University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #7

Issued: 6 Apr. 2019
Due By: 12 Apr 2019

Question 1:

A dynamic system has the transfer function

$$G(s) = \frac{7}{(s/5+1)^3}$$

Note that this system is stable. To get it to accurately track commanded behaviors $y_d(t)$, we propose to use the proportional control law $u(t) = Ke(t)$ with $K > 0$.

a.) Do a Nyquist analysis for this system, showing that the closed-loop dynamics will be stable for small values of K, unstable for large values of K. Show a representative plot for each case, count the number of -1 encirclements, and apply the stability criterion. Use this analysis, together with the Bode diagrams of the system, to determine the maximal value of K for which the closed-loop system will be stable.

b.) Determine the value of K for which $L(s)$ will have a phase margin of $55°$. Determine the closed-loop poles which result from this choice of K. Estimate from the closed-loop pole locations the expected settling time and percent overshoot of the response for $y(t)$ when $y_d(t)$ is a unit step. Use Matlab to obtain an exact step response of $T(s)$ and quantify the relevant features. Comment on the exact response vs. your initial estimates, and discuss possible reasons for any discrepancies.

c.) Use Matlab to compute the input $u(t)$ generated by the controller during the step response in b.) Determine the maximal and steady-state values of $|u(t)|$ in this case.

d.) Determine the steady-state tracking error which will be seen for the step response of the feedback loop in b.) If instead the feedback loop were asked to track $y_d(t) = A\sin(\omega t)$, for what range of $\omega > 0$ (if any) can you ensure that $|e_{ss}(t)| < A/2$?

Question 2:

Repeat Question #1 for a feedback system with

$$G(s) = \frac{s+1}{2(s-3)} \qquad\qquad H(s) = \frac{K}{s}$$

and again $K > 0$. Note that in this case the plant is unstable, and the compensator is only marginally stable. For this system, your Nyquist analysis in a.) will show the closed-loop system to be unstable for small values of K and stable for large K. Repeat the rest of Question #1.

Question 3:

Returning to our initially motivating hovercraft example, if we include the electromechanical dynamics of the propulsive fan used to move the vehicle, the motion of the hovercraft can be modeled using the transfer function

$$G(s) = \frac{12}{s^2(\tau s + 1)}$$

with $\tau > 0$.

a.) Use Nyquist to prove that it is *impossible* to stably control this system using a proportional control law $u(t) = Ke(t)$.

b.) Suppose instead that we use the "proportional+derivative" (PD) control law

$$u(t) = K_p e(t) + K_d \dot{e}(t)$$

where both K_p and K_d are positive. We showed on Problem Set #4 that, unlike the original hovercraft model with only the two poles at the origin, this new model cannot be stabilized by a PD controller without some constraints on the two gains. We are now in a position to prove this using Nyquist, indeed to show that the exact constraint for stability is $\tau K_p < K_d$. Sketch representative Nyquist plots for this system for gains which both do, and do not, satisfy this constraint (2 sketches) and apply the stability criterion in each case.

c.) Suppose specifically that $\tau = 0.17$, and for simplicity we use initially $K_d = K_p$ in b). Find the value for K_p which will result in $L(s)$ having maximal phase margin. Determine the resulting closed-loop poles and verify that they are all stable.

d.) For a unit step $y_d(t)$, about how much overshoot would you expect to see in the response for $y(t)$ based on the poles of $T(s)$ in c)? Use Matlab to obtain the exact step response and label on this plot the exact overshoot and settling time. Notice that there is more overshoot than you might have predicted. Why?

e.) What will happen to the phase margin and magnitude crossover if the gain values used in the controller were increased by a factor of 10 compared to part c.)? How do you expect this to affect the overshoot and settling time of the closed-loop step response? Generate the actual step response that results from the higher gains and discuss.

f.) Suppose that you were dissatisfied with the achievable maximum phase margin in c), and wanted to increase it by 10°. How would you modify the controller gains? How would the necessary changes affect the magnitude crossover frequency? How does this redesign affect the step response transients in d)?

Question 4:

For the compensator

$$H(s) = \frac{20(s+2)^3}{s(s+6)(s+12)}$$

Determine the derivative-free implementation equations which would be used to generate $u(t)$ from $e(t)$ in the operation of the feedback loop.