2^{nd} order min phase factors - phase

$$\left[\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right)s + 1\right]^{\pm 1}$$

$\Rightarrow$ Like magnitudes, can sketch as repeated real factor ($\zeta = 1$) for $\sqrt{2}/2 \le \zeta \le 1$

$\Rightarrow$ for $0 \le \zeta < \frac{\sqrt{2}}{2}$, a more significant correction is needed

$\Rightarrow$ when $\zeta = 0$, phase changes <u>discontinuously</u> by $\pm 180°$ at $\omega = \omega_n$

<u>Example:</u>

$$G(s) = \frac{\left(s/10 + 1\right)}{\left[s^2 + 2\zeta s + 1\right]}$$

If $\zeta = 1$:

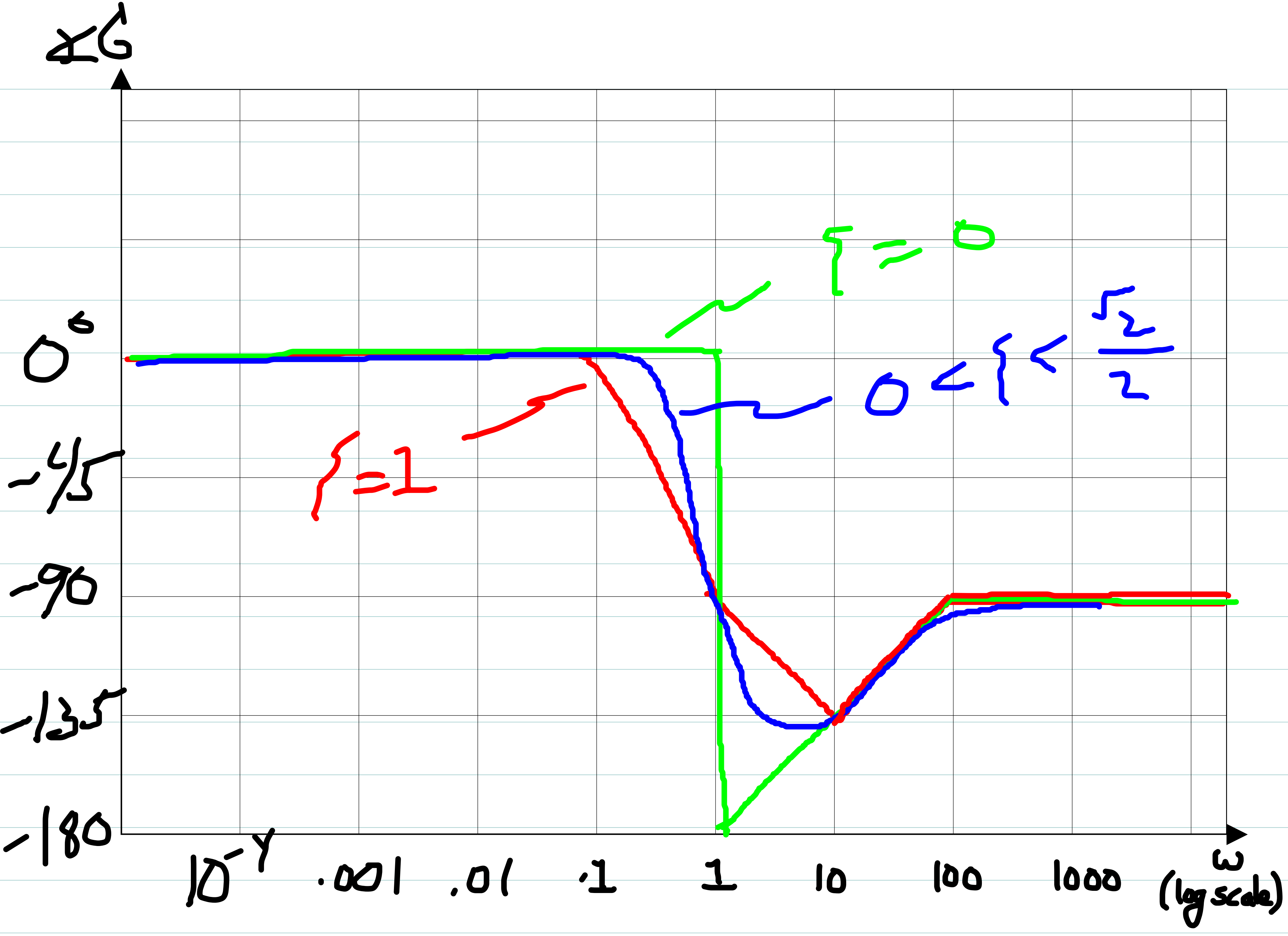
Change of $-90°$/dec in .1 to 10
Change of $+45°$/dec in 1 to 100

Net is $-90°$/dec from .1 to 1
$-45°$/dec from 1 to 10
$+45°$/dec from 10 to 100

If $\zeta = 0$

Change of $+45°$/dec in 1 to 100

$-180°$ drop at $\omega = \omega_n = 1$


∠G
0°
-45
-90
-135
-180
10^-4
.001
.01
.1
1
10
100
1000
ω
(log scale)
ζ = 0
0 < ζ < √2/2
ζ = 1

Notes: (2nd order phase, small ξ)

- Unlike magnitude, no useful simple formula to quantify "steepness" of phase drop for small ξ.

- Usually sketch something in between the $\xi = 1$ and $\xi = 0$ limits

- Necessarily qualitative - will use Matlab when precise analysis is needed.

- Note generally that we expect to see steep phase drops near frequencies where magnitude diagram shows resonant peaks!

Polar Plots

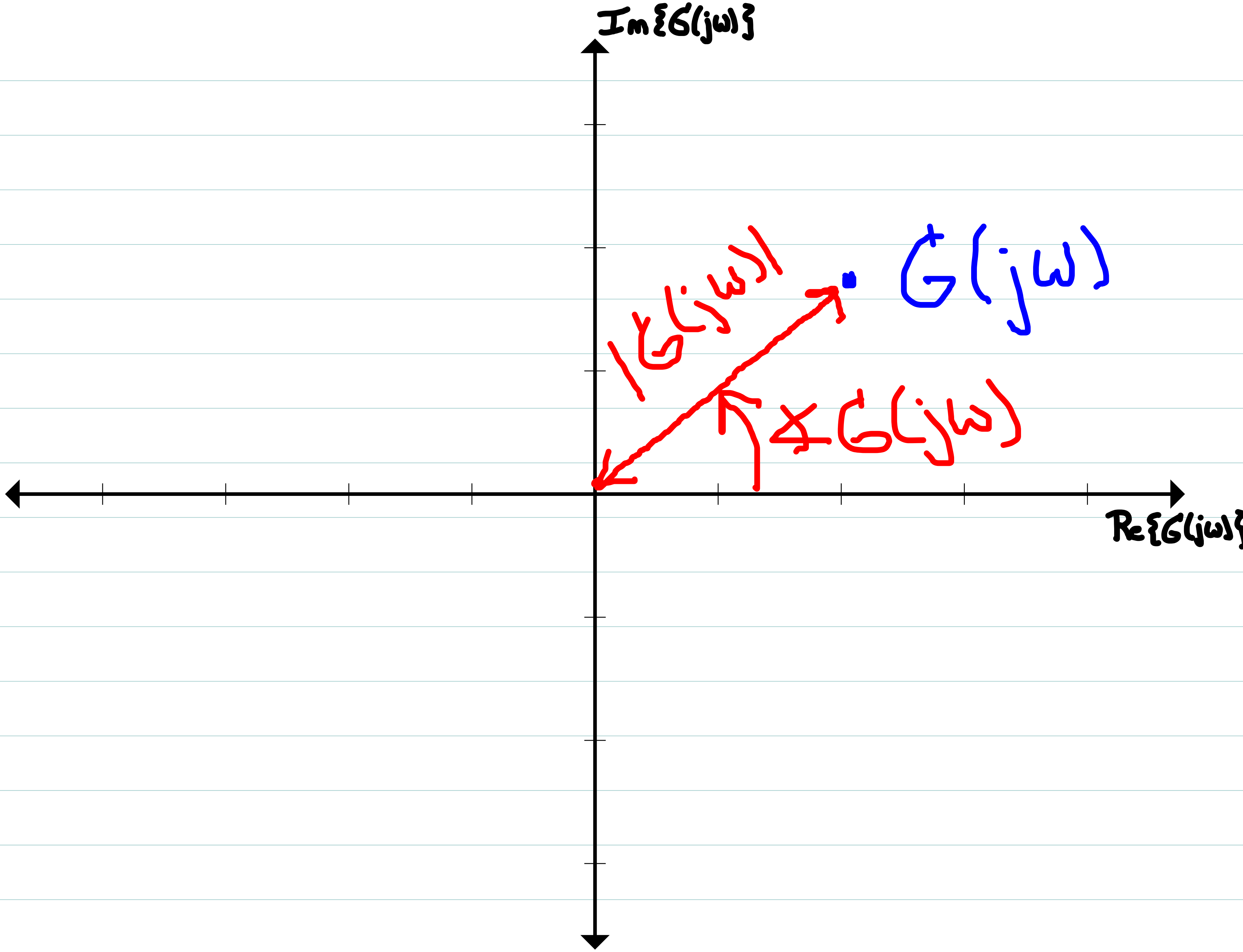
$\Rightarrow$ A different way of showing the properties of $G(j\omega)$

$\Rightarrow$ Bode plots $|G(j\omega)|$ and $\angle G(j\omega)$ vs. ω, using logarithmic scales for $0 \le \omega < \infty$

$\Rightarrow$ Polar shows $G(j\omega)$ as points on complex plane as ω VARIES from $\emptyset$ to ∞ using actual (non-logarithmic) scales

$\Rightarrow$ Learn to sketch polar from Bode

$\Rightarrow$ We are aiming for something qualitatively correct, but will deliberately distort scales to make certain critical features readily apparent.



Im{G(jω)}
Re{G(jω)}
G(jω)
|G(jω)|
∡G(jω)

⇒ For each $\omega \in [0,\infty)$, $G(j\omega)$ is a <u>different</u> point on Complex plane

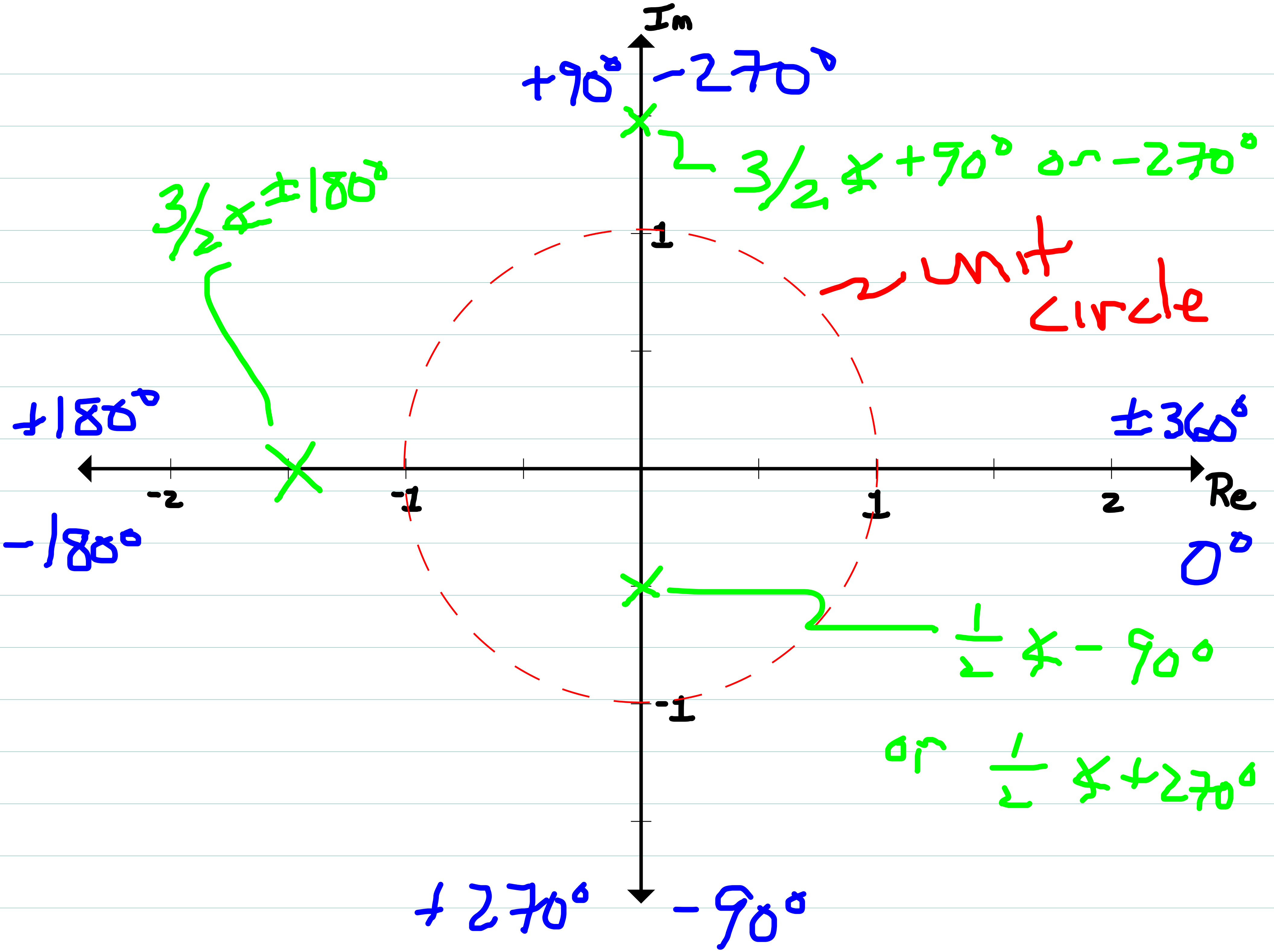
⇒ As ω varies from $\emptyset$ to ∞, these points will trace out a <u>curve</u> on complex plane.

⇒ Bode diagrams show us the polar <u>coordinates</u> of the points $G(j\omega)$ for each ω

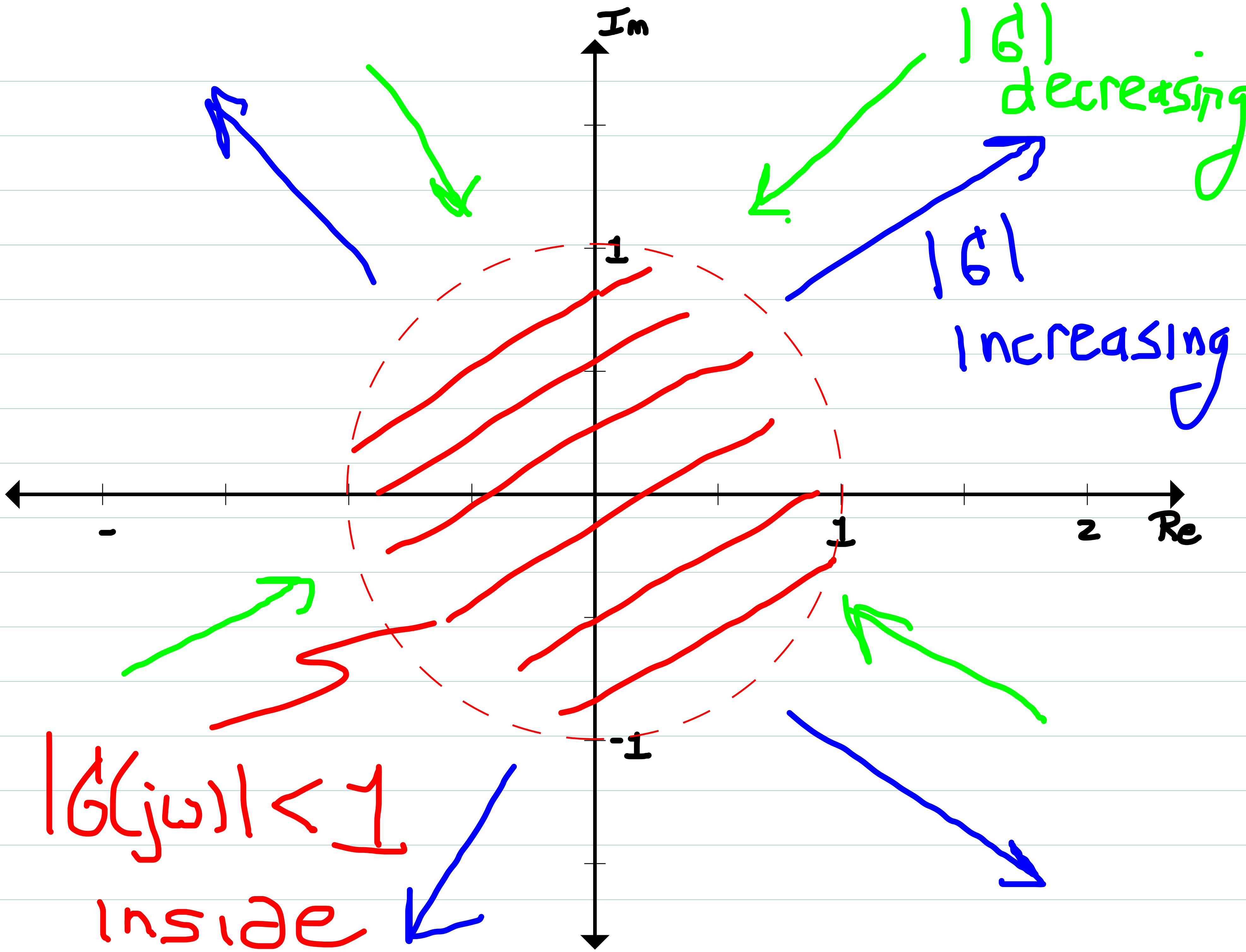
⇒ To map from Bode to polar

1.) Remember to convert magnitudes from dB back to actual.

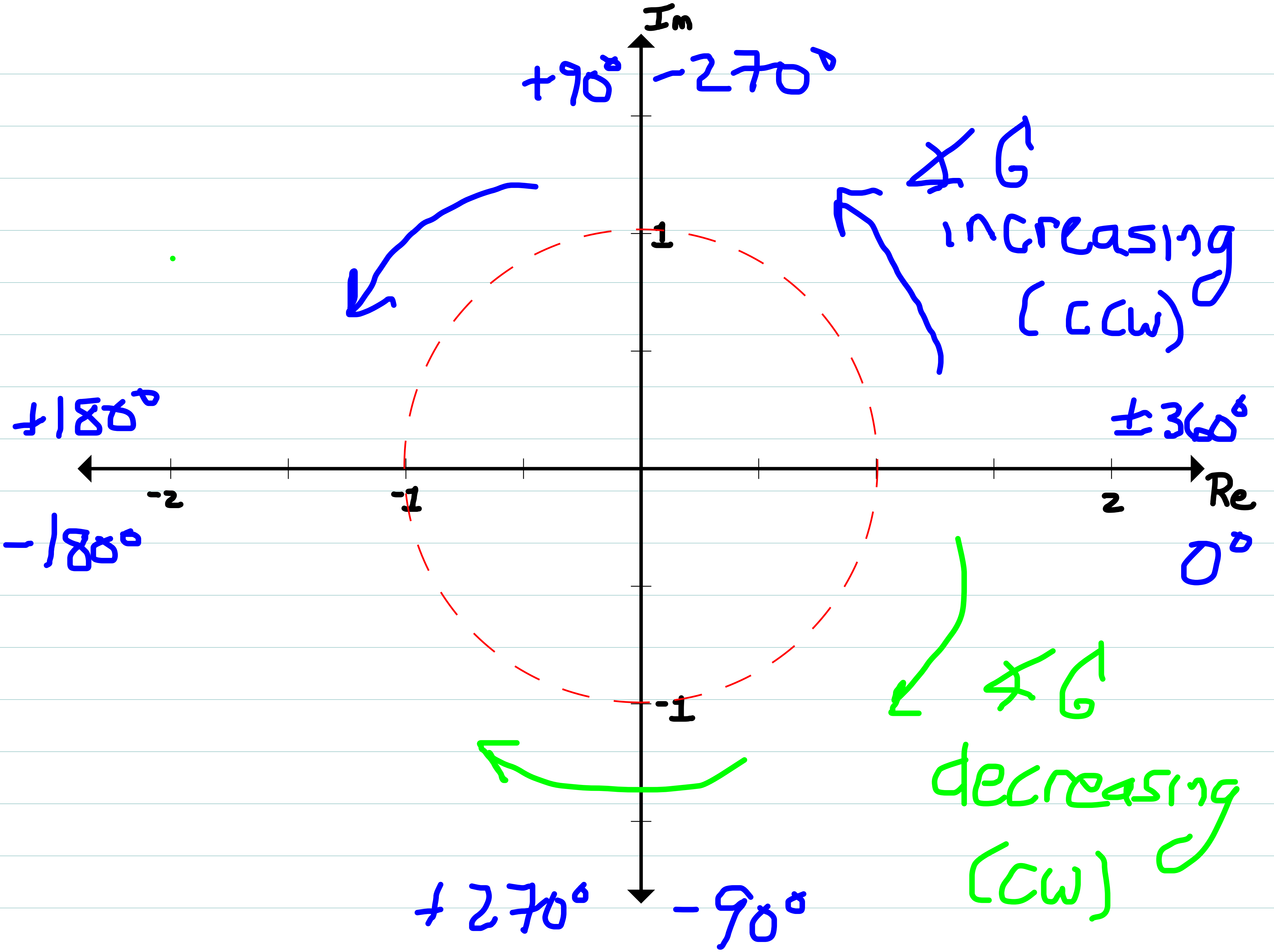
2.) Remember angle convention for Complex numbers.



Im
+90°
−270°
3/2 ∡ +90° or −270°
3/2 ∡ ±180°
unit circle
1
+180°
−180°
±360°
0°
−2
−1
1
2
Re
−1
1/2 ∡ −90°
or 1/2 ∡ +270°
+270°
−90°



Im
Re
1
-1
-
1
2
|G|
decreasing
|G|
Increasing
|G(jω)| < 1
inside



Im
+90°
−270°
∡G
increasing
(ccw)
+180°
±360°
−2
−1
1
2
Re
−180°
0°
−1
∡G
decreasing
(cw)
+270°
−90°

A simple Example

$$G(s) = \frac{K_B}{(1+\tau s)} \qquad \begin{array}{l}\tau > 0 \text{ (min phase)} \\ K_B > 1\end{array}$$

Alway start by thinking about low/high freq. limiting behavior:

for $\omega << \frac{1}{|\tau|}$: Mag slope =

Phase =

for $\omega >> \frac{1}{|\tau|}$: Mag slope =

Phase =

A simple Example

$$G(s) = \frac{K_B}{(1+Ts)}$$

$T > \emptyset$ (min phase)
$K_B > 1$

Alway start by thinking about low/high freq. limiting behavior:

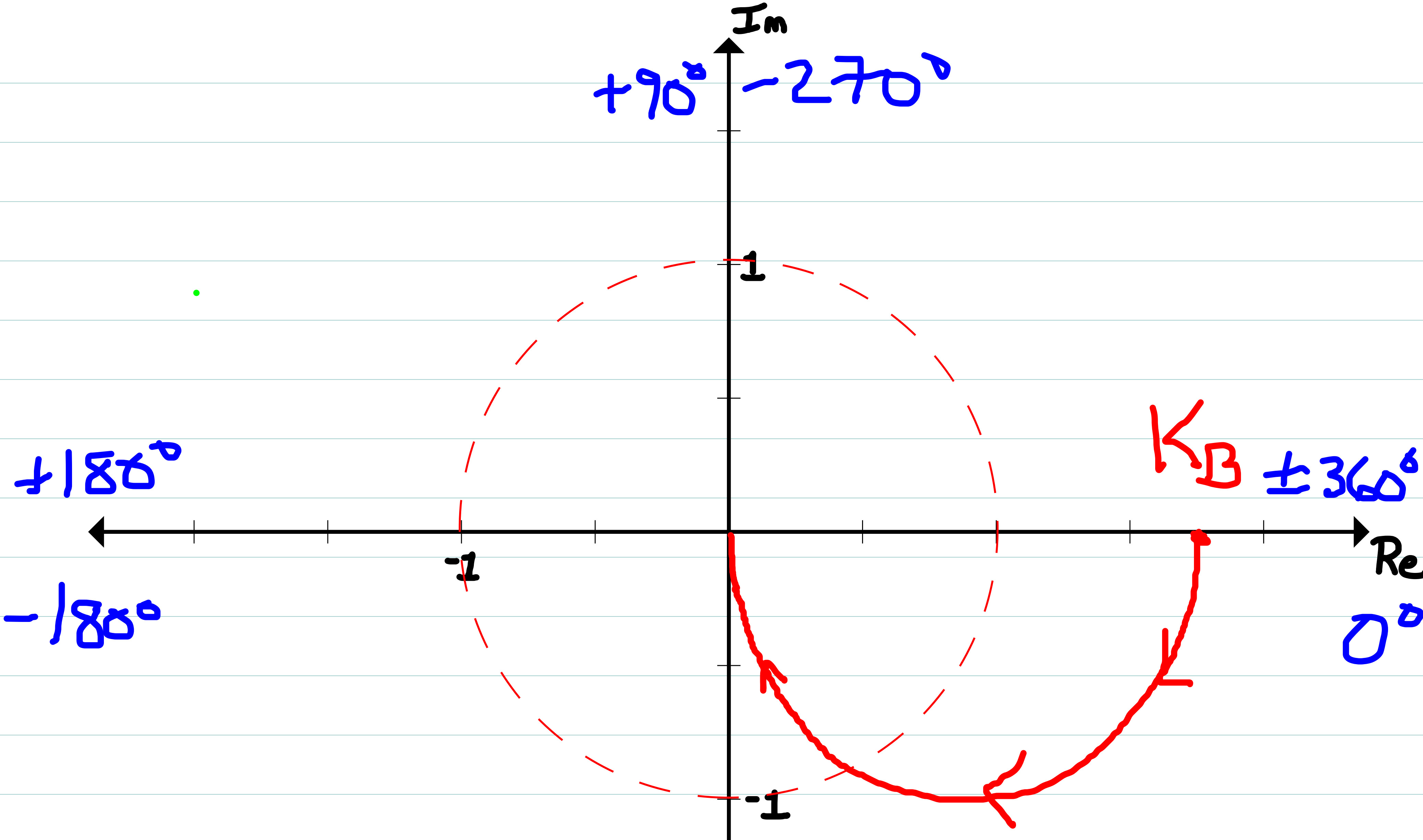
for $\omega \ll \frac{1}{|T|}$: Mag slope = 0 dB/dec (constant)

Phase = $0°$ (constant)

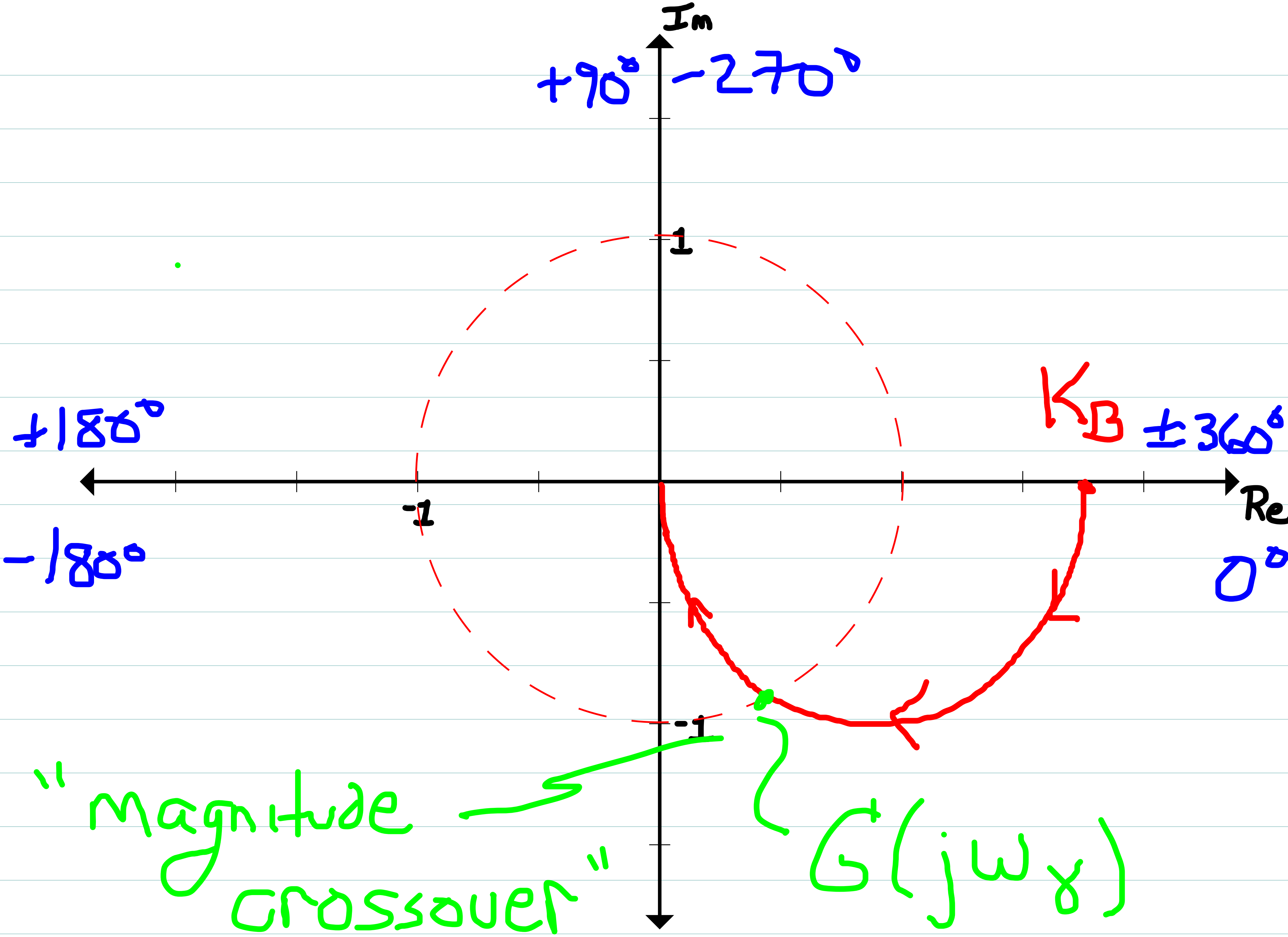
for $\omega \gg \frac{1}{|T|}$: Mag slope = -20 dB/dec

Phase = $-90°$

Low freq. magnitude is $|K_B| > 1$, high freq. magnitude is $\emptyset$: $\lim_{\omega \to \infty} |G(j\omega)| = \emptyset$



Arrows indicate increasing freq.



Im
+90°
-270°
1
+180°
-180°
-1
K_B
±360°
Re
0°
-1
"magnitude crossover"
$G(j\omega_\gamma)$

Magnitude Crossover

"Magnitude crossover" occurs where polar plot "punctures" the unit circle

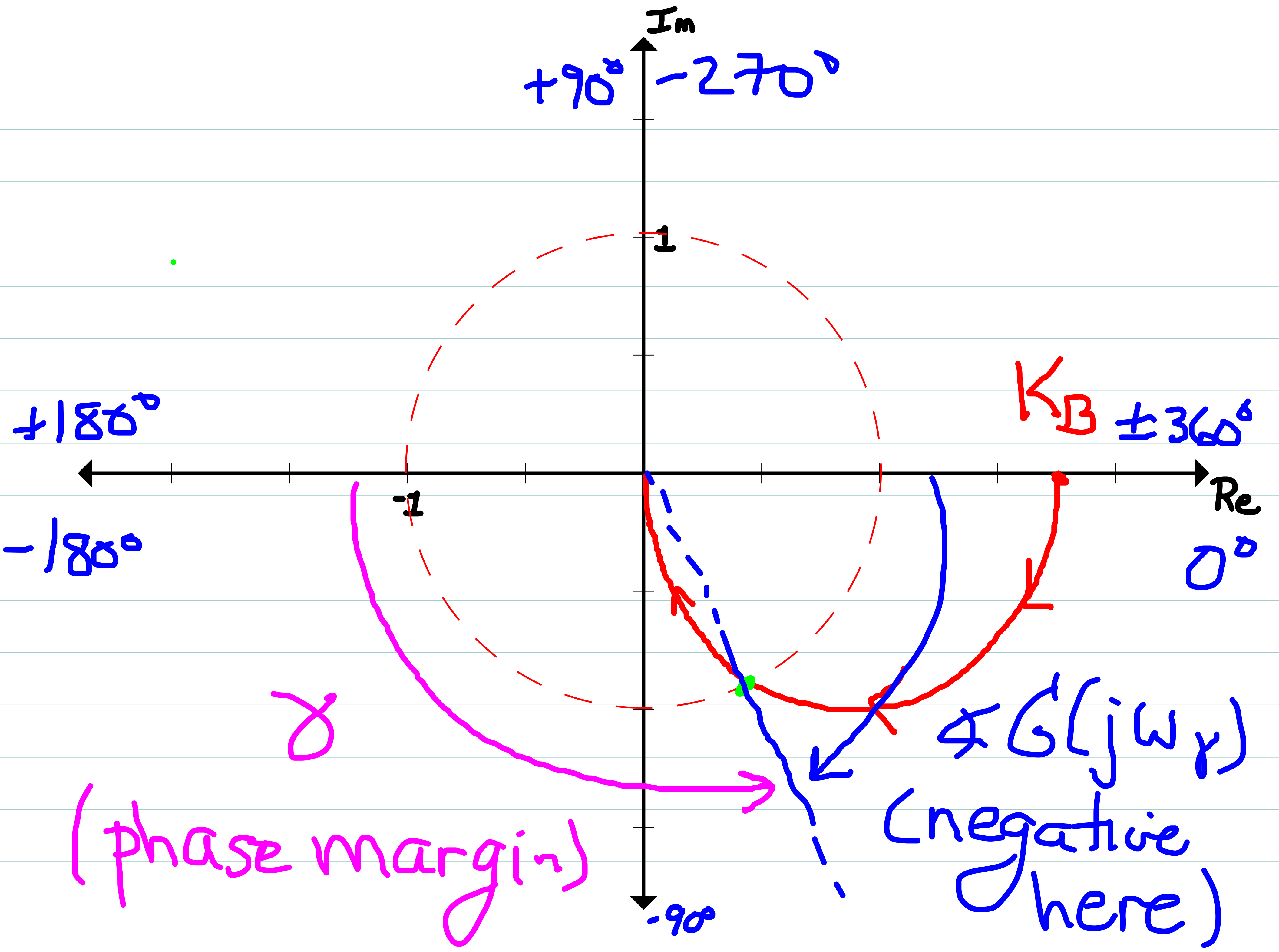
$|G(j\omega)| = 1$ at this point.

The frequency at which this occurs is the "magnitude crossover freq", termed ω_γ

Easily seen on Bode: ω_γ is the frequency where $|G(j\omega)| = \emptyset$ dB

<u>Note:</u> depending on the system there may be one, many, or no magnitude xover freq.

Important quantity: $\measuredangle G(j\omega_\gamma)$: phase at magnitude xover



Im
+90°
-270°
1
+180°
-180°
-1
±360°
0°
Re
K_B
-90°
γ
(phase margin)
$\measuredangle G(j\omega_\gamma)$
(negative here)

Phase Margin

The **phase margin** is the angle around the unit circle from -1 to magnitude crossover point, measured positive counter-clockwise from -1 (or, equiv, CW from mag xover to -1)

The phase margin angle, γ, is expressed in deg (although later it will be convenient to express in rad).

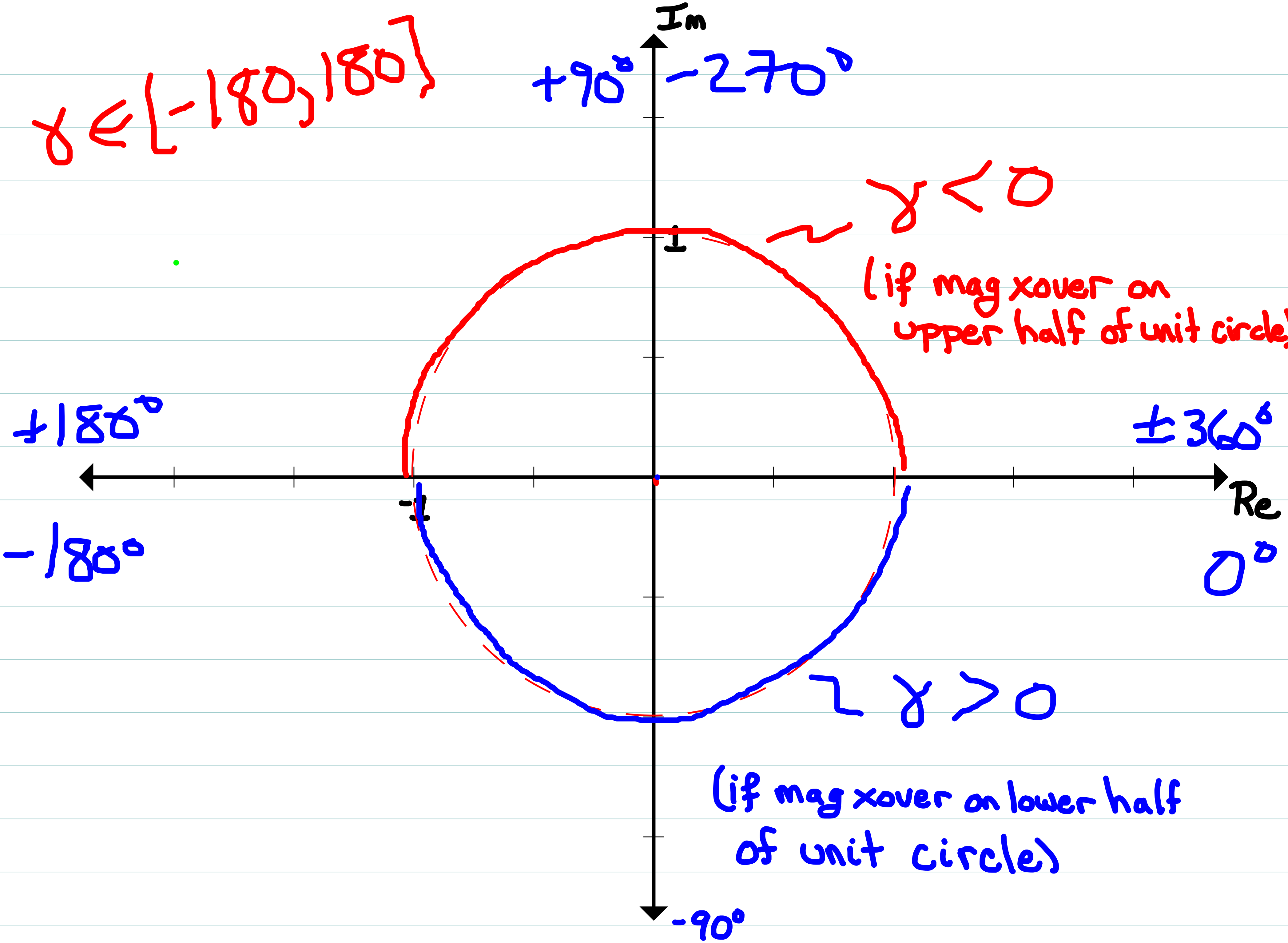
Assuming we write $\measuredangle G(j\omega_\gamma)$ in range $[0°, -360°]$ an expression for γ is:

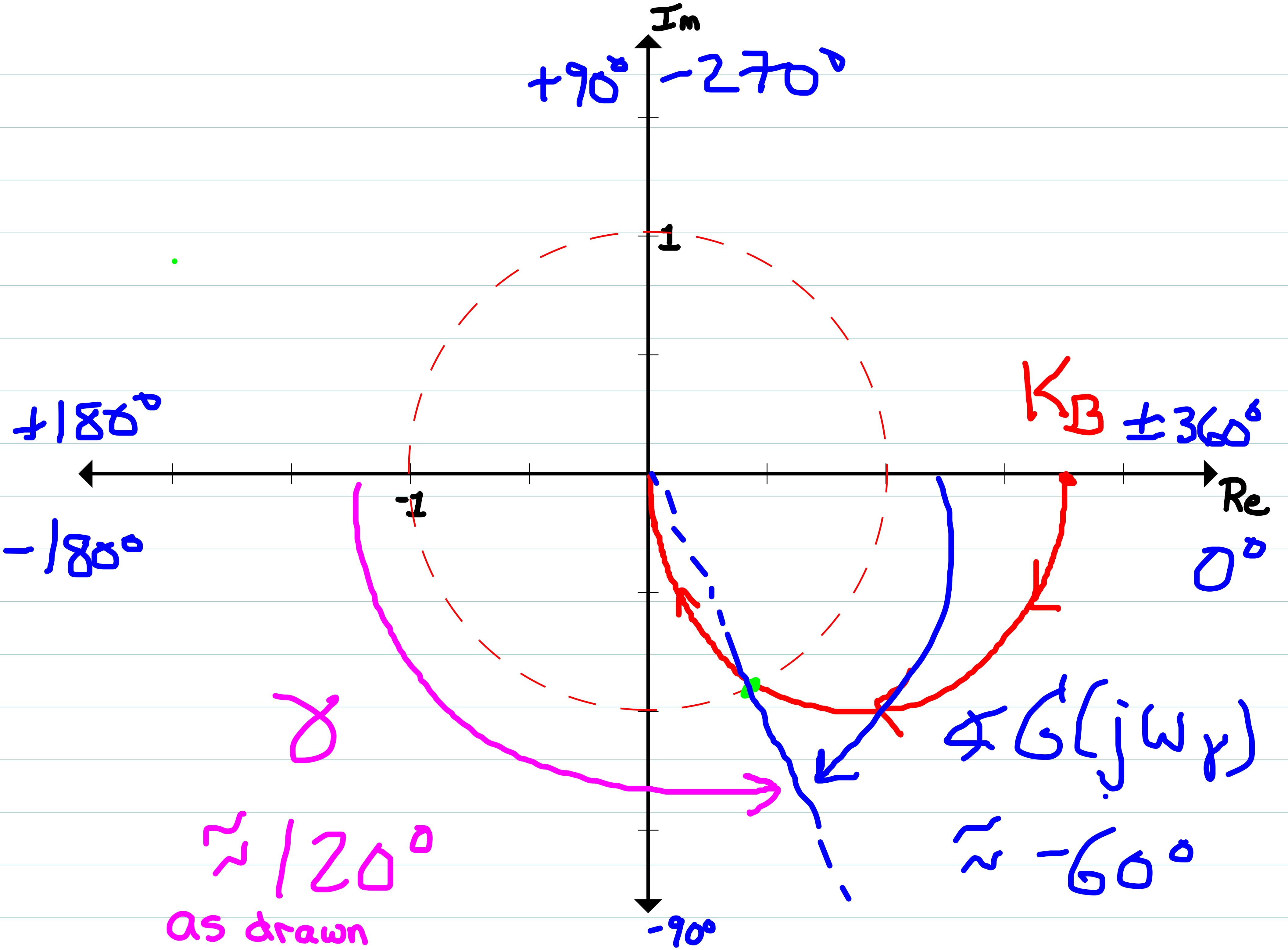
$$\gamma = 180° + \measuredangle G(j\omega_\gamma)$$

$\Rightarrow \gamma > 0$ if $\measuredangle G(j\omega_\gamma) > -180°$

Note: Matlab will usually try to wrap phase plot $\measuredangle G(j\omega)$ so that $\measuredangle G(j\omega_\gamma)$ is in this range. Sometimes it doesn't. You can always manually add or subtract a multiple of $360°$ to get $\measuredangle G(j\omega_\gamma)$ in this range.

$\gamma \in [-180, 180]$




Im
+90°
-270°
1
+180°
-180°
-1
Re
K_B
±360°
0°
γ
~120°
as drawn
$\measuredangle G(j\omega_\gamma)$
~ -60°
-90°