University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #10

Issued: 4 May 2019
Due By: 10 May 2019

Question 1:

The transfer function for the dynamics of a particular system is given by

$$G(s) = \frac{7}{(s+2)^2}$$

To accurately track at least constant y_d, you decide initially to try an integral control strategy with $\dot{u}(t) = Ke(t)$ (equivalently $u(t) = K\int e(\tau)d\tau$).

a.) Sketch by hand, but as accurately as possible, the locus of possible closed-loop poles as $K > 0$ increases. Determine the real axis portions of the locus, the asymptotes and their intercept. Do the exact calculation to determine the real-axis break-out point.

b.) Now, use the Matlab `rlocus` command to generate the exact root locus. Use the data cursor in the plot window to identify: i) the value of K for which the closed-loop system will have poles with a damping ratio of $\sqrt{2}/2$; and ii) the largest value of K for which the closed-loop system will be stable. For the gain in i), what settling time do you expect for a step response? For the gain in ii), where does the locus intersect the imaginary axis? How does and why does this point relate to the margin/crossover characteristics of $L(j\omega)$?

Question 2:

To improve the transient performance of the feedback loop in Question #1, you instead implement the PI compensator $H(s) = K_p + (K_i/s)$, or equivalently $H(s) = K(s-z)/s$ where $K = K_p$ and $z = -K_i/K_p$.

a.) Relative to the system poles there are 3 possible locations for the (real) compensator zero z: i.) to the left of the plant poles at -2; ii.) between the poles at -2 and the origin, and finally iii.) to the right of the origin. Sketch by hand, as accurately as possible, the locus of possible closed-loop poles as $K > 0$ increases in *each* of these three cases. Determine the real axis portions of the locus, the asymptotes and their intercept.

b.) One of the possibilities above will result in the closed-loop system being unstable for any K. Identify which case (i.-iii.) and explain why.

c.) The other two cases show that the closed-loop system with this compensator can be stable for high K (quite unlike the controller in Question #1) provided that the zero is placed appropriately. Identify a simple root-locus derived constraint on the location of the zero which will guarantee a stable closed-loop system for large values of K. Verify that this condition is also equivalent to ensuring that $L(s)$ has positive phase margin for any K.

d.) Determine values of K_p and K_i so that $T(s)$ is an ideal second-order transfer function without zeros, whose poles have damping ratio $\sqrt{2}/2$ and the fastest possible settling time. Do you expect better transient response characteristics compared to #1bi? Explain.

Question 3:

For the system

$$G(s) = \frac{4(s-1)}{s-5}$$

a.) Use a root locus argument to show that it is possible to stabilize this system using a compensator with a single *unstable* pole (but *without* unstable cancellation!!). Sketch the resulting locus. What constraint must the compensator pole satisfy to ensure $T(s)$ can be stable for some choice of controller gain?

b.) Specify the complete details for the design of such a compensator $H(s)$ that ensures $T(s)$ has double real poles at -1. Show the complete resulting root locus for your design.

c.) Determine the input $u(t)$ your controller would produce for this system when $y_d(t)$ is a unit step and the compensator in b) is used. Show (analytically and numerically) that $u(t)$ is bounded, find its peak magnitude and its (finite) steady-state value.

d.) Suppose the sensor used to implement the compensator in b.) has noise which can be modeled as "tonal" (single frequency): $n(t) = A_N \sin(30t)$. (Noise with many different frequency components is called "broadband". This is a more realistic model in practice, but requires more advanced techniques to analyze.) Determine, as a fraction of A_N, the amplitude of the additional steady-state tracking error this noise would create. Determine also the amplitude of the control inputs this noise would induce when the system is in operation.

Question 4:

For the system in Question #3, with the controller designed in #3b:

a.) If there were $\pm 10\%$ uncertainty on the gain of $G(s)$ (nominally 4), can you guarantee closed-loop stability when your design in #3b.) is used in practice? NOTE: this question can be answered entirely in the context of the root locus for this problem. Indicate on your locus from #3b.) the range of closed-loop poles that would be possible given this uncertainty.

b.) Suppose instead it is the pole of $G(s)$ (nominally at +5) that is subject to $\pm 10\%$ uncertainty. Answer a.) using the same technique. Note that here you need to do a root locus analysis, but treating the pole of G as the variable parameter instead of the gain. To do this, write out the characteristic equation $1 + L(s) = 0$, using the exact compensator (pole and gain) from #3b, but treating the pole of $G(s)$ as variable, call it p say. Manipulate the characteristic equation to get this in the form $1 + pL_p(s) = 0$ for some rational function $L_p(s)$. Now you can apply root locus techniques to the transfer function $L_p(s)$, but with p instead of K as the variable parameter (the rules are unchanged, but apply now to $L_p(s)$ not the original $L(s)$). Mark the nominal closed-loop poles on this locus (these correspond to $p = 5$, and should agree with the poles of $T(s)$ in #3b) then show the range of possible closed-loop poles that would be possible given the assumed uncertainty on p.

c.) Repeat b.), but instead for $\pm 10\%$ variation in the location of the zero of $G(s)$ (nominally at 1).