University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #8

Issued: 13 Apr. 2019
Due By: 19 Apr. 2019

Question 1:

To illustrate that "rules of thumb" are just that – often useful but lacking the mathematical precision of a formal theorem like Nyquist – consider a feedback loop with

$$G(s) = \frac{-1.5(s+1)}{s(s-1)^2} \qquad\qquad H(s) = K$$

with $K > 0$. Note that the plant is quite unstable here, and moreover has negative gain.

a.) What is the phase margin for $L(s)$ when $K = 1$?

b.) Use Nyquist to determine if the closed-loop system is stable when $K = 1$. How many unstable closed-loop poles (if any) does the Nyquist analysis predict?

c.) How does your answer in a.) change as K varies between 0 and ∞? Are there any such values of K for which the closed-loop system is stable?

c.) Repeat a.)-c.) if you instead use negative values for K.

Question 2:

a.) Show using Nyquist that $H(s) = K(s+1)$ with $K < 0$ can result in a stable closed-loop system for the $G(s)$ in Question #1. Find the range of K for which stability will occur.

b.) Find the value of K for which the loop will have a phase margin of 30° using the compensator in a.). What is the corresponding magnitude crossover frequency?

c.) Use Matlab to obtain a step response for $T(s)$ and quantify the settling time, overshoot, and steady-state. What happens if you try to do the same for $R(s)$? Why do you suppose this happens?

d.) Generate the Bode magnitude diagram of the sensitivity transfer function $S(s)$ for the feedback loop using the controller in c.) Determine from this (i) the steady-state value of $e(t)$ when $y_d(t)$ is constant, (ii) the tracking bandwidth, and (iii) $|S(j\omega_\gamma)|$. Compare the latter with the exact analytical result.

Question 3:

a.) Since the controller in Question #2 is not implementable using only measurement of $y(t)$, design instead a lead compensator for $H(s)$ which will achieve the same phase margin and crossover as in #2b. What value of β and τ are required?

b.) Repeat #2c for the lead compensator in part a.) Determine also $\max_t |u(t)|$ and the steady-state value of $u(t)$ generated by the controller using this new design.

c.) Redesign the compensator in part a.), if you instead want to move the magnitude crossover to 2 rad/sec while maintaining the 30° of phase margin.

Question 4:

Motor speed controllers are very common applications, controlling propeller speeds on quadcopters, momentum wheel speeds on spacecraft, etc. A simple model for a DC motor is

$$I\dot{\omega}(t) = -b\omega(t) + K_m u(t).$$

where the $b\omega$ term is due to drag or friction. Suppose for this problem that $I = 5$, $b = .5$ and $K_m = 1.3$.

a.) Show that the feedback control law $u(t) = Ke(t)$, where $e(t) = \omega_d(t) - \omega(t)$ can produce a stable closed-loop system, with any desired settling time and no transient oscillation. (Nyquist should hardly be needed for this).

b.) Choose a specific value of K which will result in a 1 second settling time for the closed-loop transient response. Determine the resulting steady-state tracking error when $\omega_d(t) = 10$ (constant).

c.) To ensure that the speed perfectly tracks at least constant ω_d, the PI ("proportional+integral") controller

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

is commonly used. This can be alternately written as

$$\begin{aligned} u(t) &= K_p e(t) + K_i x_1(t) \\ \dot{x}_1(t) &= e(t) \end{aligned}$$

Determine the corresponding transfer function $H(s)$ and prove that this controller will indeed ensure perfect tracking for constant ω_d.

d.) Another common rule of thumb in feedback control design is "make sure the slope at magnitude crossover is at least -20 dB/dec". Having a slope which is too shallow near crossover means that the actual crossover point (and hence the actual phase margin) can be very sensitive to small (but inevitable) deviations in the model for $G(s)$. For this problem, adhering to this rule constains the choice of the two gains. Derive at least one set of constraints which will ensure the desired property is satisfied.

e.) Choose values for the two gains which both respect the rule in d), and achieve the same result as the design in a) and b) using repeated real closed-loop poles. Will a step response of the resulting closed-loop system show overshoot? Why or why not?