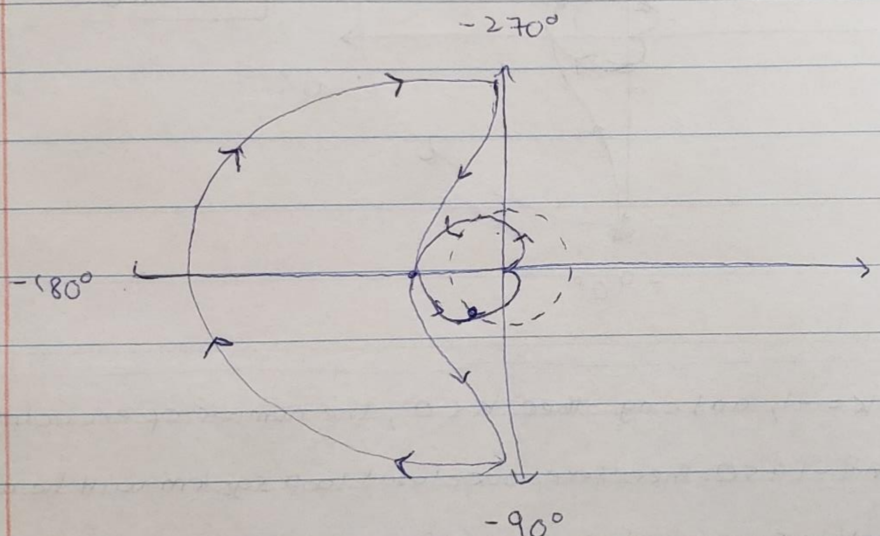
PS8 Q1 Solns

$$G(s) = \frac{-1.5(s+1)}{s(1-s)^2} \qquad H(s) = K \quad , \quad K>0$$

a) Find $\angle L(j\omega)|_{K=1}$ $\Rightarrow$ $L(s) = \dfrac{-1.5K(s+1)}{s(1-s)^2}$

$\boxed{\gamma = 48.3^\circ \quad \text{(see matlab)}}$

b) Nyquist for $L_0(s) = \dfrac{-1.5(s+1)}{s(1-s)^2}$



$P_R(T) = P_R(L) + N_{cw}$

$= 2 - 1 = 1$

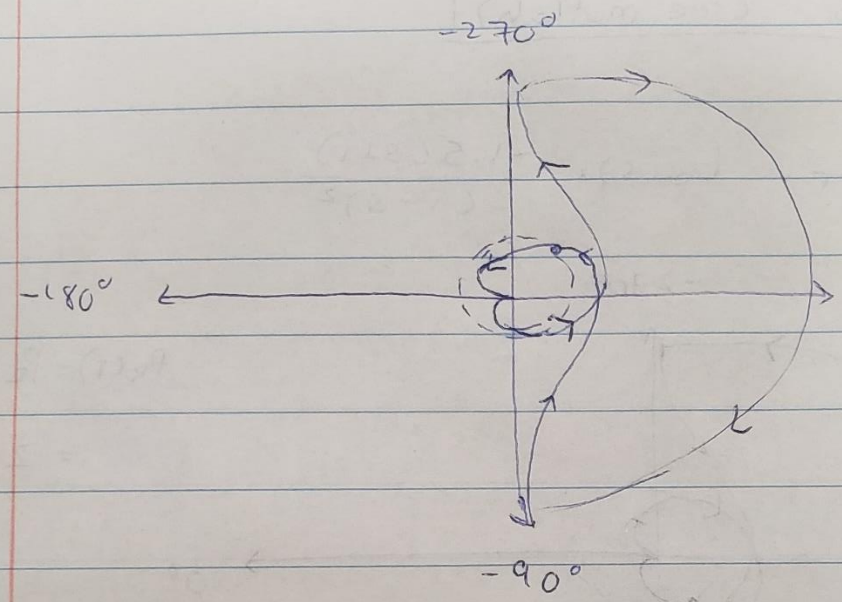
→ Nyquist analysis shows $P_R(T) = 1$, with one unstable pole predicted in the closed loop transfer function $T(s)$

c) The bode plot for part a) shows gain margin $a = 10^{-7.04/20} = .4446$
Thus, for $K < .4446$, $N_{cw} = 1$ and we would expect
$P_R(T) = P_R(L) + N_{cw} = 3$ unstable CL poles. For $K > .4446$,
$N_{cw} = -1$ and we would expect 1 unstable pole in $T(s)$. Under
no $K>0$ do we expect $T(s)$ to be stabilized.

d) a) $L = \dfrac{-1.5K(s+1)}{s(1-s)^2}$, $K = -1 \Rightarrow L = \dfrac{1.5(s+1)}{s(1-s)^2}$

$\boxed{\gamma = -132^\circ \text{ (see Matlab)}}$

b) Nyquist for $L = \dfrac{1.5(s+1)}{s(1-s)^2}$



c)

$N_{CW} = 0 \;\forall K < 0$

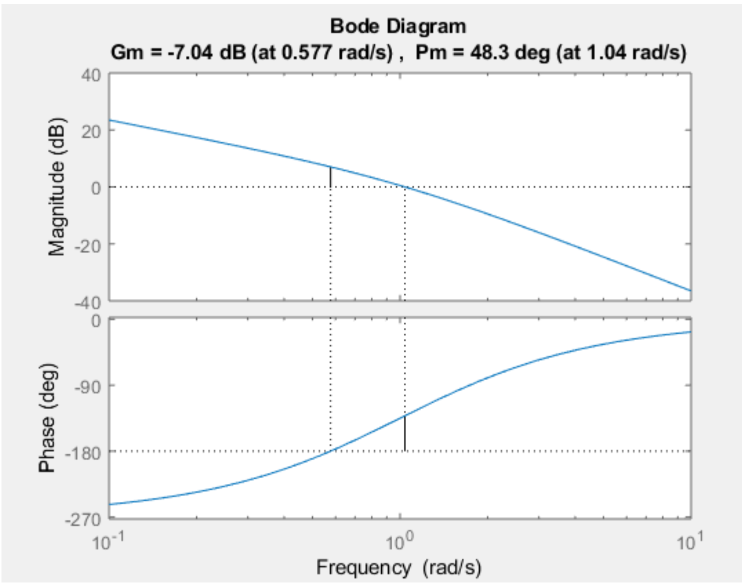
$\therefore P_R(T) = P_R(L) = 2 \;\forall K < 0$

unstable!

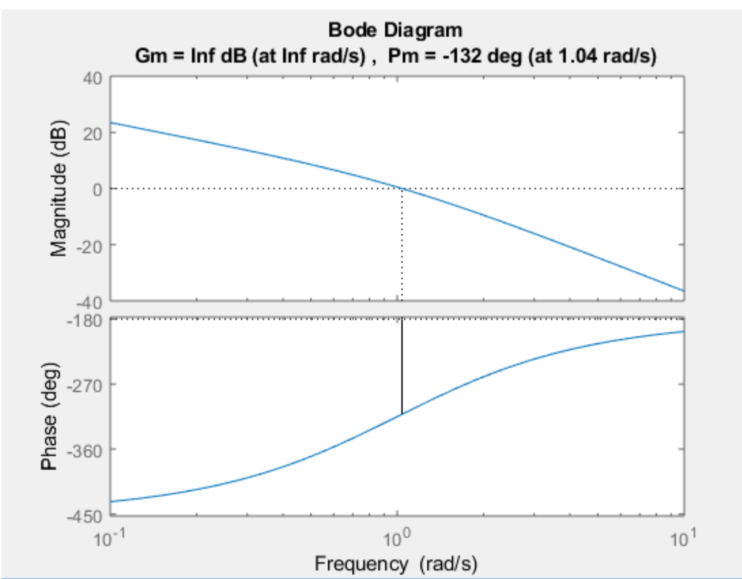
for $K = -1$, and any other $K < 0$, the number of encirclements around -1 is 0. Therefore, our closed loop system will be unstable regardless of our choice of $K < 0$

Q1 part a

```
%1a find phase margin of L @ K=1
s = tf('s');
G = -1.5*(s+1)/(s*(s-1)^2);
margin(G)
```




```
%1d find phase margin of L @ K=-1
s = tf('s');
G = 1.5*(s+1)/(s*(s-1)^2);
margin(G)
```



Question 2

$G(s) = \frac{-1.5\ (s+1)}{s(s-1)^2}$ $\qquad$ $H(s) = K(s+1)$ with $K<0$.

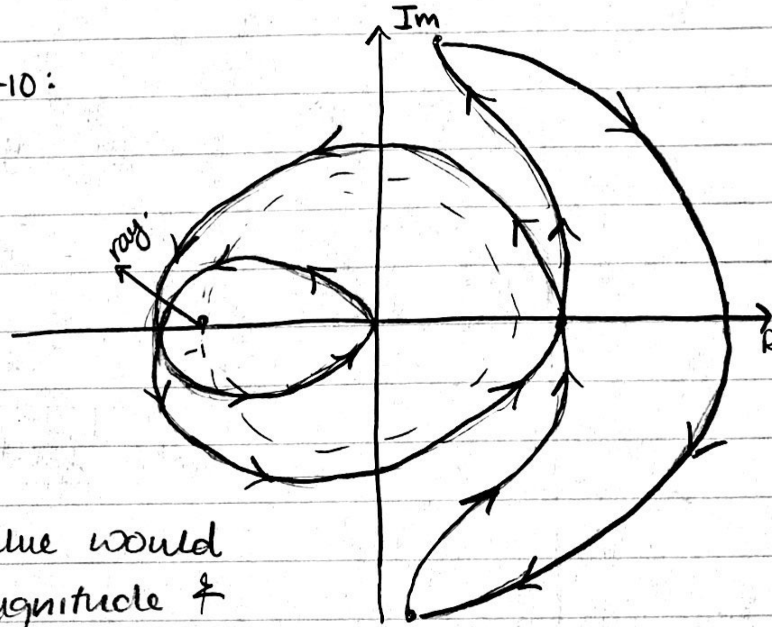
$L(s) = G(s)H(s) = \frac{-1.5K(s+1)^2}{s(s-1)^2}$

a.) Nyquist analysis: $\qquad$ if $K=-10$:

$N_{cw}(L) = -P_R(L)$

$-2 = -2$ ✓

$\boxed{\text{STABLE}}$



Find range of K for stability.

⟹ The minimum negative K value would shift $\|L(j\omega)\|$ up so the magnitude & phase crossover occur @ the same location.

$L_o(s) = \frac{1.5(s+1)^2}{s(s-1)^2}$ (with $K=-1$) $\Rightarrow$ $a_{dB} = 1.6107\text{ dB} = 1.2038.$

$|K_{min}| = 1.2038$

$\boxed{\therefore \text{ Since } K<0, \text{ the range of } K \text{ for stability is } K < -1.2038.}$

b) want $\gamma = 30°$.

$\gamma = 30° = 180 + \measuredangle L(j\omega)$

$\therefore \measuredangle L(j\omega) = -150°$

@ $\measuredangle L_o(j\omega) = -150°$: $\omega = 3.73\ \frac{rad}{s}$

$\|L(j\omega)\| = -7.92\text{ dB}$
$= 2.4889.$

$\therefore$ $\boxed{K_{new} = -2.48}$ $\qquad$ $\boxed{\omega = 3.73\ \frac{rad}{s}}$

c.) Using matlab, from the step response of $T(s)$:

$\boxed{M_p = 64.1\%}$ $\boxed{t_s = 6.9\,s}$ $\boxed{y_{ss} = 1}$

Plotting the step response of $R(s)$ fails because the compensator used here cause $R(s)$ to have more zeros than poles ⟹ Bad!

d.) i.) $e_{ss}(t)$ when $y_d(t) = A$ (constant)

From the $\|S(j\omega)\|$ diagram, the low frequency magnitude approaches $-\infty$ dB, which means $S(0) = 0$. Thus, e_{ss} is also 0 with a constant input $y_d(t)$.

$\boxed{e_{ss}(t) = 0}$

ii.) tracking bandwidth.

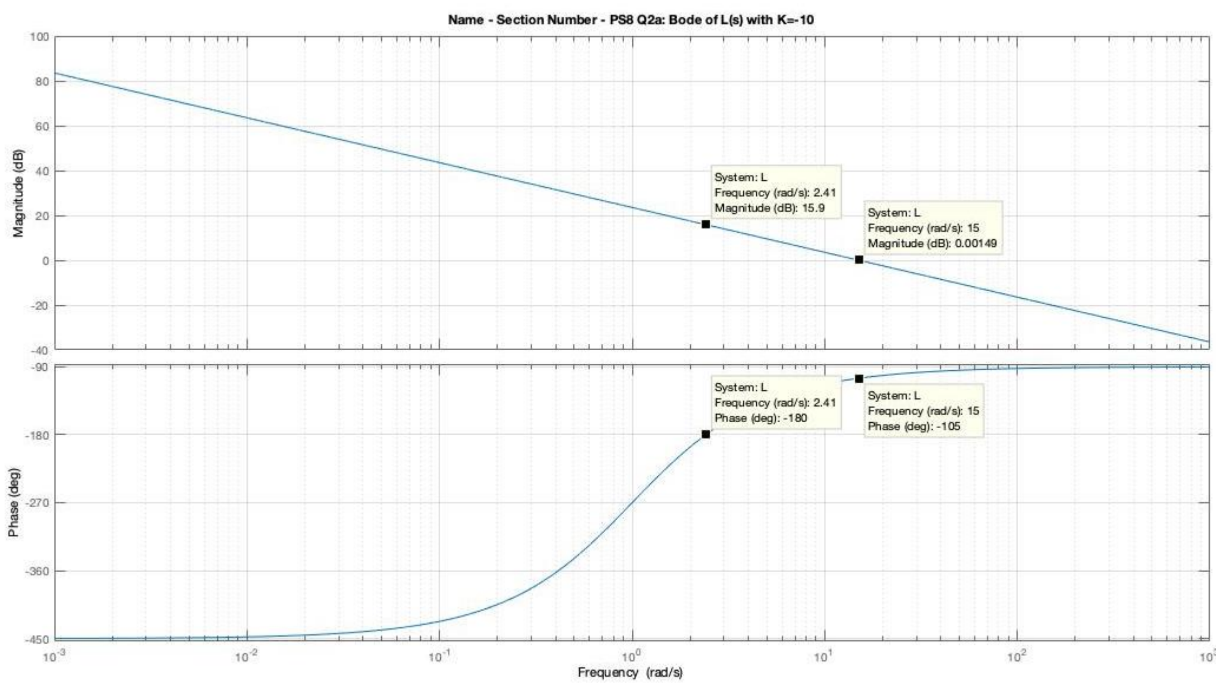
$|S(j\omega_B)| \approx -3$ dB. From the $\|S(j\omega)\|$ diagram, $\boxed{\omega_B = 1.72 \frac{rad}{s}}$

iii.) $|S(j\omega_\gamma)|$ (graphically vs. analytically)

From the $\|S(j\omega)\|$ diagram, $\boxed{|S(j\omega_\gamma)| = 5.23 \text{ dB}}$. They match!

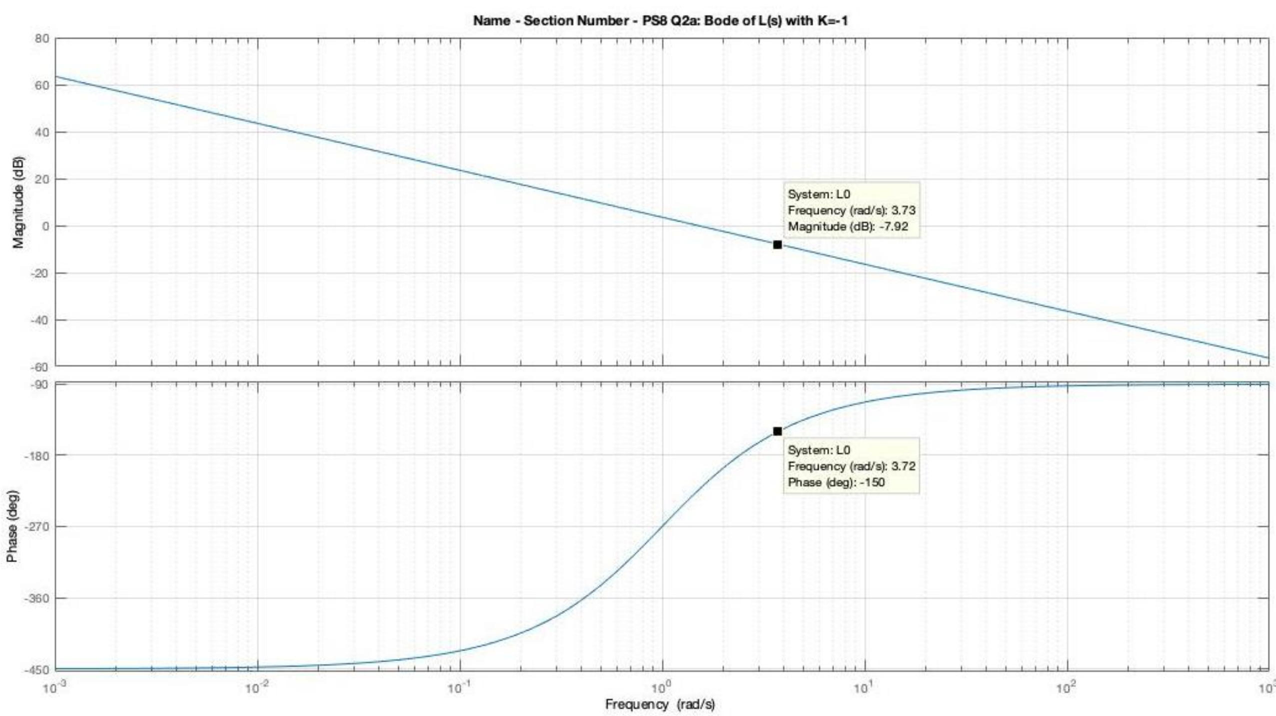
Analytically, matlab gives $\boxed{|S(j\omega_\gamma)| = 5.2694 \text{ dB}}$

```
% ENAE 432, Spring 2019
% TA Solutions
% PS8, Question 2

% Part A
s = tf('s');
w = logspace(-3,3,250000);
G = -1.5*(s+1)/(s*(s-1)^2);
K = -1;
H = K*(s+1);
L0 = minreal(G*H);
[Gm_db, Pm_deg, Wcg, Wcp] = margin(L0)
Kmin = -10^(Gm_db/20)

K = -10;
H = K*(s+1);
L = minreal(G*H);
figure(1)
bode(L,w); grid on;
title('Name - Section Number - PS8 Q2a: Bode of L(s) with K=-10');
```



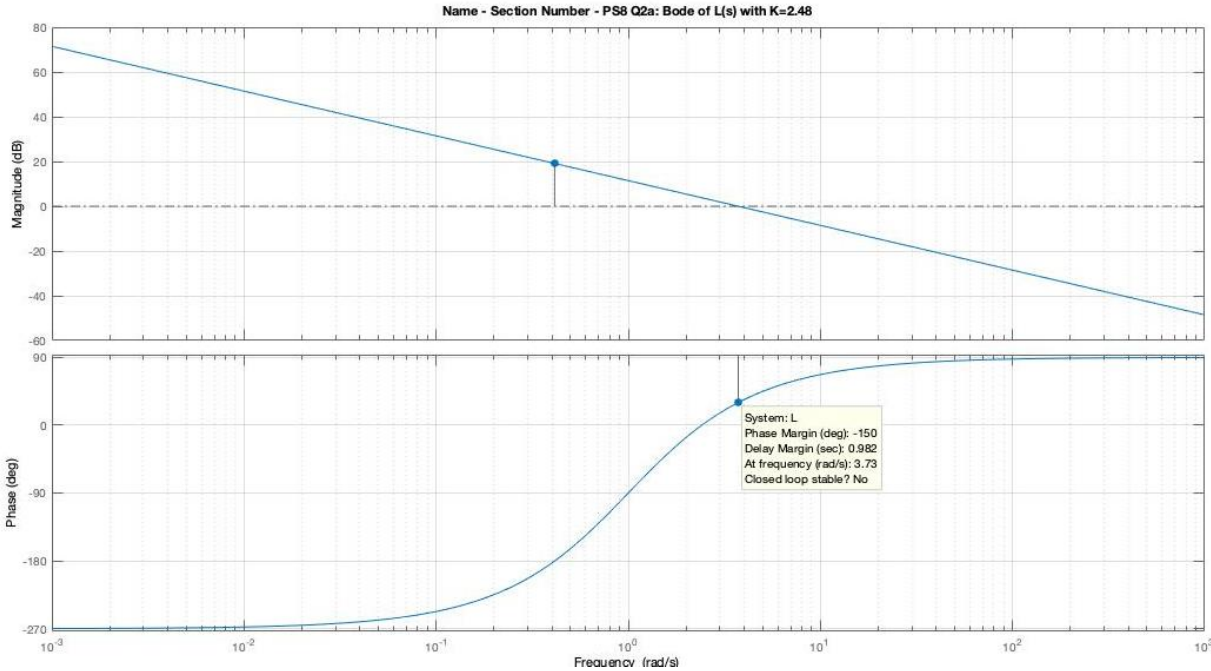

```
Gm_db =

    1.6107


Pm_deg =

  -44.7603
```

```
Wcg =

    2.4161


Wcp =

    1.5000

Kmin =

   -1.2038



% Part B
figure(2)
bode(L0,w); grid on;
title('Name - Section Number - PS8 Q2a: Bode of L(s) with K=-1');
Gm_db_b = -7.92;  % value of shift needed (obtained from Bode diagram)
Knew = -10^(-Gm_db_b/20)
Hnew = Knew*(s+1);
Lnew = minreal(G*Hnew);
figure(3)
bode(Lnew,w); grid on;
title('Name - Section Number - PS8 Q2a: Bode of L(s) with K=2.48');
```

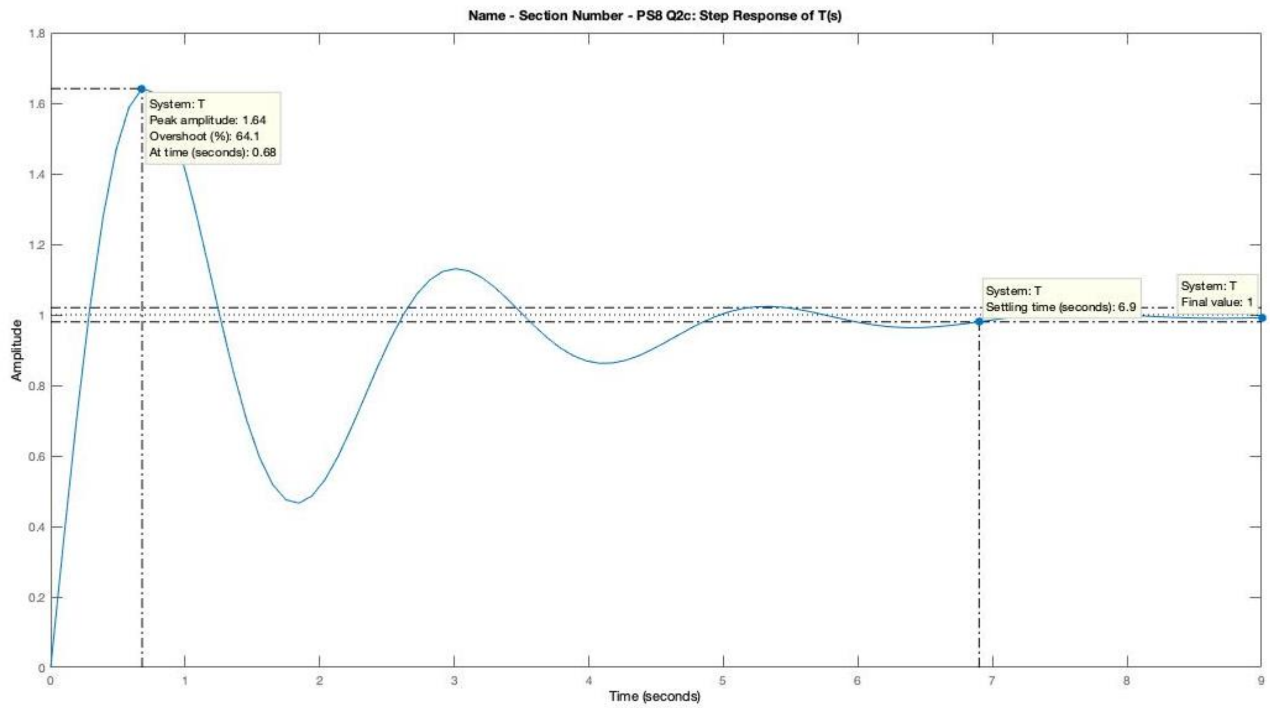



```
Knew =
```

```
   -2.4889
```



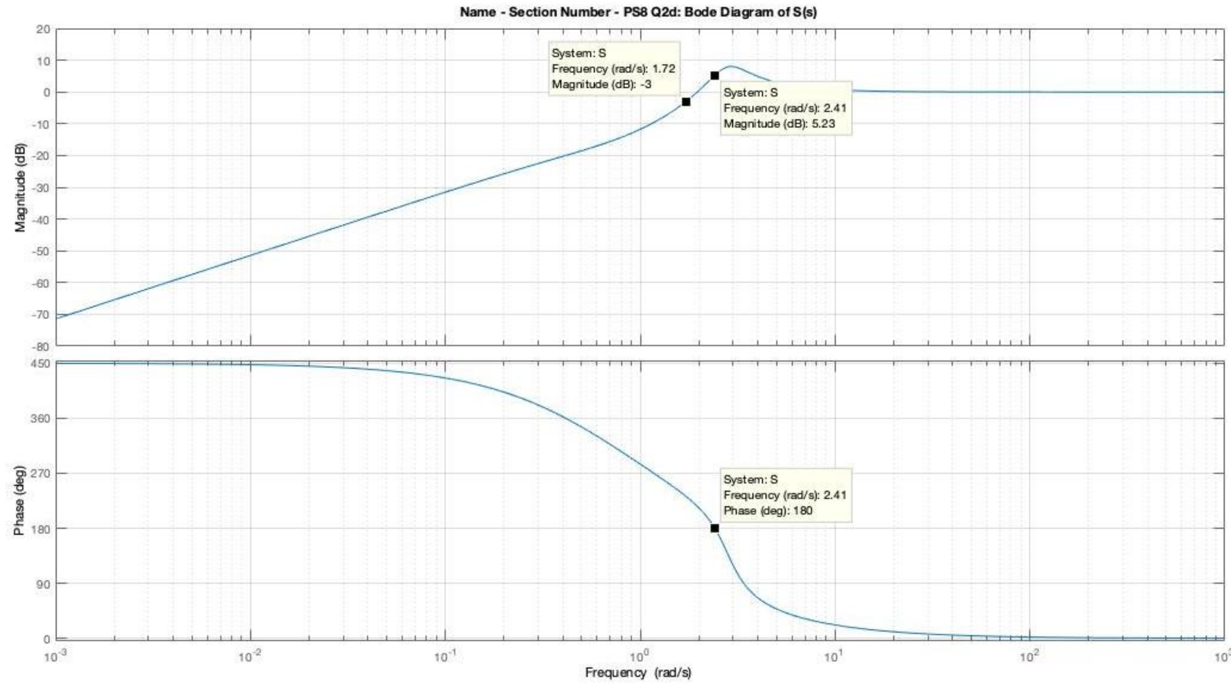

```
% Part C
T = feedback(Lnew,1)
figure(4)
step(T); title('Name - Section Number - PS8 Q2c: Step Response of
T(s)');
R = feedback(Hnew,G)
%figure(5)
%step(R); title('Name - Section Number - PS8 Q2c: Step Response of
R(s)');

T =

     3.733 s^2 + 7.467 s + 3.733
  ---------------------------------
  s^3 + 1.733 s^2 + 8.467 s + 3.733

Continuous-time transfer function.
```




```
R =

  -2.489 s^4 + 2.489 s^3 + 2.489 s^2 - 2.489 s
  --------------------------------------------
       s^3 + 1.733 s^2 + 8.467 s + 3.733

Continuous-time transfer function.
```

```
% Part D
S = feedback(1,Lnew)
bode(S,w); grid on;
title('Name - Section Number - PS8 Q2d: Bode Diagram of S(s)');
[Gm_db, Pm_deg, Wcg, Wcp] = margin(L);
S_wcg = abs(evalfr(S,j*Wcg))
S_wcg_db = mag2db(S_wcg)
```

```
S =

           s^3 - 2 s^2 + s
  ---------------------------------
  s^3 + 1.733 s^2 + 8.467 s + 3.733

Continuous-time transfer function.
```




```
S_wcg =

    1.8343


S_wcg_db =

    5.2694
```

PS8 Solutions

Question #3

desired characteristics

$\gamma = 30^\circ$

$\omega_\gamma = 3.73$ rad/s

$$G(s) = \frac{-1.5(s+1)}{s(s-1)^2}$$

a) Design A Lead Compensator that will achieve the above

- $\gamma_{des} = 30^\circ$
- $\omega_{\gamma des} = 3.73$ rad/s

General Form of a Lead Compensator

$$H(s) = K\frac{(\beta\tau s+1)}{(\tau s+1)}$$

* Determine ϕ_{req}

$$\measuredangle H(s) = \phi_{req} = \gamma_{des} - 180 - \measuredangle G(j\omega_{des})$$

$$= 30^\circ - 180^\circ - [\measuredangle(-1.5) + \measuredangle(3.73j+1) - \measuredangle(3.73j) - 2\measuredangle(3.73j-1)]$$

can use evalfr() for G and then angle()

$$\phi_{req} = 30^\circ - 180^\circ - [-180^\circ + \operatorname{atan}(\tfrac{3.73}{1}) - 90^\circ - \underbrace{2\operatorname{atan}(\tfrac{3.73}{-1})}_{\text{2nd quadrant!}}]$$

$$\underline{\phi_{req} = -105^\circ}$$

* notice that the ϕ_{req} cannot be achieved with just a single pole and zero as the maximum angle contribution will be $\pm 90^\circ$; however we know that $K<0$, thus we have the following relationship

$$\measuredangle H(\omega j) = \phi_{req} \qquad H_0 = \frac{1}{-K}H$$

$$\measuredangle H_0(j\omega) + \measuredangle(-K) = \phi_{req}$$

$$\measuredangle H_0(j\omega) - 180^\circ = -104$$

$$\underline{\measuredangle H_0(j\omega) = 75^\circ = \phi_{req,new}}$$

* This $\phi_{req,new}$ is the angle that our pole and zero must provide

Find β

$$\sin\phi = \left[\frac{\beta-1}{\beta+1}\right]$$

$$\beta\sin\phi + \sin\phi = \beta - 1$$

$$\sin\phi + 1 = \beta - \beta\sin\phi$$

$$\beta = \frac{1+\sin\phi}{1-\sin\phi}$$

$$\boxed{\beta = 57.7}$$

Find τ

$$\tau = \frac{1}{\omega\sqrt{\beta}}$$

$$\boxed{\tau = .0354}$$

Find K

let $L_0 = L|_{K=-1}$

$$|K| = \frac{1}{||L_0(j\omega_{des})||}$$

$$|K| = 1.26$$

$$\boxed{K = -1.26}$$

not required for this part but useful for determining τ

b) Characterize the step Response of $T(s)$

$T(s)$
$$\boxed{\begin{aligned} t_s &= 8.92s \\ \%OS &= 64.3\% \\ y_{ss} &= 1 \end{aligned}}$$

When we do the same for $R(s)$, we do not get an error like on problem #2, since we now have an implementable design

i.e, we have # poles $\geq$ # zeros

$$\boxed{\begin{aligned} |U_{max}| &= 72.7 \\ U_{ss}(t) &= 0 \end{aligned}}$$

c) Redesign the compensator such that.

$W_{des} = 2 \text{ rad/s}$
$\gamma = 30^\circ$
, we follow the same steps in part a)

$$\phi_{req} = 30^\circ - 180^\circ - \measuredangle G(jW_{des})$$
$$\boxed{\phi_{req} = -70^\circ}$$

- taking into account of the requirement, we have a $\phi_{req,new}$, which is the angle contribution of the poles and zeros

$$\phi_{req,new} = -70^\circ + 180^\circ$$
$$\boxed{\phi_{req,new} = 110^\circ}$$

- notice that $\phi_{req,new} > 90^\circ$, which means that a single pole and zero will not be enough to fulfill this ϕ_{req}. so we have to modify the lead compensator

$$H(s) = K\frac{(\beta T s + 1)^2}{(Ts+1)^2} \qquad H_c(s) = \frac{(\beta T s + 1)^2}{(Ts+1)^2}$$

- notice that our new $H(s)$ is just two lead compensators multiplied to one another, which allows us to make this relationship

$$\phi_{req,new} = \measuredangle \frac{(\beta T s + 1)}{(Ts+1)} + \measuredangle \frac{(\beta T s + 1)}{(Ts+1)}$$

- arbitrarily split $\phi_{req,new}$ in half to each angle contribution as such

$$55^\circ = \frac{\phi_{req,new}}{2} = \measuredangle \frac{(\beta T s + 1)}{(Ts+1)} \qquad 55^\circ = \frac{\phi_{req,new}}{2} = \measuredangle \frac{(\beta T s + 1)}{(Ts+1)}$$

Finally we can use lead compensator designs to find β and τ

$$\beta = \frac{1+\sin\phi}{1-\sin\phi} \qquad \tau = \frac{1}{\omega\sqrt{\beta}}$$

$$\boxed{\beta = 10.06} \qquad \boxed{\tau = .1576}$$

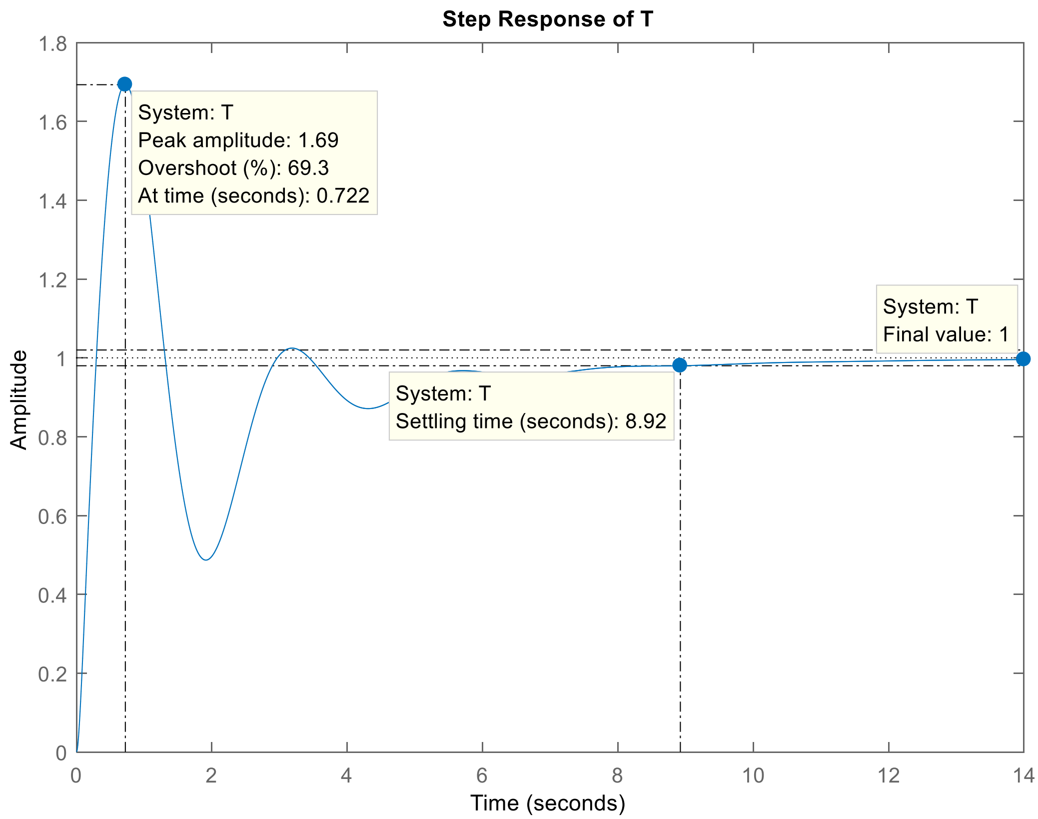
$$K = \frac{1}{\| L_0(j\omega_{des}) \|}$$

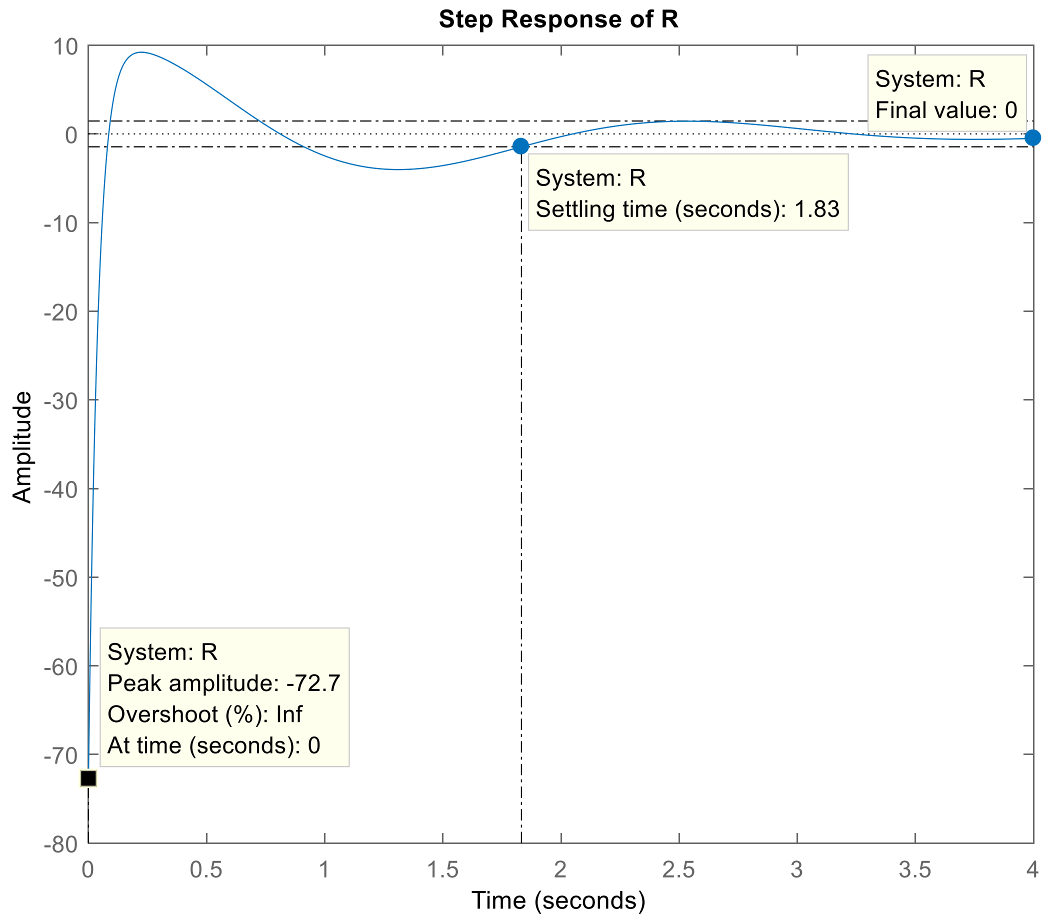
$$|K| = .2965$$

$$\boxed{K = -.2965}$$

Thus $H(s) = -.2965 \dfrac{((10.06)(.1576)s+1)^2}{(.1576s+1)^2}$

$$\boxed{H(s) = -.2965 \frac{(1.586s+1)^2}{(.1576s+1)^2}}$$


Step Response of T
System: T
Peak amplitude: 1.69
Overshoot (%): 69.3
At time (seconds): 0.722
System: T
Settling time (seconds): 8.92
System: T
Final value: 1
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
2
4
6
8
10
12
14



Step Response of R
System: R
Final value: 0
System: R
Settling time (seconds): 1.83
System: R
Peak amplitude: -72.7
Overshoot (%): Inf
At time (seconds): 0
Amplitude
Time (seconds)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
0.5
1
1.5
2
2.5
3
3.5
4

PS Q.

Q4. $I\dot{\omega}(t) = -b\omega(t) + K_m u(t)$

$I = 5,\ b = 0.5,\ K_m = 1.3$

(A) $G(s) = \dfrac{K_m}{Is + b}$

$u(t) = Ke(t)$

$U(s) = KE(s) \implies H(s) = K$

$L(s) = G(s)H(s) = \dfrac{KK_m}{Is + b}$

$T(s) = \dfrac{L(s)}{1 + L(s)} = \dfrac{KK_m}{Is + b + KK_m}$

pole of $T(s) \implies -\dfrac{(b + KK_m)}{I}$

- single real pole
- no transient oscillation

The pole will always be in the LHP as long as $b + KK_m > 0$

OR

$K > -\dfrac{b}{K_m}$ $\boxed{K > -0.385}$

(B) (i) FOR A SETTLING TIME, $t_s = 1$ second

$t_s = \dfrac{4}{|\sigma|} = \dfrac{4}{\left(\dfrac{b + KK_m}{I}\right)} \implies \boxed{K = (4I - b)\dfrac{1}{K_m} = 15}$

(ii) $e_{ss}(t) = S(0)\,\omega_d(t) = S(0)\,\underline{10}$

$S(s) = \dfrac{1}{1 + L(s)} = \dfrac{Is + b}{Is + b + KK_m}$; $S(0) = 0.025$

$\boxed{e_{ss}(t) = 0.25}$

(C) $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau$

$$H(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s}$$

$$L(s) = G(s)H(s) = \frac{K_m(K_p s + K_i)}{s(Is+b)}$$

$L(s)$ has 1 pole at the origin, therefore this controller will ensure perfect tracking for constant w_d.

(D) $$L(s) = \frac{K_m(K_p s + K_i)}{s(Is+b)}$$

In bode form:
$$\frac{\frac{K_m K_I}{b}\left(\frac{K_p s}{K_I} + 1\right)}{s\left(\frac{I s}{b} + 1\right)}$$

- pole at the origin ensures that the low frequency magnitude slope will always be $-20\,dB/dec$
- Notice that the pole is $\left(-\frac{b}{I}\right)$ and location of the zero is $\left(-\frac{K_i}{K_p}\right)$
- As long as the pole (corner frequency) occurs before the zero, there will always be at least $-20\,dB/dec$ slope for all range of frequencies

i.e. $$\boxed{\left|\frac{b}{I}\right| < \left|\frac{K_i}{K_p}\right| \quad \text{OR} \quad \frac{K_i}{K_p} > \frac{b}{I}}$$

(E) $T(s) = \dfrac{L(s)}{1+L(s)} = \dfrac{K_m(K_p s + K_i)}{s(Is+b) + K_m(K_p s + K_i)}$

$$= \frac{K_m(K_p s + K_i)}{Is^2 + bs + K_m K_p s + K_m K_i}$$

$$T(s) = \frac{K_m(K_p s + K_i)}{Is^2 + (b + K_m K_p)s + K_m K_i}$$

- We need repeated real closed-loop poles. This means that the $t_s = \dfrac{6}{|\sigma|}$ and for $t_s = 1 \Rightarrow \underline{\sigma = -6}$

- let's take a look at the denominator of $T(s)$

$$Is^2 + (b + K_m K_p)s + K_m K_i$$

$$\Rightarrow \quad s^2 + \left(\frac{b + K_m K_p}{I}\right)s + \frac{K_m K_i}{I}$$

Compare this with $s^2 + 2\sigma s + \underline{\sigma^2 + w_d^2}$ $\qquad \underline{w_d = 0}$ (repeated real poles)

$$-2\sigma = \frac{b + K_m K_p}{I} \longrightarrow (1) \qquad \frac{K_m K_i}{I} = \sigma^2$$

$$\boxed{\begin{aligned} K_p &= \underline{45.7692} \\ K_i &= \underline{138.4615} \end{aligned}}$$

Yes, the step response will show overshoot due to the presence of LHP zero $\left|\left(-\dfrac{K_i}{K_p}\right)\right|$ which is less than the repeated poles at $|\underline{-6}|$

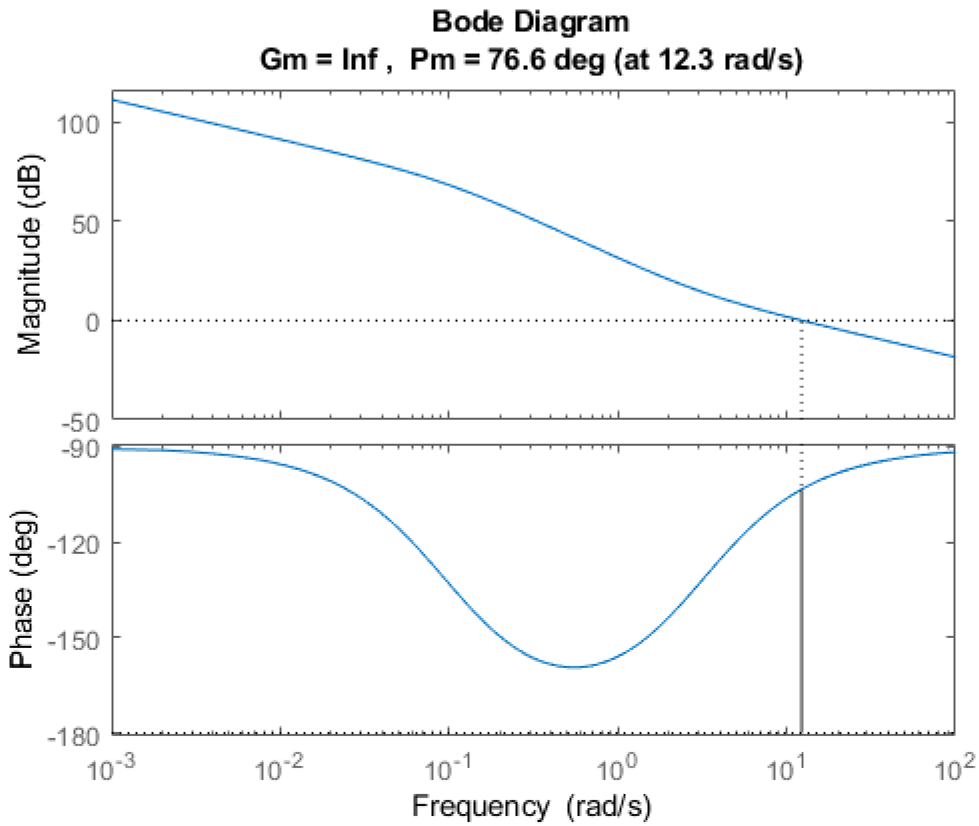
ENAE 432 PS8

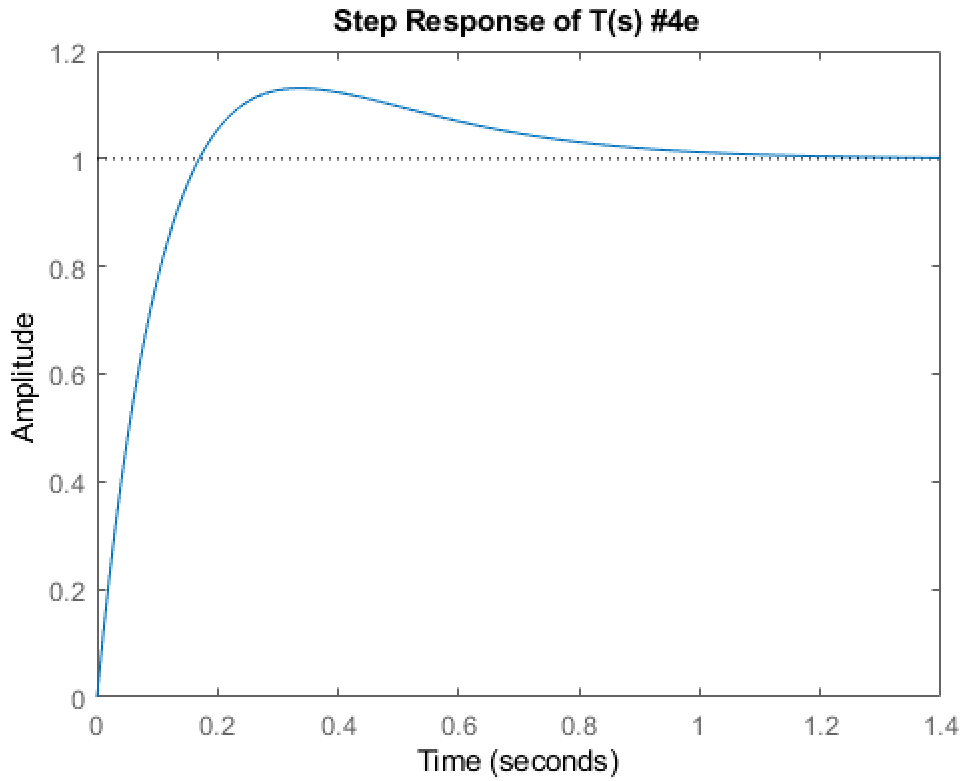
Question 4

```
clear all
clc
s = tf('s');
I = 5; b = 0.5; Km = 1.3;
G = Km/(I*s + b);
K = (4*I - b)*1/Km
L = G*K;
T = minreal(L/(1+L));
poles = pole(T)
S = minreal(1/(1+L));
esst = abs(evalfr(S,0))*10
%
% Parts C, D, E
Ki = 36*I/Km
D = b+Km;
Kp = (12*I - b)/Km
H = Kp + Ki/s;
L = G*H;
S = minreal(1/(1+L));

figure(1)
bode(L)
title('Bode of L(s) confirming at least -20dB/dec for all range of
 \omega')
grid on
margin(L)
T = minreal(L /(1+L));
pole(T)

figure(2)
step(T)
title('Step Response of T(s) #4e')
```

```
K =

    15


poles =

   -4.0000


esst =

    0.2500
```

```
Ki =

  138.4615


Kp =

   45.7692


ans =

   -6.0000
   -6.0000
```





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