Another Example

$$G(s) = \frac{K_B}{(1+\tau s)^3} \qquad K_B > 1 \qquad \tau > \emptyset$$

Low freq mag: Constant at K_B

Low freq. phase: Constant at $0°$

High freq. mag slope:

High freq. phase:

Another Example

$$G(s) = \frac{K_B}{(1+\tau s)^3} \qquad K_B > 1 \quad \tau > \emptyset$$

Low freq mag: Constant at K_B

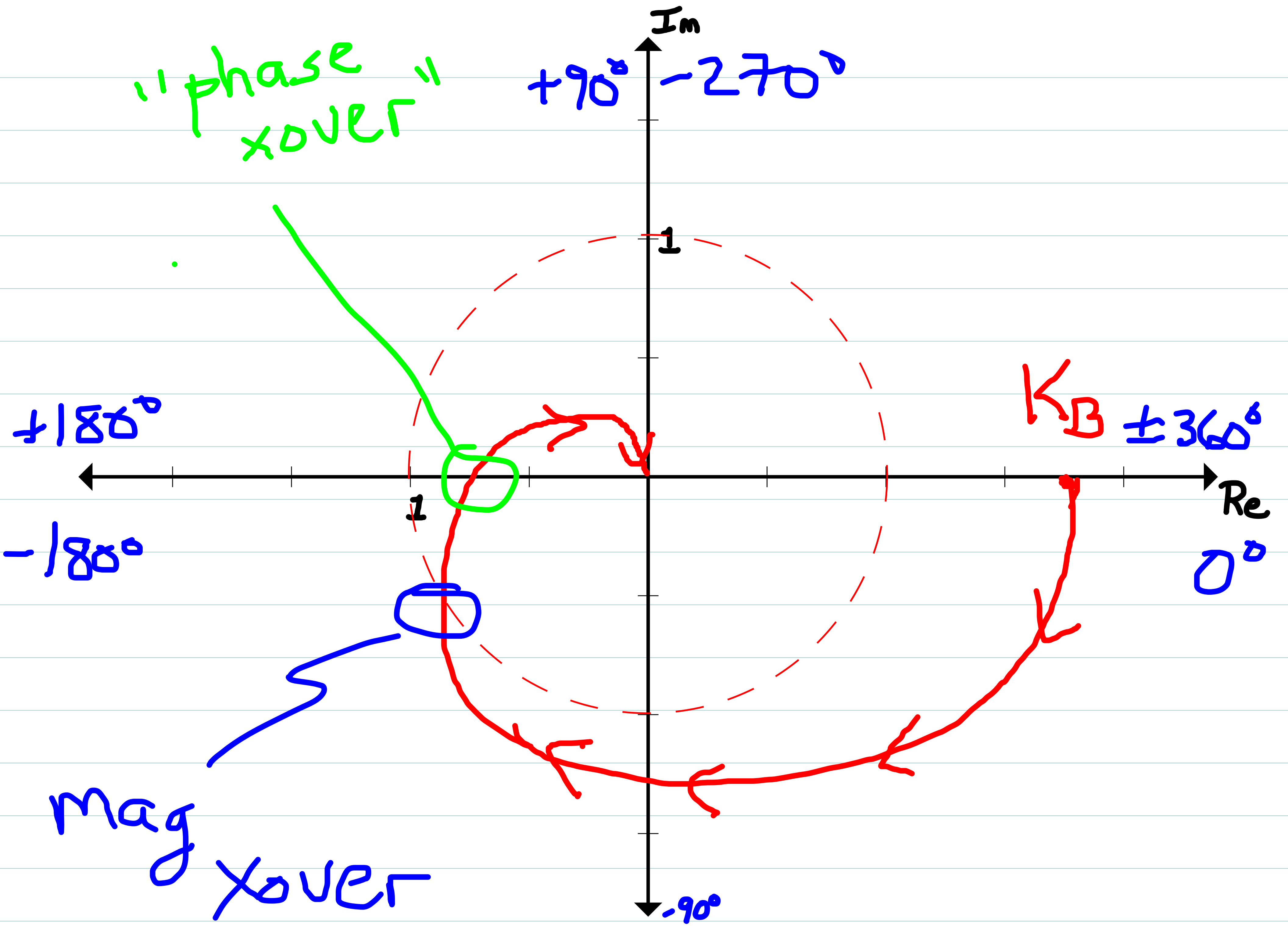
Low freq. phase: Constant at $0°$

High freq. mag slope: -60 dB/dec

High freq. phase: $-270°$

Recall: negative high freq. slope means

$|G(j\omega)| \rightarrow \emptyset$ as $\omega \rightarrow \infty$


"phase xover"
Im
+90°
-270°
1
+180°
-180°
KB
±360°
0°
Re
1
mag xover
-90°

Phase Crossover

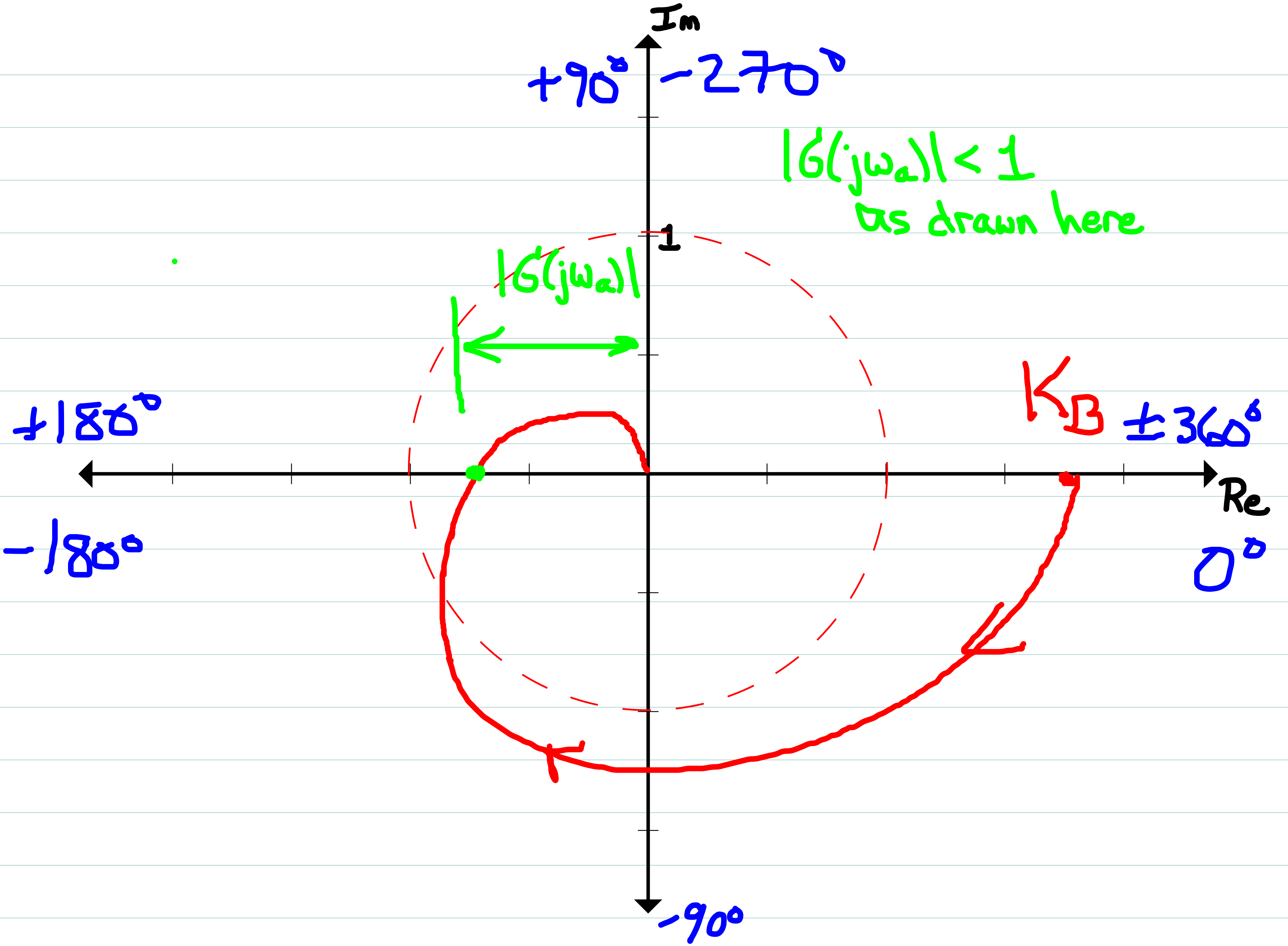
The "phase crossover" of a polar plot is the point where the plot crosses through the negative real axis.

This corresponds to the point where $\angle G(j\omega) = -180°$

Again, easily seen from Bode phase diagrams: call ω_a "phase crossover freq." the value of ω for which $\angle G(j\omega) = -180°$.

Note: May be one, none, or many ω_a depending on system.

Important quantity: $|G(j\omega_a)|$ magnitude at phase xover frequency


Im
Re
+90°
-270°
1
|G(jω_a)| < 1
as drawn here
|G(jω_a)|
+180°
-180°
K_B
±360°
0°
-90°

Gain Margin

The gain margin, a, is defined as:

$$a = \frac{1}{|G(j\omega_a)|}$$

Gain margin is commonly expressed in dB:

$$a_{dB} = 20 \log a$$
$$= -|G(j\omega_a)|_{dB}$$

So gain margin in dB is negative of Bode magnitude at phase crossover freq.

Meaning of Gain and phase margins

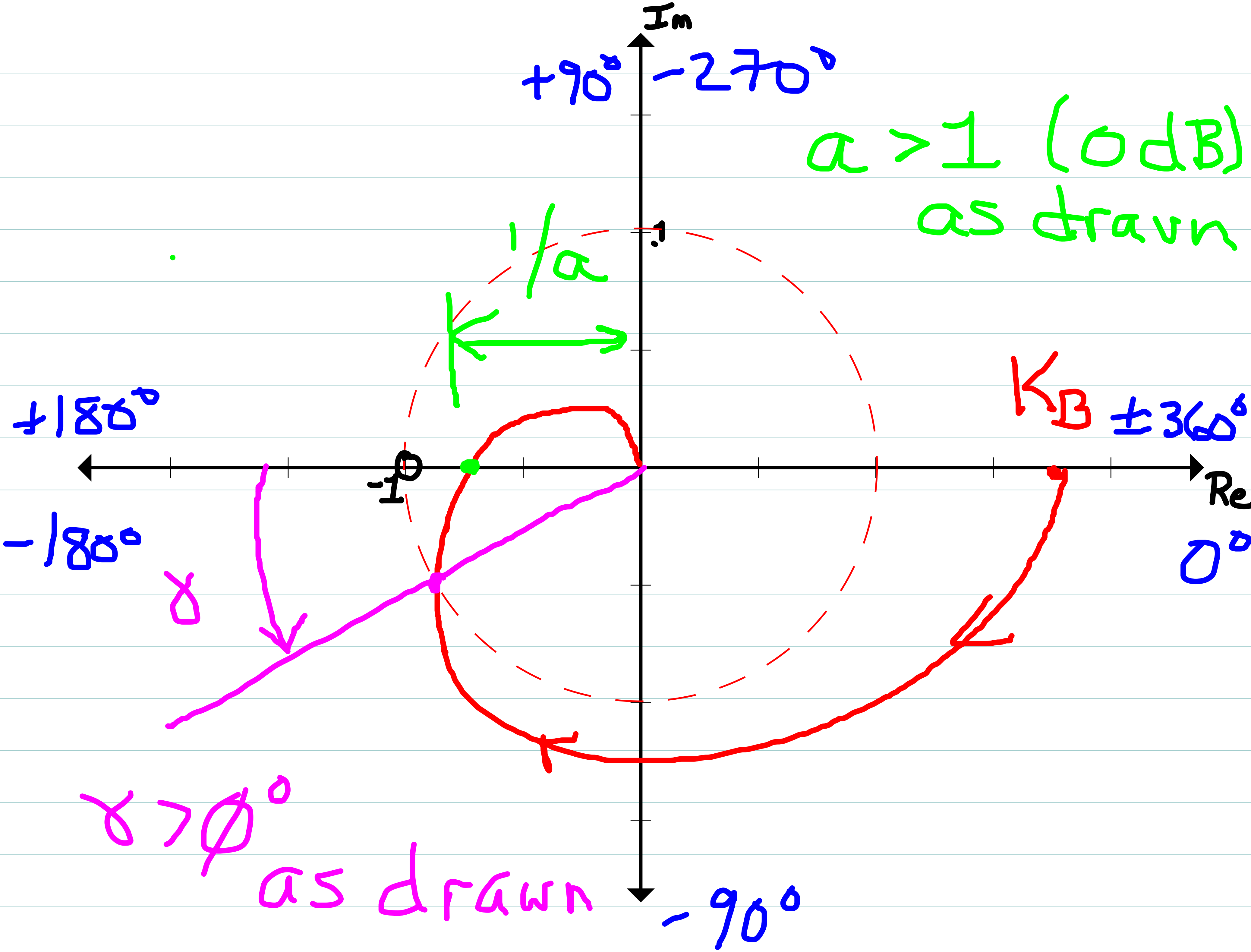
a, γ measure how close polar plot comes to point $-1+\phi j$ ("-1 point") in complex plane. Recall $-1+\phi j = 1 \measuredangle -180°$

Two "pseudo-orthogonal" directions

→ a measures distance to -1 along real Axis as a ratio $1/|G(j\omega_a)|$

⇒ γ measures distance to -1 as an angle around unit circle.

Note: $a > 1$ ($a > \phi$ dB) means phase crossover occurs <u>inside</u> unit circle. $a < 1$ ($a < \phi$ dB) means phase crossover is outside unit circle

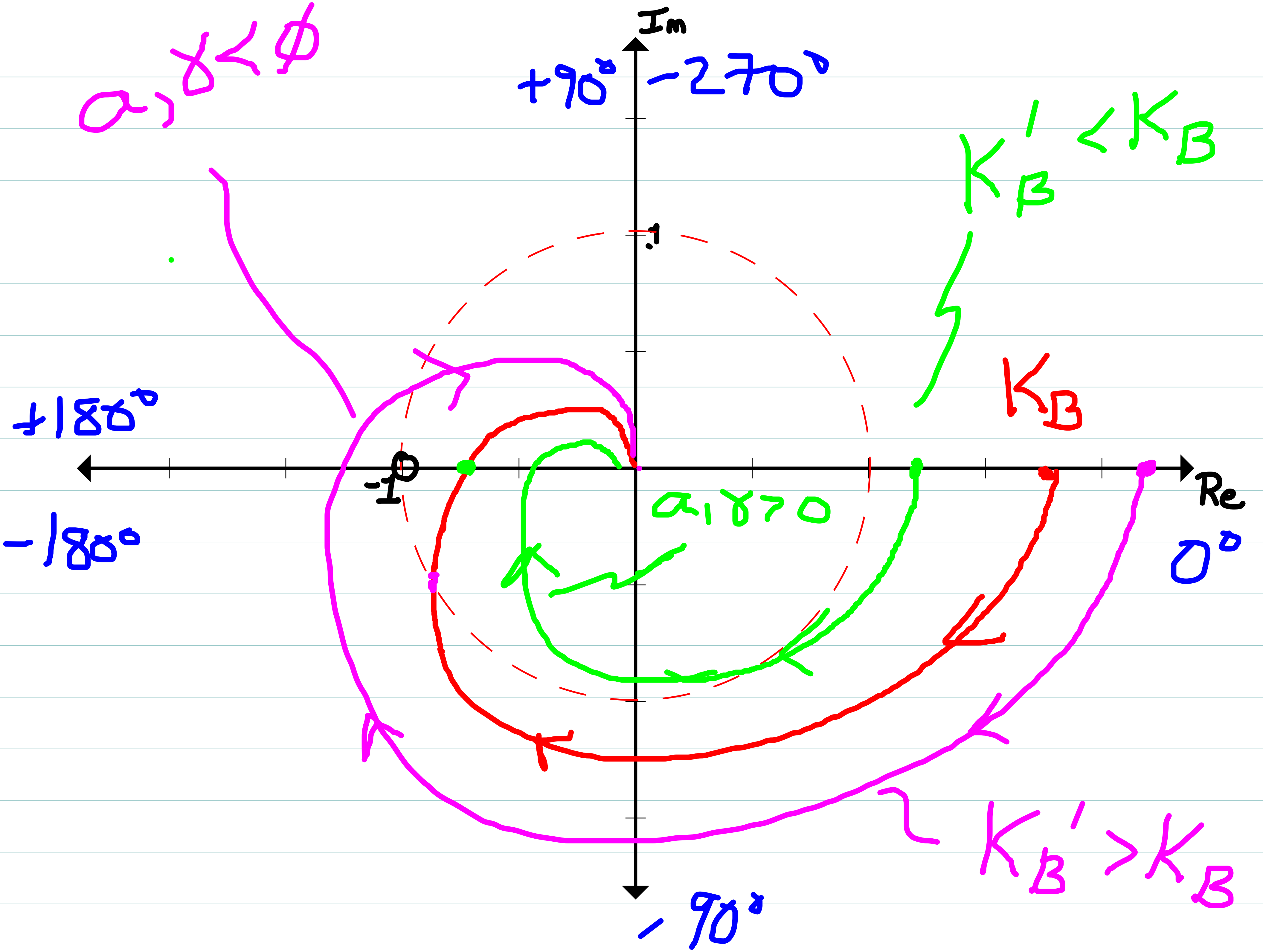


Im
+90°
-270°
a > 1 (0dB)
as drawn
1
1/a
+180°
-180°
-1
Re
K_B
±360°
0°
γ
γ > 0°
as drawn
-90°

Effect of Gain Changes

Increasing or decreasing K_B uniformly expands or contracts polar plot about the origin

$\Rightarrow$ Will generally change crossovers and margins


Im
Re
+90°
-270°
+180°
-180°
0°
-90°
1
-1
a, γ < ∅
a, γ > 0
K_B' < K_B
K_B
K_B' > K_B

Effect of Zeros

Since they affect magnitude and phase, zeros will change shape of polar plot.

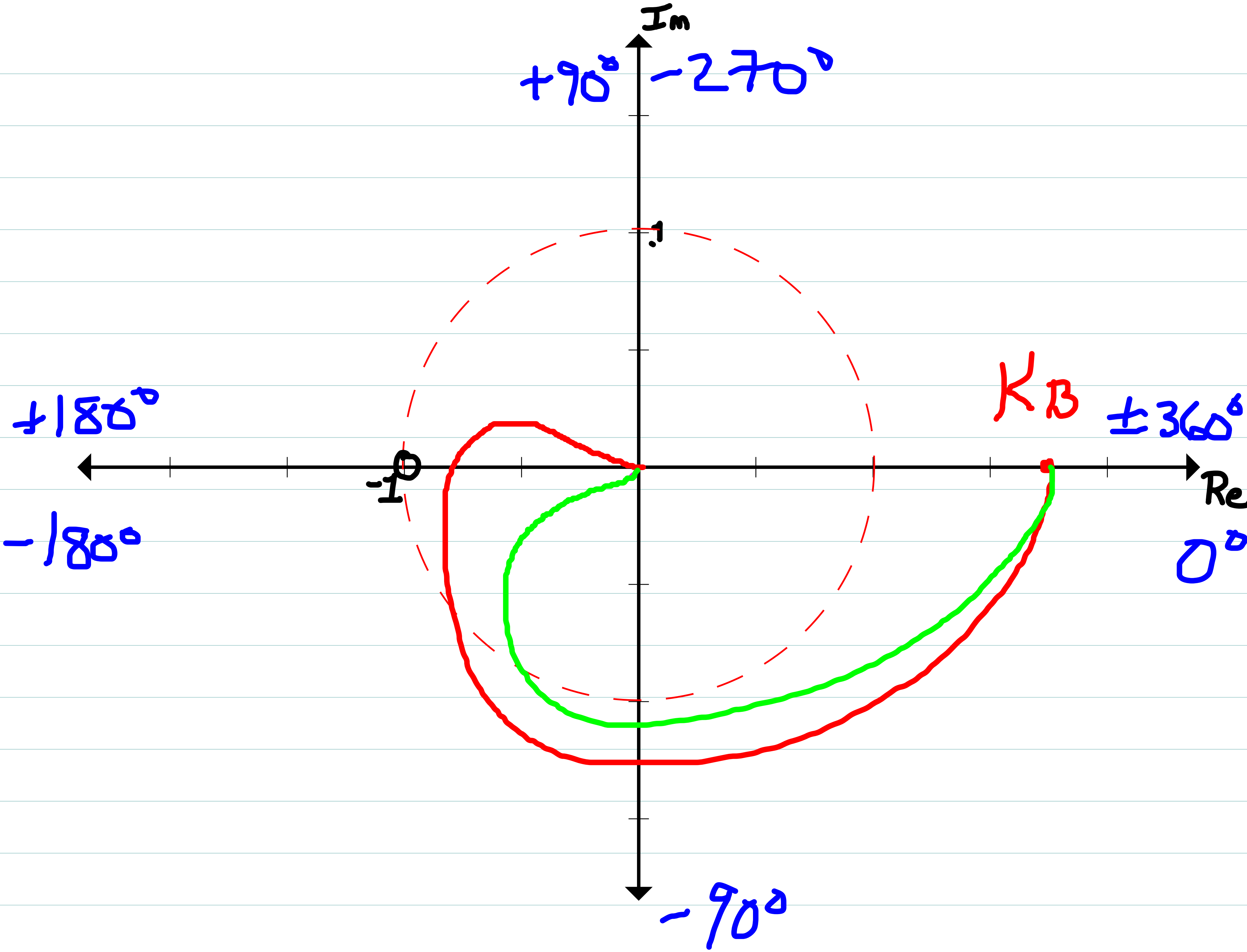
Example:

$$G(s) = K_B \frac{(T_1 s + 1)}{(T_2 s + 1)^3} \qquad K_B > 1 \quad T_1, T_2 > \emptyset$$

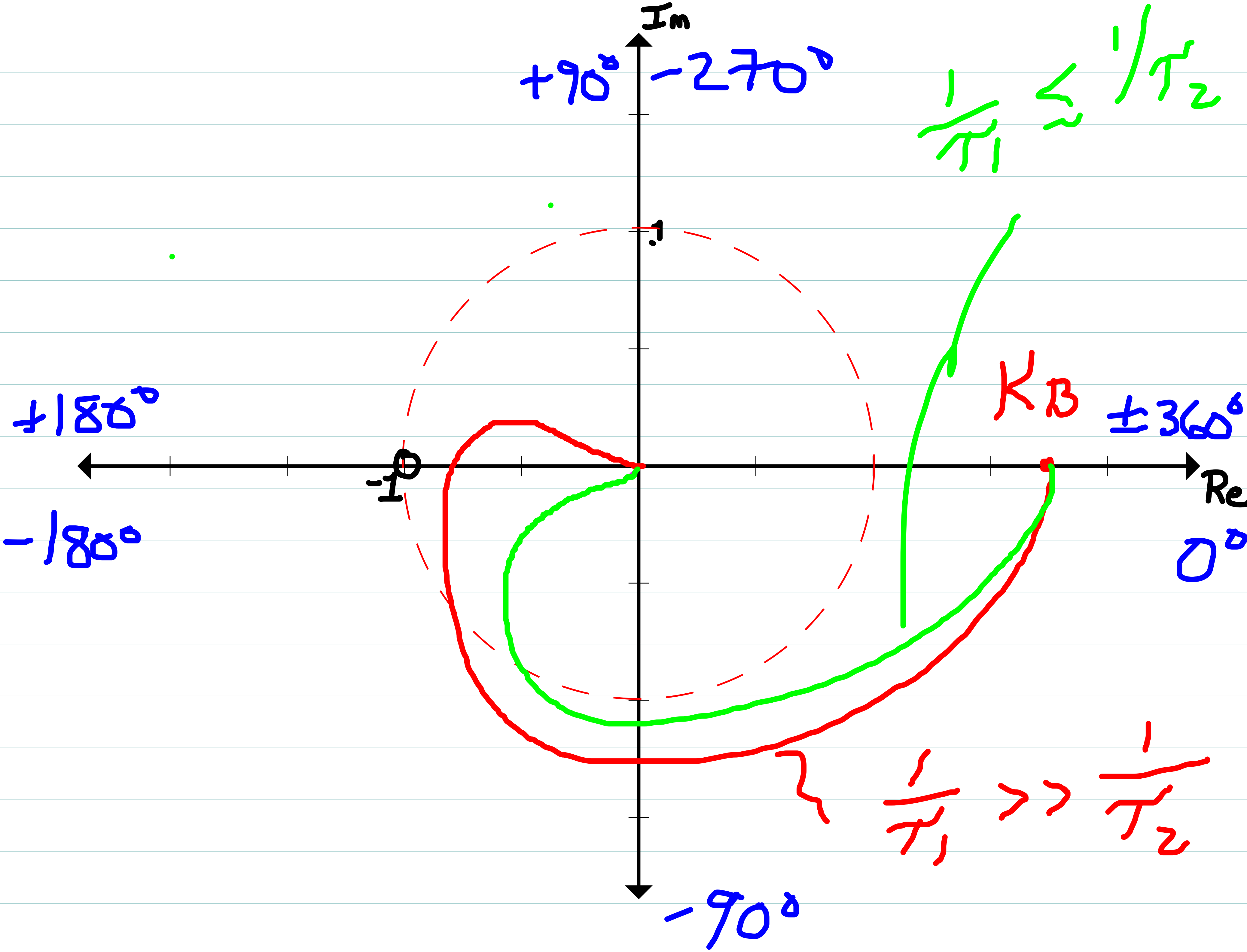
high freq phase: $-180°$ here (Why??)

But this limit may be asymptotically approached from above or below as $\omega \to \infty$

This difference can have a profound impact on shape of plot. Need to check Bode for accuracy, but can often "reason it out" for simple cases.



Im
+90°
-270°
1
+180°
-180°
-1
Re
KB
±360°
0°
-90°



Im
Re
+90°
-270°
+180°
-180°
±360°
0°
-90°
-1
1
KB
1/τ1 ≤ 1/τ2
1/τ1 >> 1/τ2

Poles at origin

Poles at origin will introduce a unique feature to a polar plot.

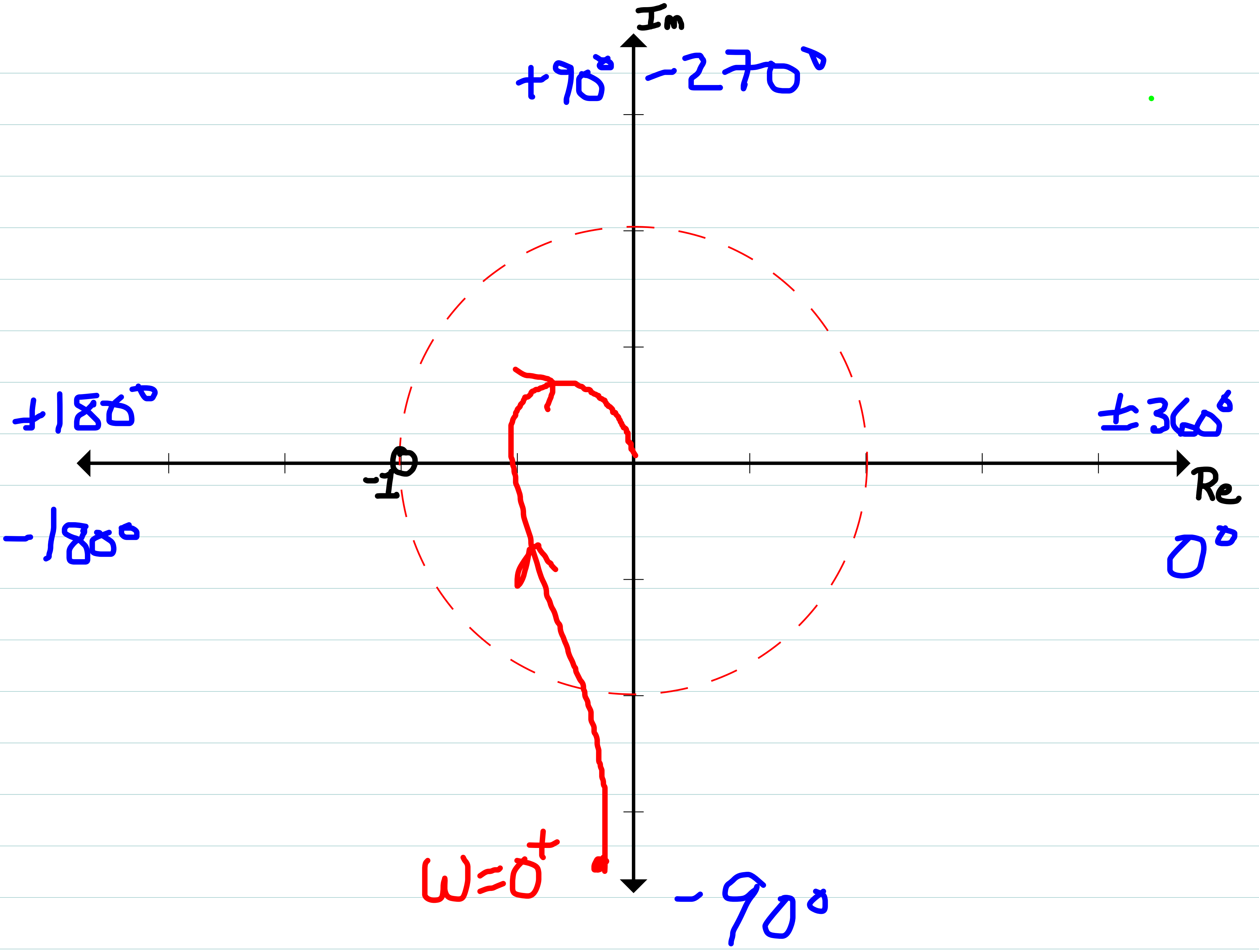
$$\lim_{\omega \to 0} \measuredangle G(j\omega) = \measuredangle K_B - N\,90^\circ$$

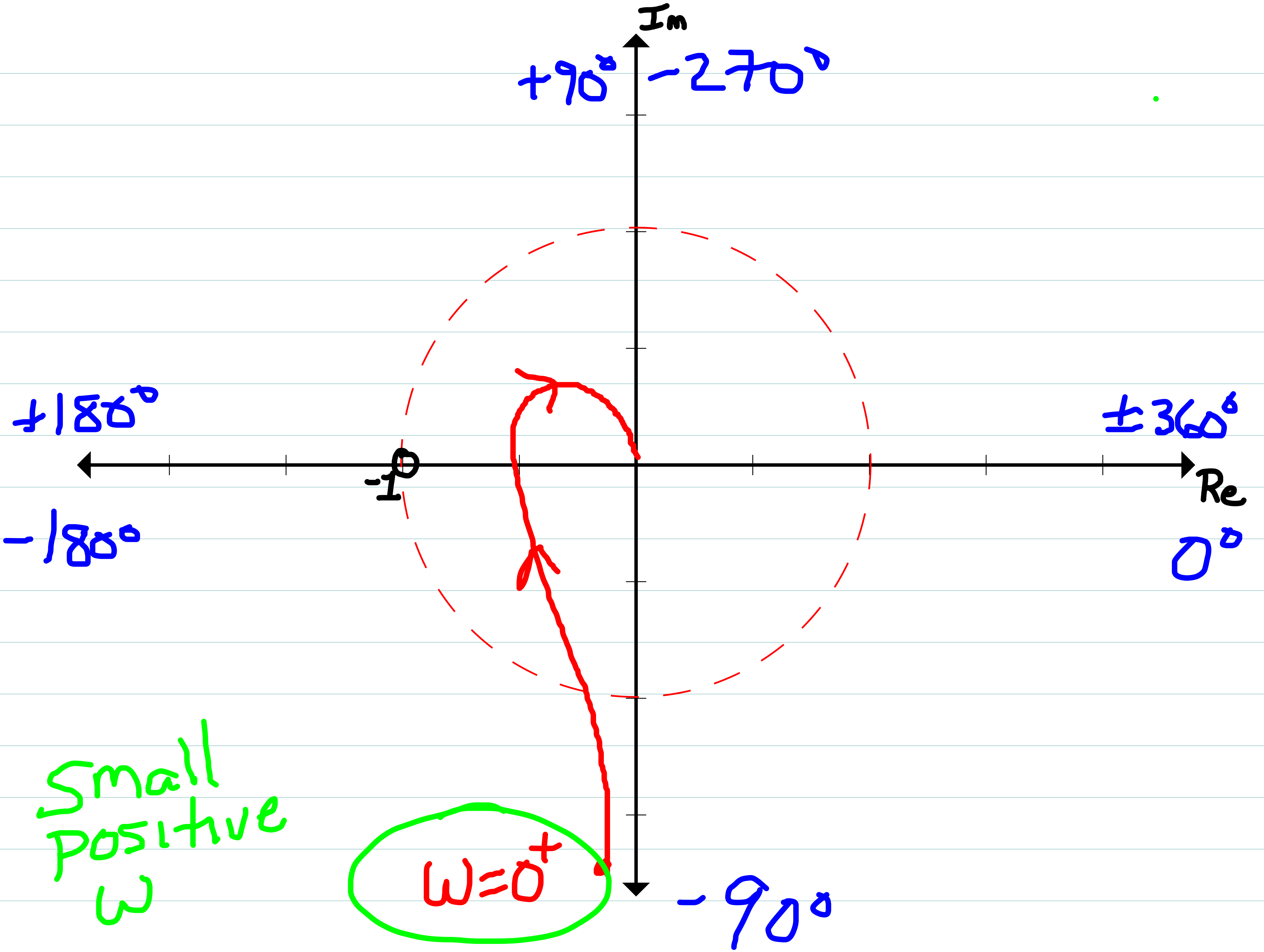
and $\lim_{\omega \to \emptyset} |G(j\omega)| = \infty$ in these cases

$\Rightarrow$ Polar plot will exhibit a "tail" along one of the coordinate axes.

Example:

$$G(s) = \frac{K_B}{s(Ts+1)^2} \qquad T, K_B > \emptyset$$


Im
+90°
-270°
+180°
-180°
-1
±360°
0°
Re
ω=0+
-90°


Im
+90°
-270°
+180°
-180°
-1
±360°
0°
Re
-90°
ω=0+
Small positive ω

Note: Which side of a coordinate axis the tail lies on is sometimes important.

$\Rightarrow$ Determined by asymptotic behavior of phase as $\omega \to \phi$.

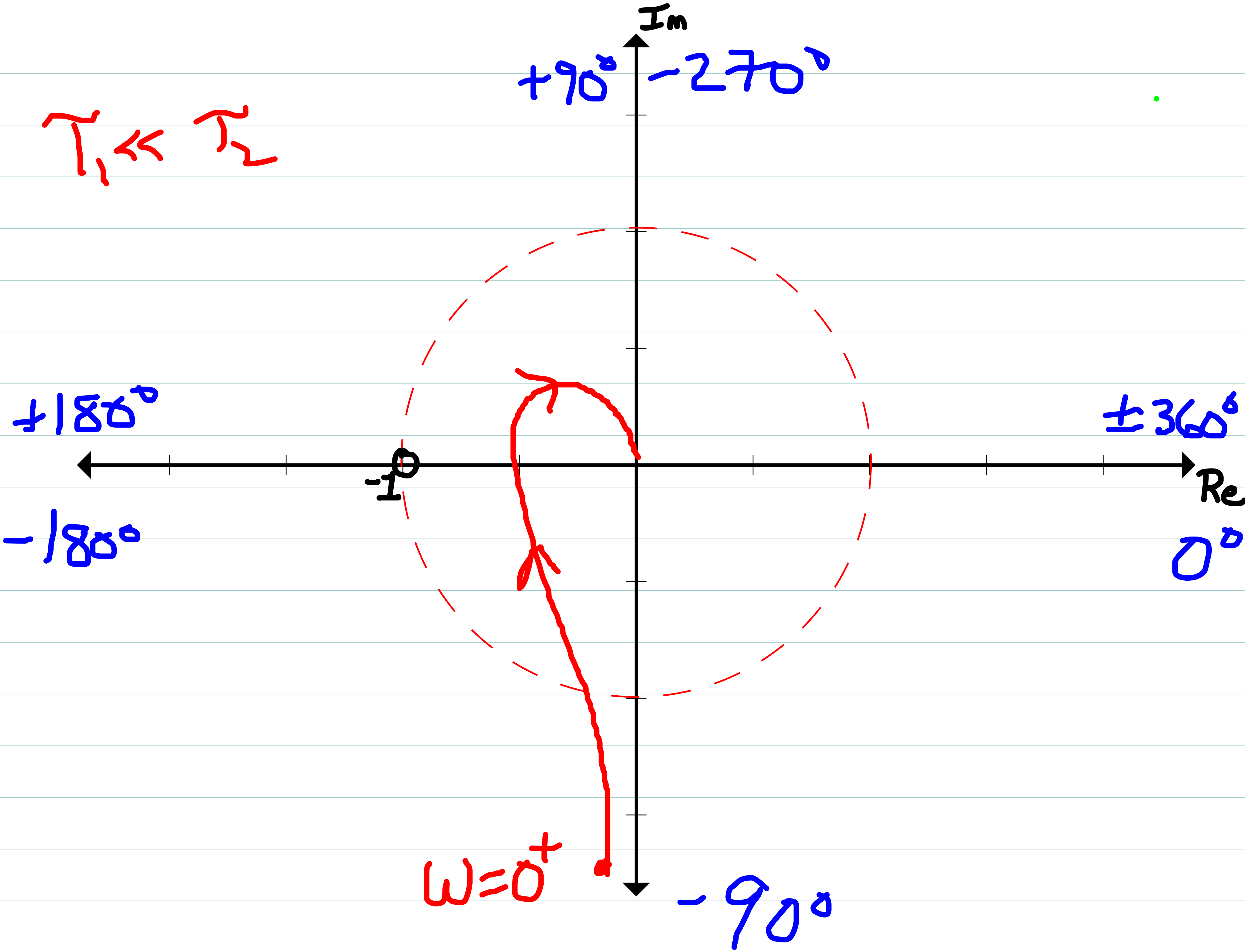
Example:

$$G(s) = K_B\left[\frac{(T_1 s+1)}{s(T_2 s+1)^3}\right]$$

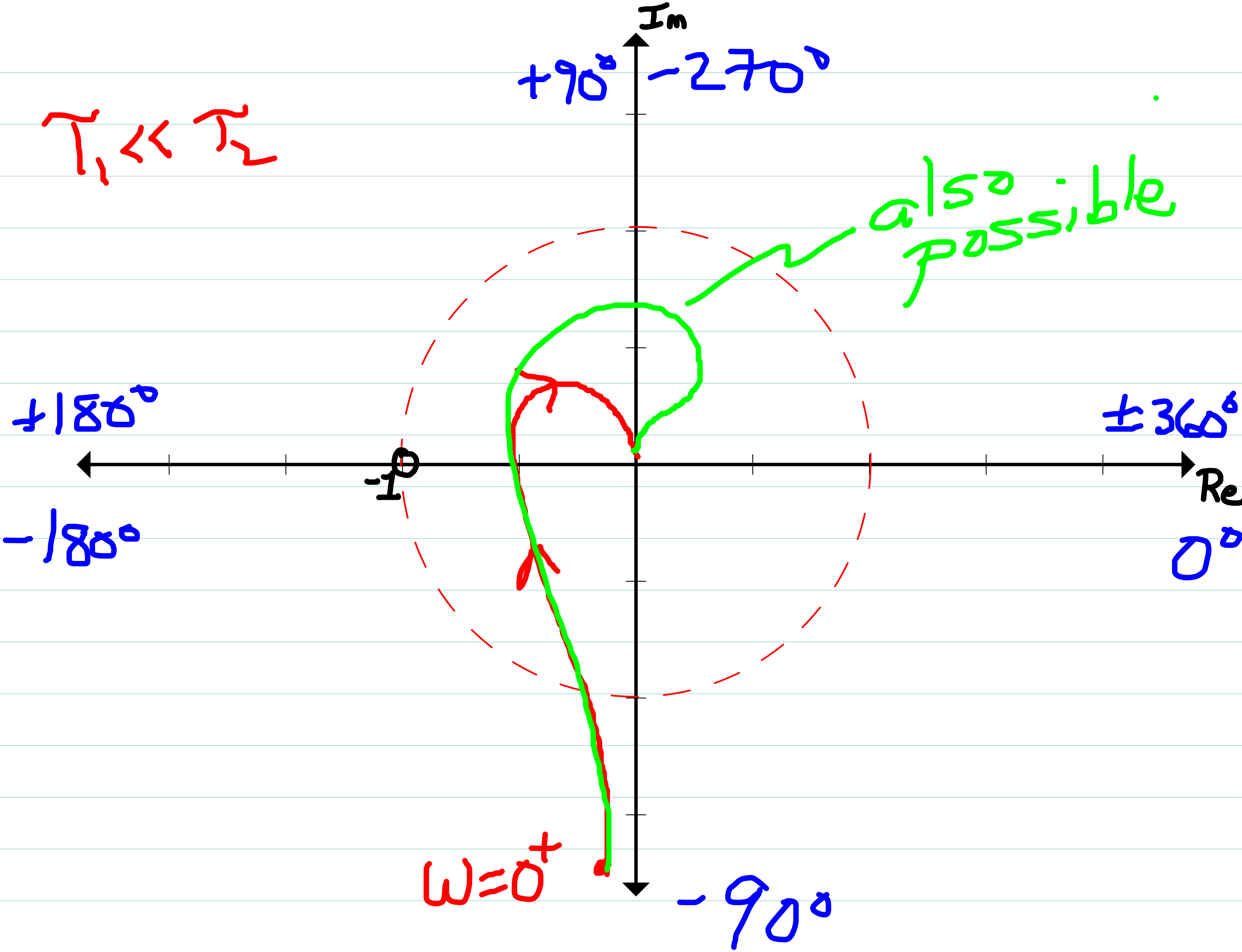
if $T_1 << T_2$ (so $\frac{1}{T_1} >> \frac{1}{T_2}$) then as $\omega \to \phi$ phase approaches $-90°$ *from below* (equivalently, phase is decreasing as ω increases from ϕ).

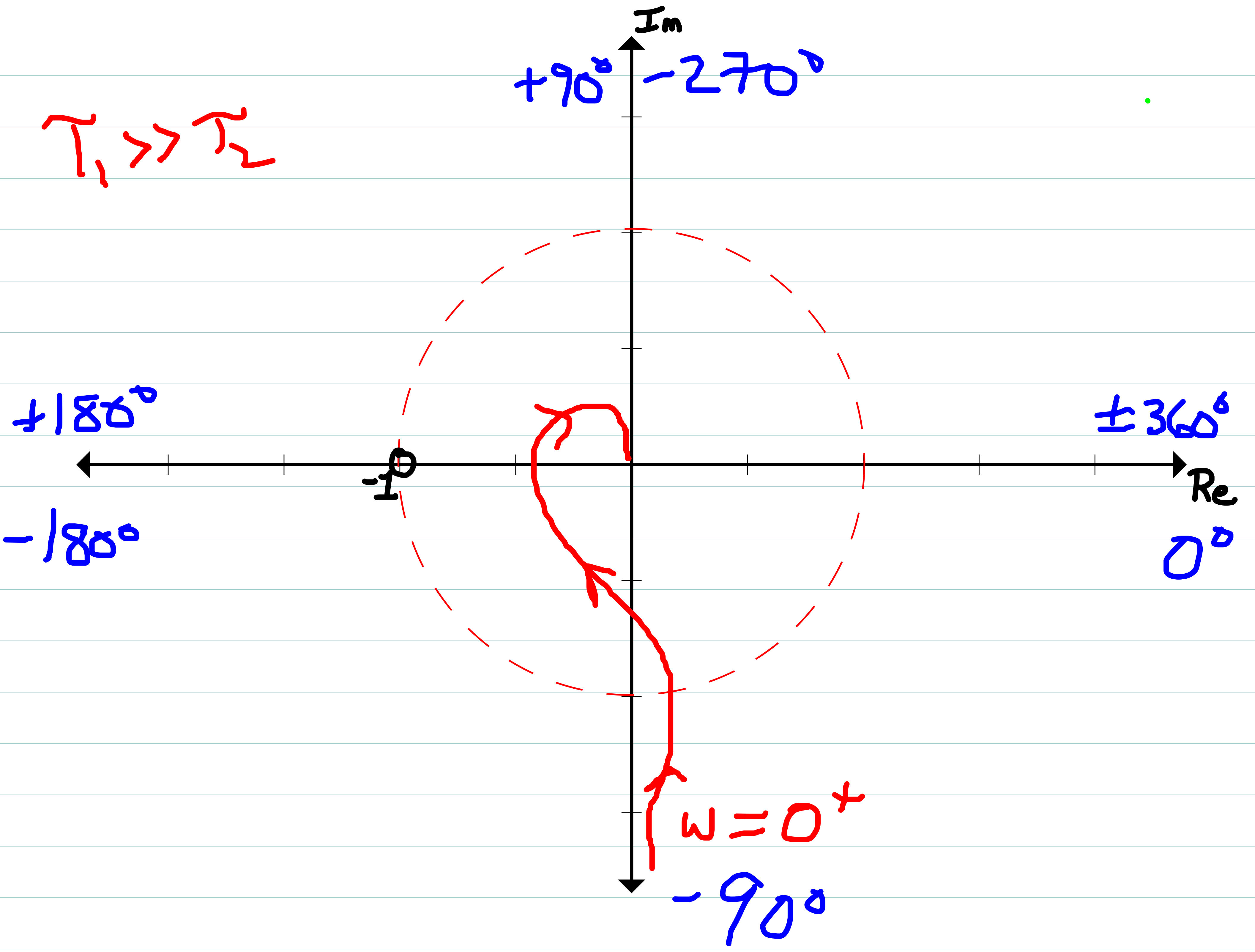
Conversely, if $T_1 >> T_2$, phase approaches $-90°$ *from above* as $\omega \to 0$.

$\tau_1 \ll \tau_2$



$T_1 \ll T_2$



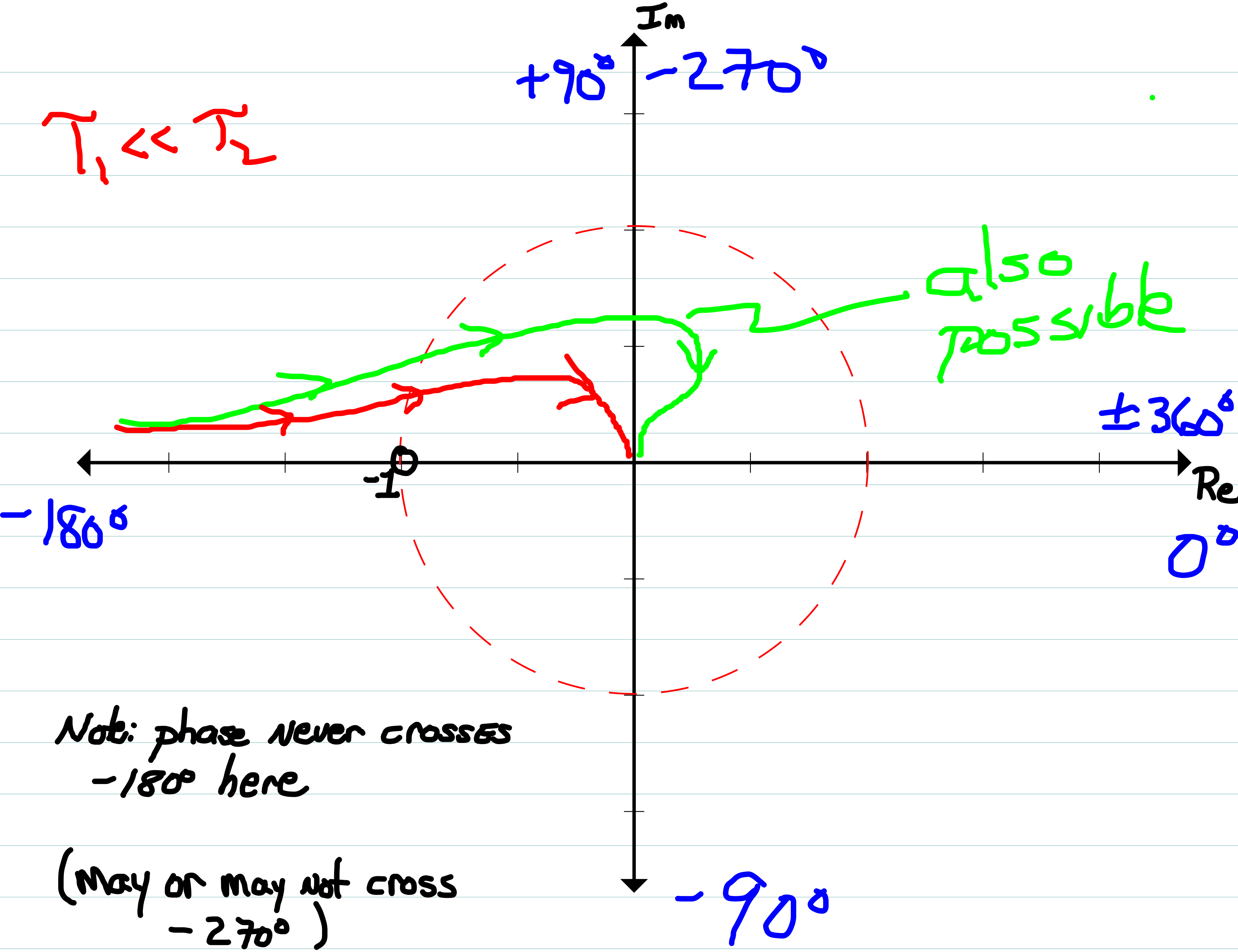

$\tau_1 >> \tau_2$
Im
+90°
-270°
+180°
-180°
±360°
0°
Re
-1
$\omega = 0^+$
-90°

Add'l poles at origin change the coordinate axis the tail lies along.

Example:

$$G(s) = K_B\left[\frac{T_1 s+1}{s^2(T_2 s+1)^2}\right]$$

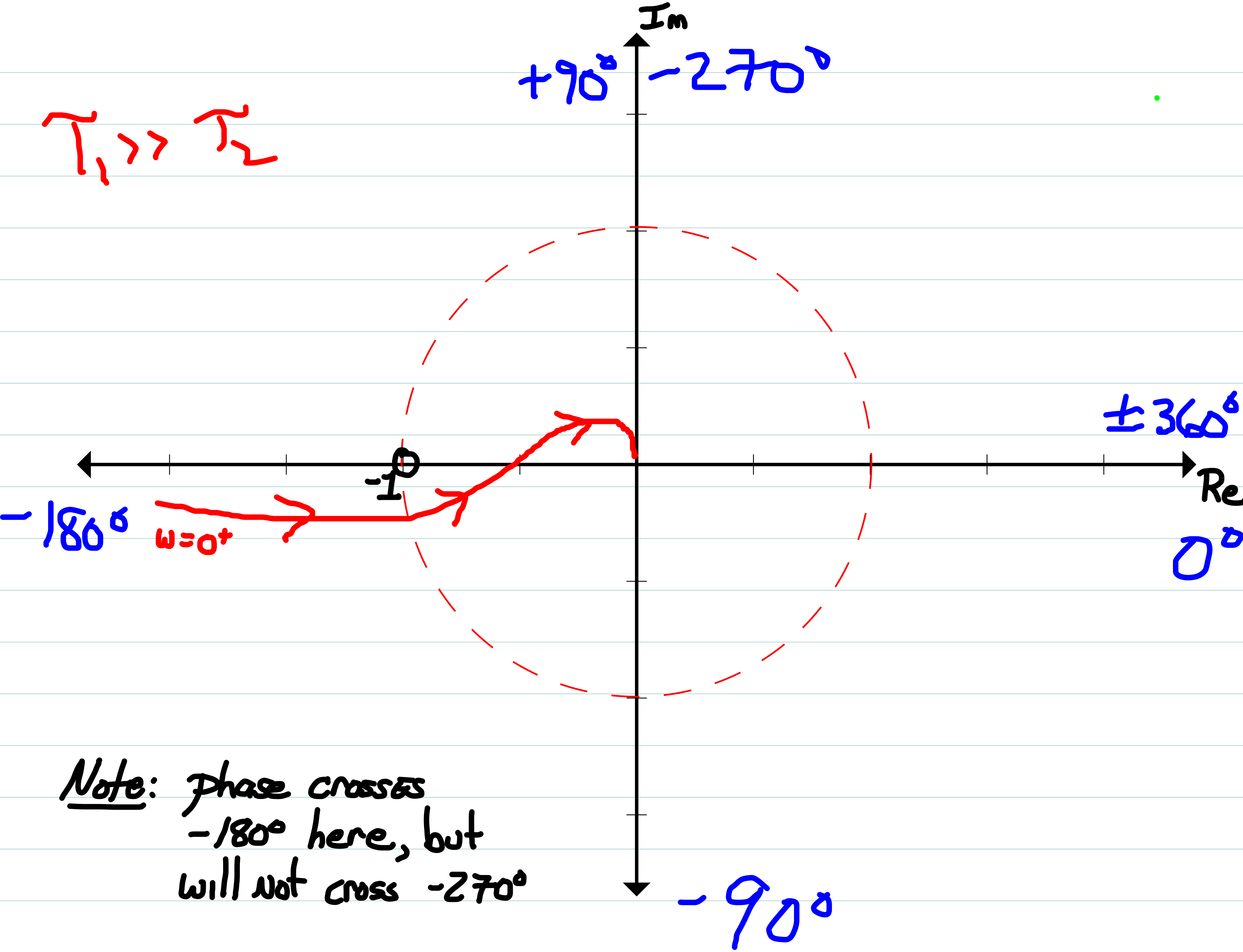
$T_1 << T_2$



Note: phase never crosses -180° here

(may or may not cross -270°)

$T_1 >> T_2$



<u>Note</u>: Phase crosses $-180°$ here, but will not cross $-270°$

A more complicated example

$$G(s) = K_B \left[\frac{(T_2 s+1)^2}{s^2 (T_1 s+1)(T_3 s+1)^2} \right]$$

With $T_1 >> T_2 >> T_3 > \phi$ $\left(\frac{1}{T_1} << \frac{1}{T_2} << \frac{1}{T_3} \right)$

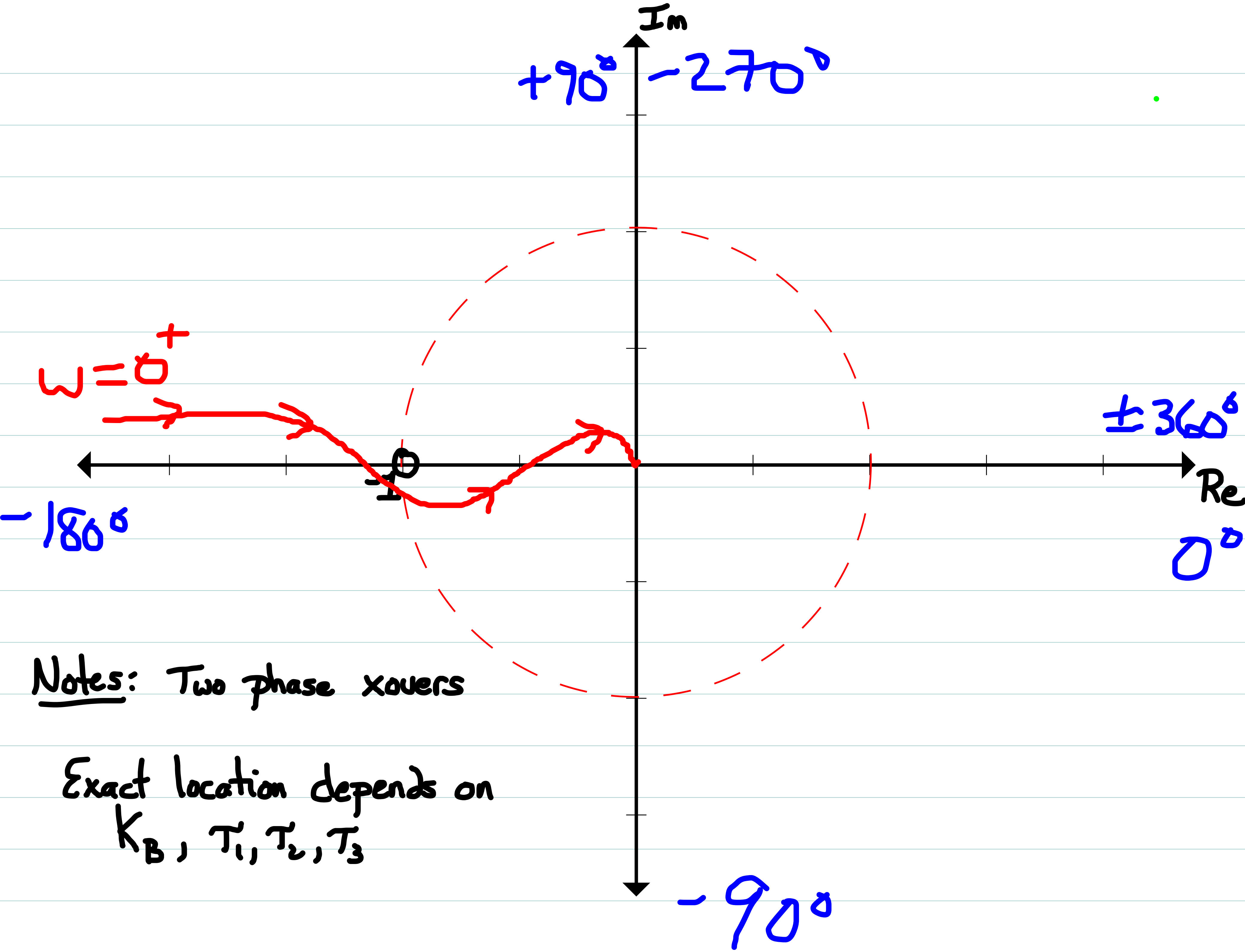
Low freq phase: $-180°$

high freq phase: $-270°$

phase initially decreases from pole at $-1/T_1$

Then increases due to double zero at $-1/T_2$

Then falls again due to double pole at $-1/T_3$



Notes: Two phase xovers

Exact location depends on K_B, T_1, T_2, T_3