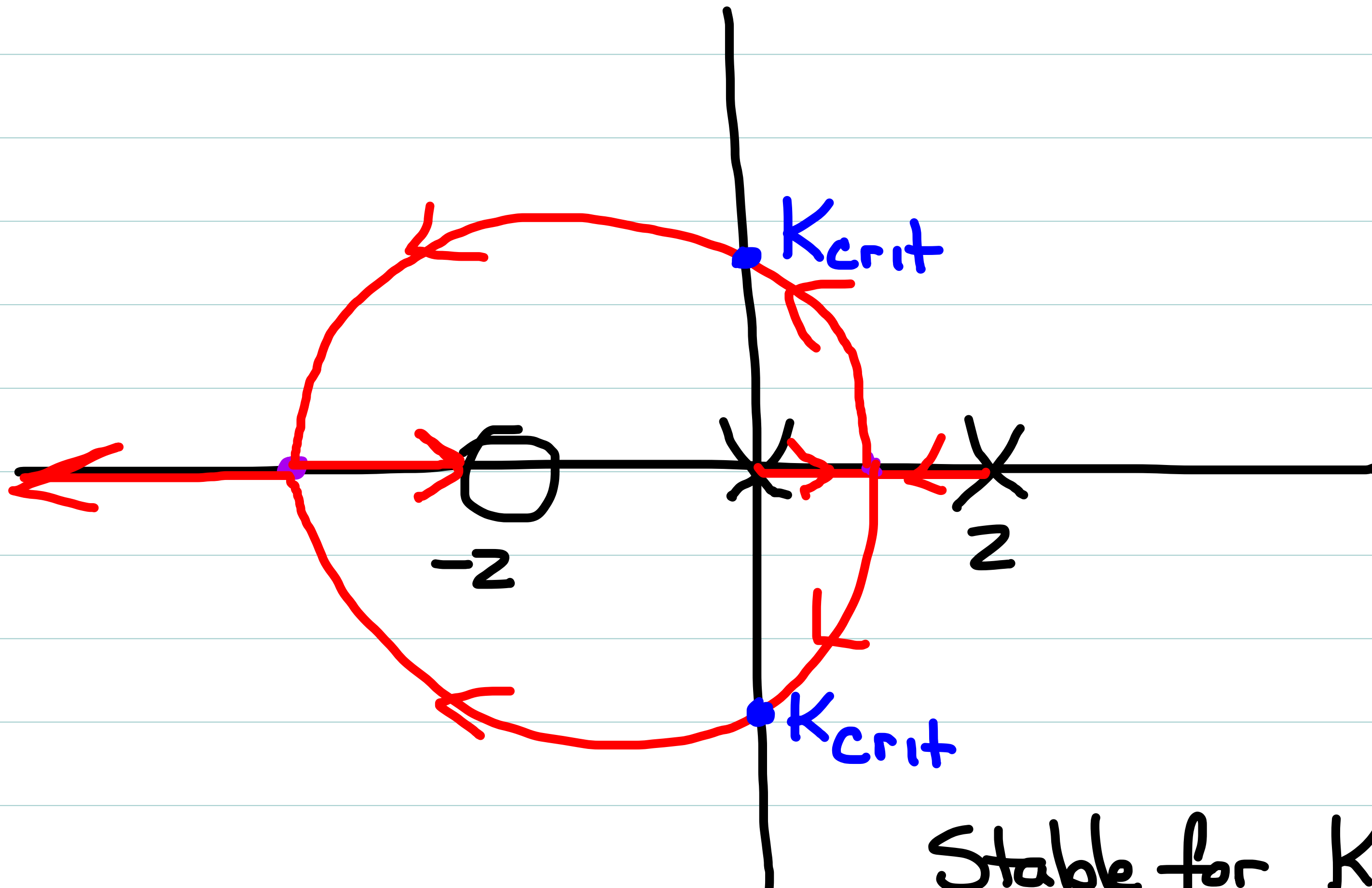
# Example #6, cont



# Example #7

$$L(s) = \frac{K(s+2)}{s(s-2)}$$

Similar analysis to above

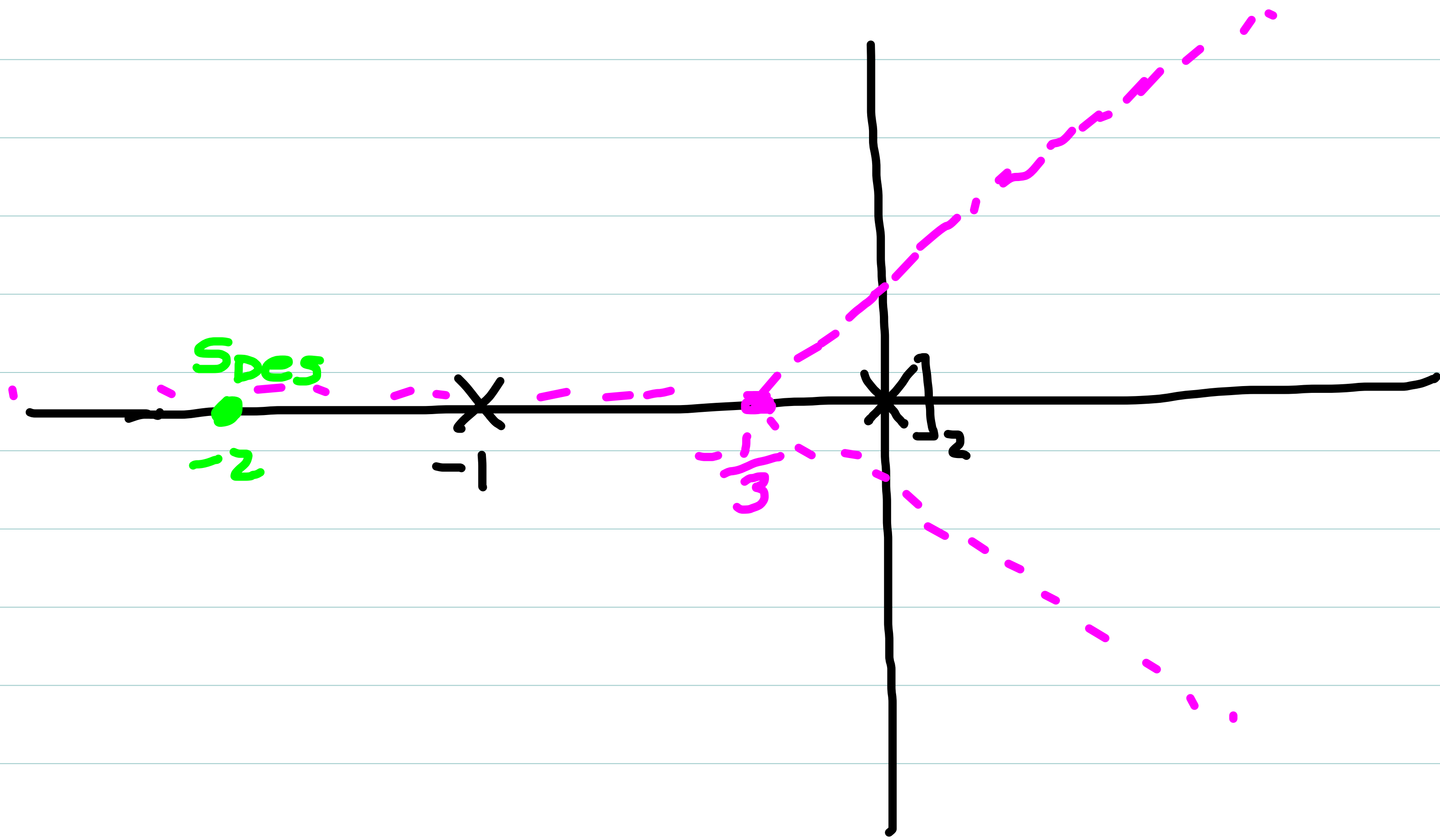



Stable for $K > K_{crit}$

# Example #3

This is where we originally started our investigation

with $H(s)=K$, $L(s)=\dfrac{K}{s^2(s+1)}$



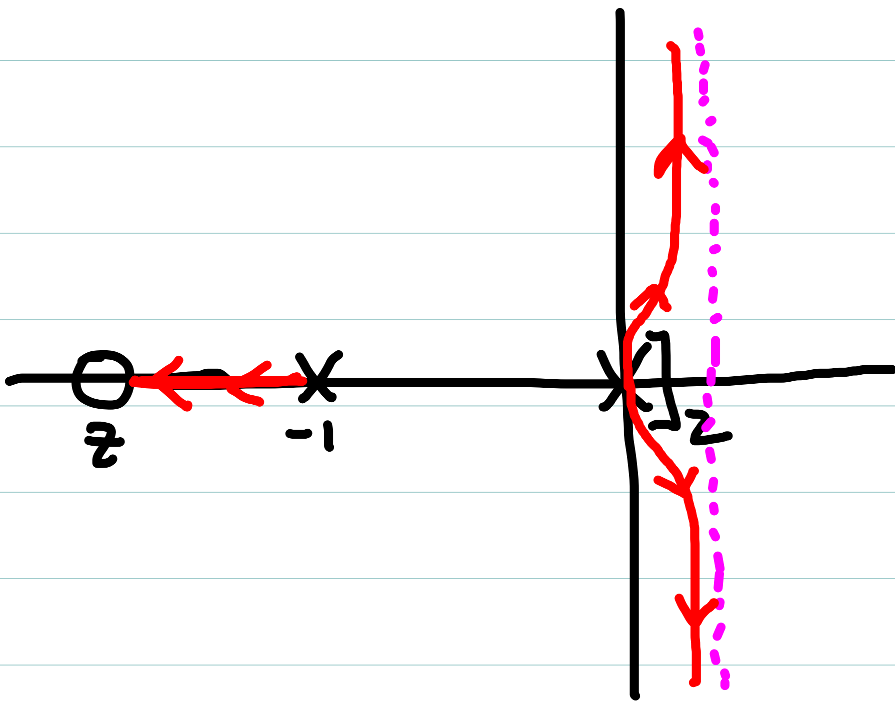
# Example #3

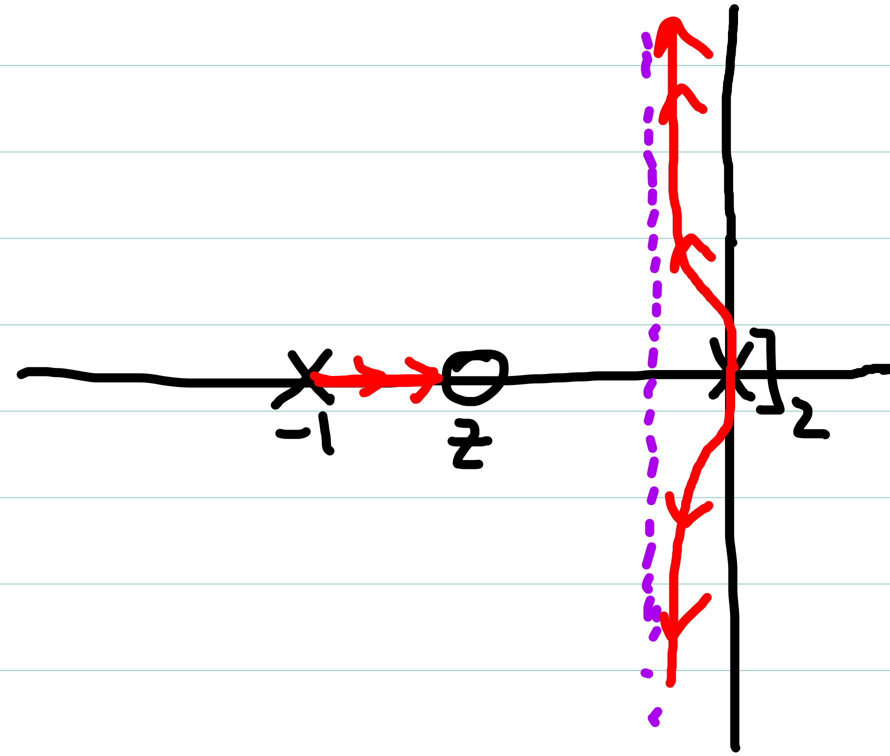
This is where we originally started our investigation

$$L(s) = \frac{K}{s^2(s+1)}$$


We can get the desired pole at $-2$, but will inevitably have poles of $T(s)$ in RHP

With instead $H(s) = K(s-z)$




$z < -1 \Rightarrow \alpha_\ell = \pm 90^\circ$

$0 < z < -1$

$$\sigma_a = \frac{-1}{2}(1+z) > 0$$

$\Rightarrow \sigma_a < 0$

So, with $H(s) = K(s-z)$ we can stabilize the system as long as $|z| < 1$ (which would agree with a Nyquist/phase margin analysis)

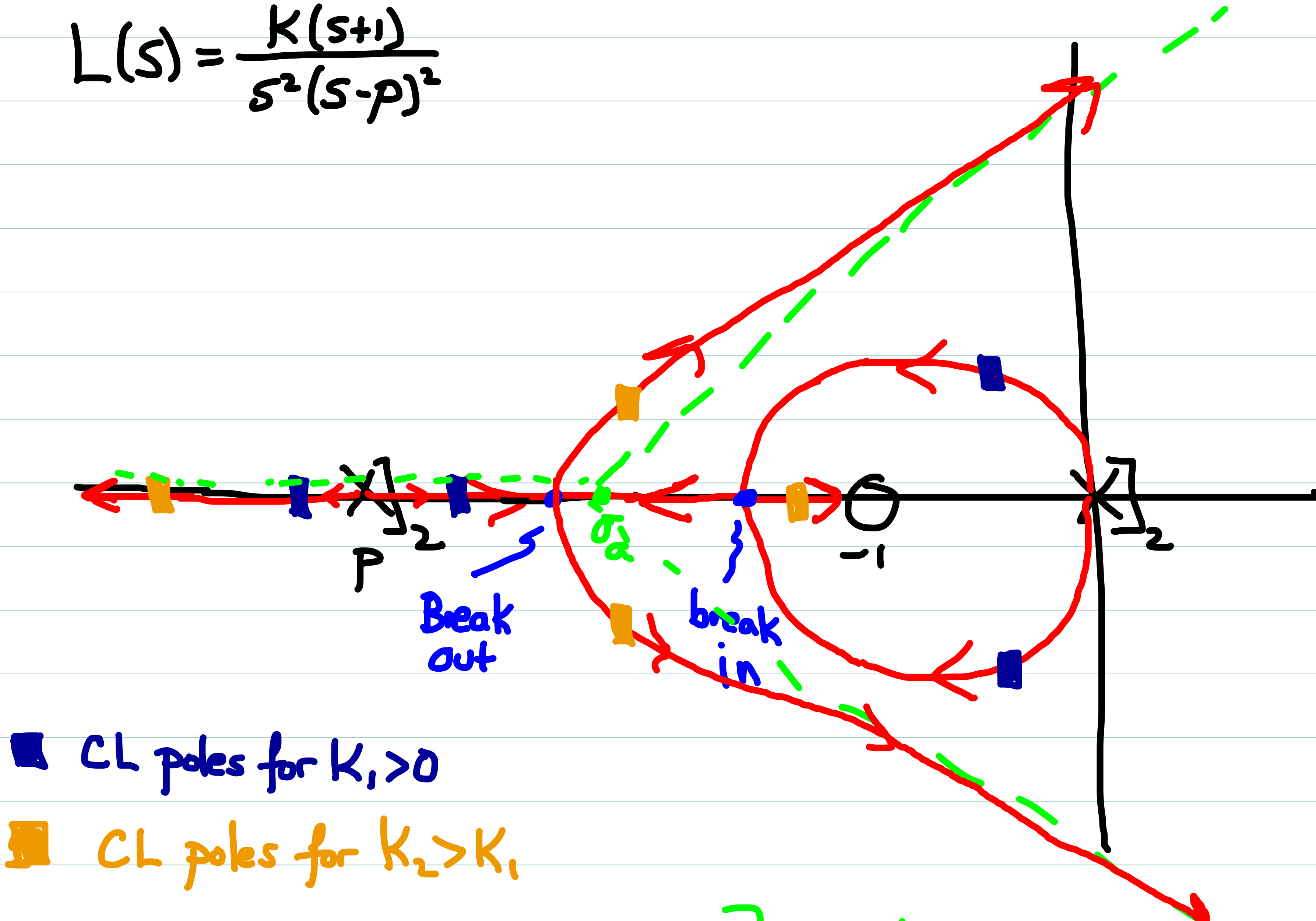
<u>But</u> we would have to accept a real pole $> -1$, and moreover this pole would not be dominant

An implementable compensator which could allow a real dominant CL pole near $-2$ would be

$$H(s) = K\left[\frac{(s+1)^2}{(s-p)^2}\right]$$

which has an interesting locus (next page)

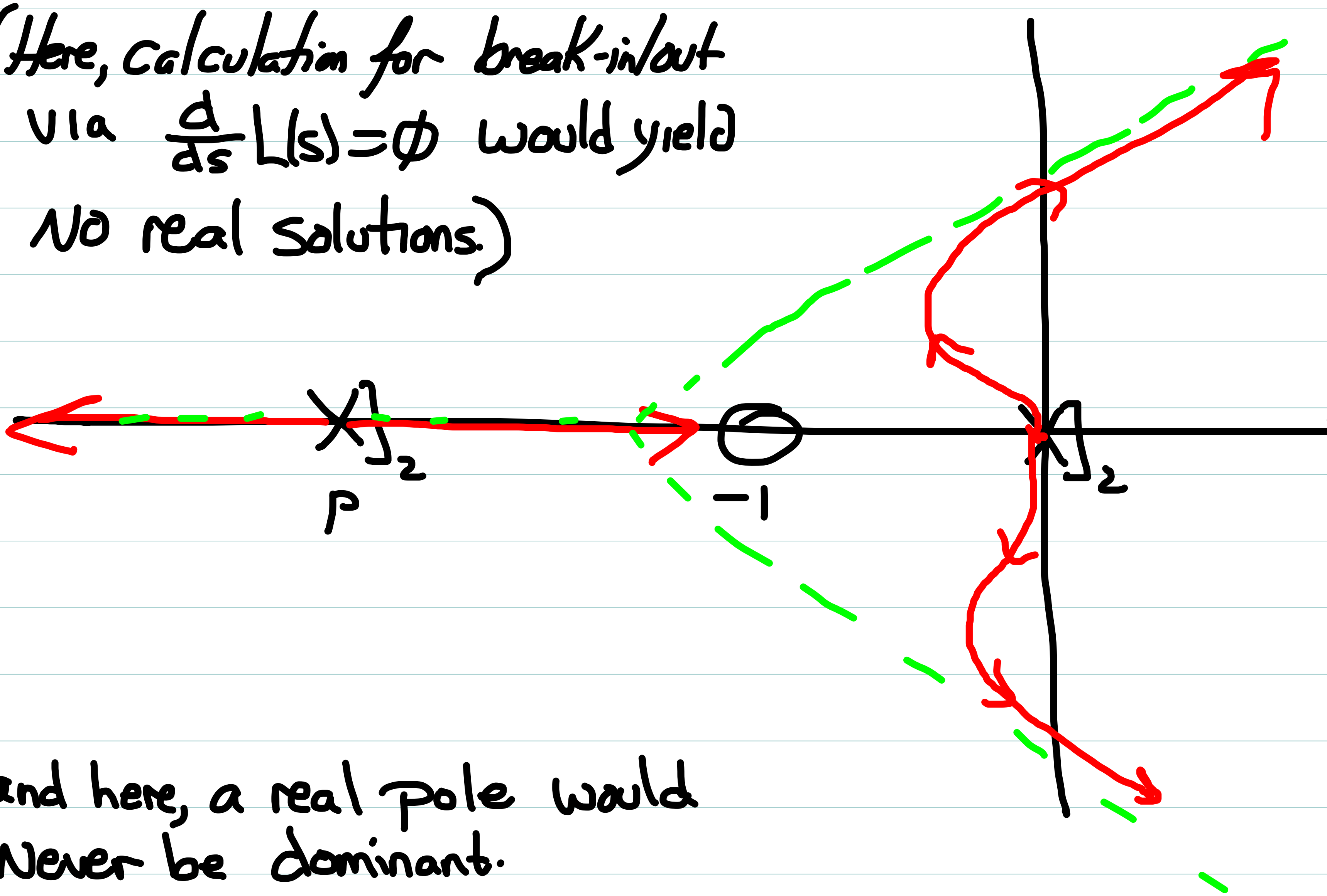
$$L(s) = \frac{K(s+1)}{s^2(s-p)^2}$$




$$\sigma_a = \frac{2p+1}{3}$$

However, depending on exact value of $p$, this is also possible:

(Here, calculation for break-in/out via $\frac{d}{ds}L(s) = \emptyset$ would yield No real solutions.)




and here, a real pole would Never be dominant.

# Comments on root locus method

⇒ Rules are not determinative; there may be many locus shapes consistent with calculations (although Matlab rlocus command will show you an exact plot).

⇒ Cannot adapt method to account for effects of time delay

⇒ Can adapt method only for very simple kinds of robustness analysis.

⇒ Bode/Nyquist methods preferred in professional practice.

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⇒ But root locus does provide useful additional insights which are not available using freq. methods

⇒ Familiarity with both gives "best of both worlds"