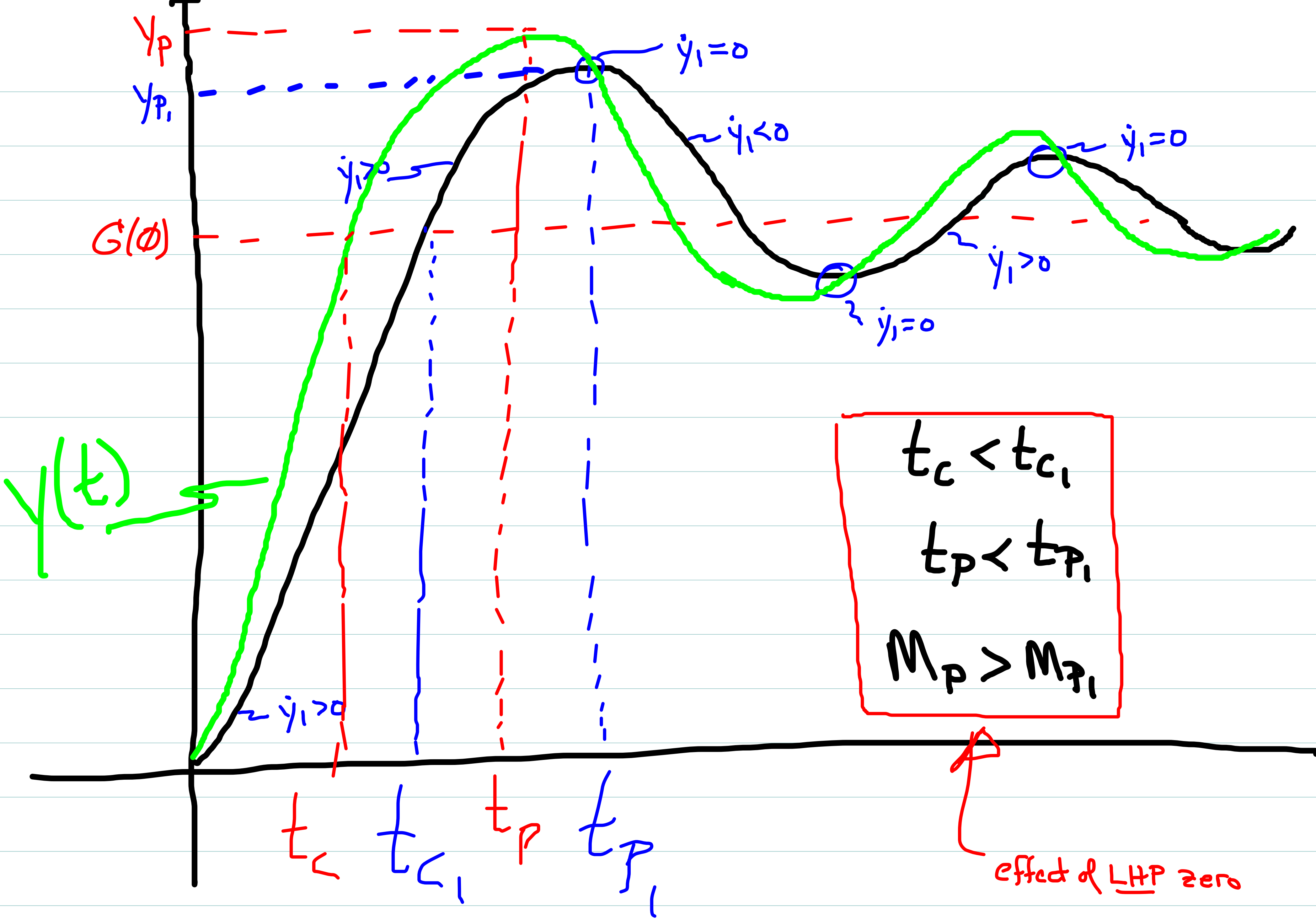


y_p
y_{p_1}
$G(\phi)$
$y(t)$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$\dot{y}_1 < 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
t_c
t_{c_1}
t_p
t_{p_1}
$t_c < t_{c_1}$
$t_p < t_{p_1}$
$M_p > M_{p_1}$
effect of LHP zero

<u>Summary of observations</u>

A LHP zero changes a 2^{nd} order step response by:

$\Rightarrow$ <u>Increasing</u> overshoot y_p and M_p

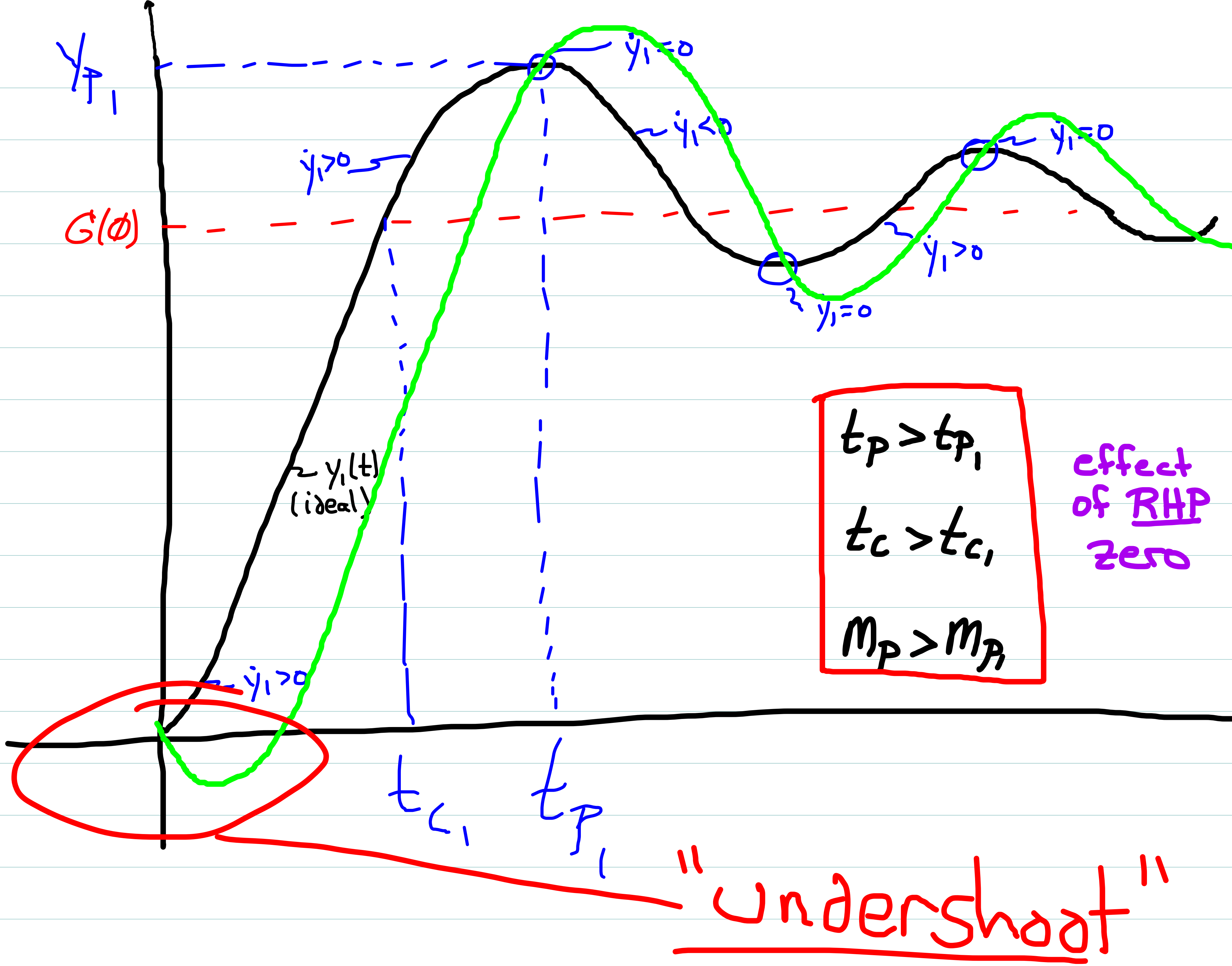
$\Rightarrow$ <u>decreasing</u> t_c and t_p

In a sense, system "responds" faster (crosses y_{ss} more quickly), but price is greater overshoot.

$\Rightarrow$ <u>Note:</u> tricky to quantify exact changes to t_c, t_p, y_p based on z_1

$\Rightarrow$ However, note change from "ideal" response is proportional to $\frac{1}{|z_1|}$

$\Rightarrow$ The further z_1 is from imag axis, the smaller the effect



y_{P_1}
$G(\emptyset)$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 < 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 = 0$
$y_1(t)$ (ideal)
$\dot{y}_1 > 0$
t_{c_1}
t_{P_1}
$t_P > t_{P_1}$
$t_c > t_{c_1}$
$M_P > M_{P_1}$
effect of RHP zero
"undershoot"

Observations (RHP zero)

- ⇒ Again, the peak response is greater
- ⇒ However, t_c and t_p have increased
- ⇒ Appearence of a new feature : "undershoot"
- ⇒ Response initially heads "in wrong direction" before ultimately returning to the same steady-state
- ⇒ Such behavior is NOT UNstable
- ⇒ It is, however, very tricky to design controllers for such systems.

Performance Specifications

⇒ Step inputs representative for many desired behaviors

- move to new pointing angle (spacraft)
- Move to new altitude or heading (aircraft)

⇒ Required performance often specified as upper limits on acceptable t_s, M_p

- System must settle quickly enough, and not overshoot too much.

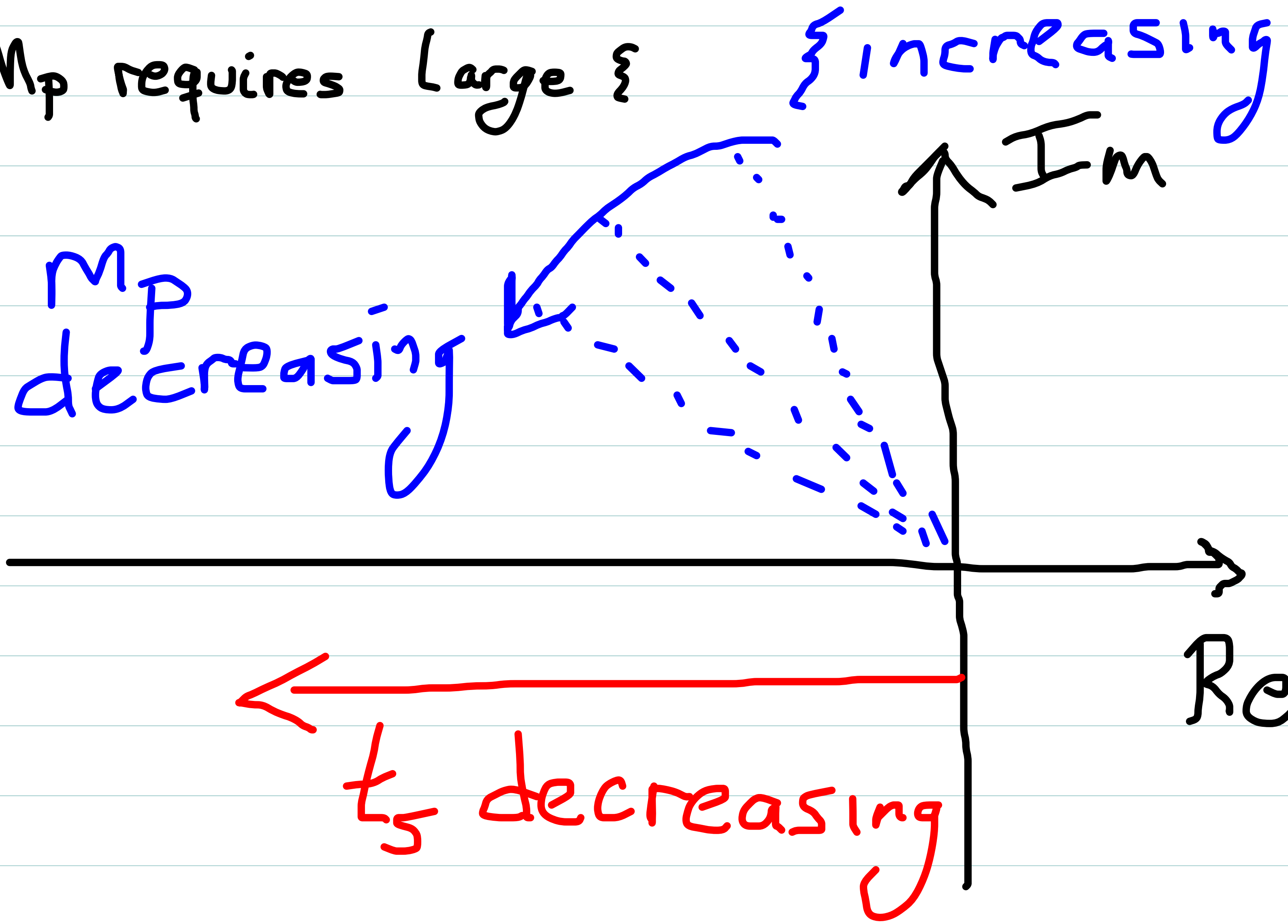
⇒ Recall:

- t_s inversely proportional to $|Re\{p_i\}|$
- M_p a decreasing function of ζ

$$t_s \approx \frac{4}{|\mathrm{Re}\{p_1\}|} \quad , \quad M_p = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right]$$

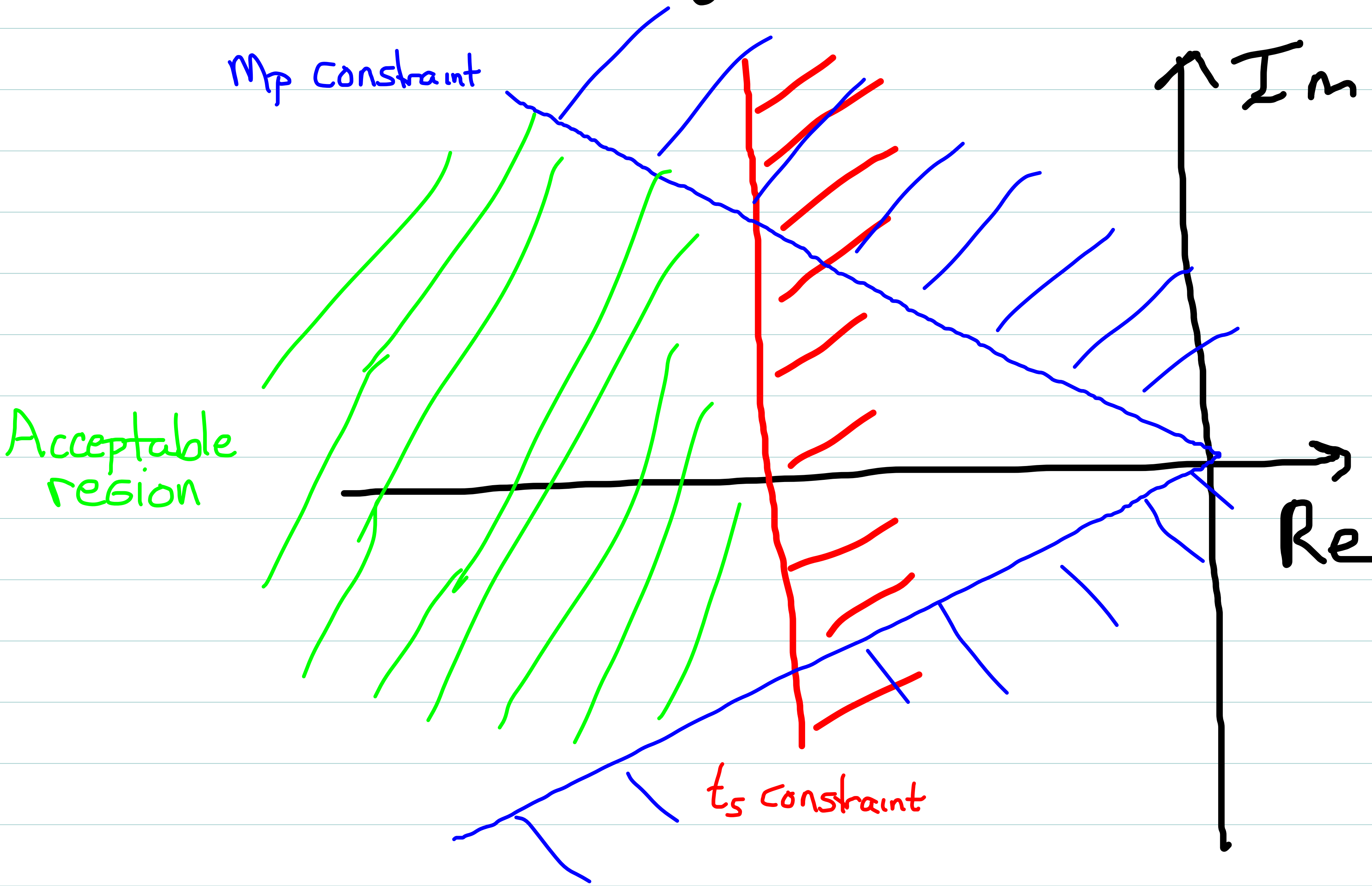
$\Rightarrow$ small t_s requires large $|\mathrm{Re}\{p_1\}|$

$\Rightarrow$ small M_p requires large ζ

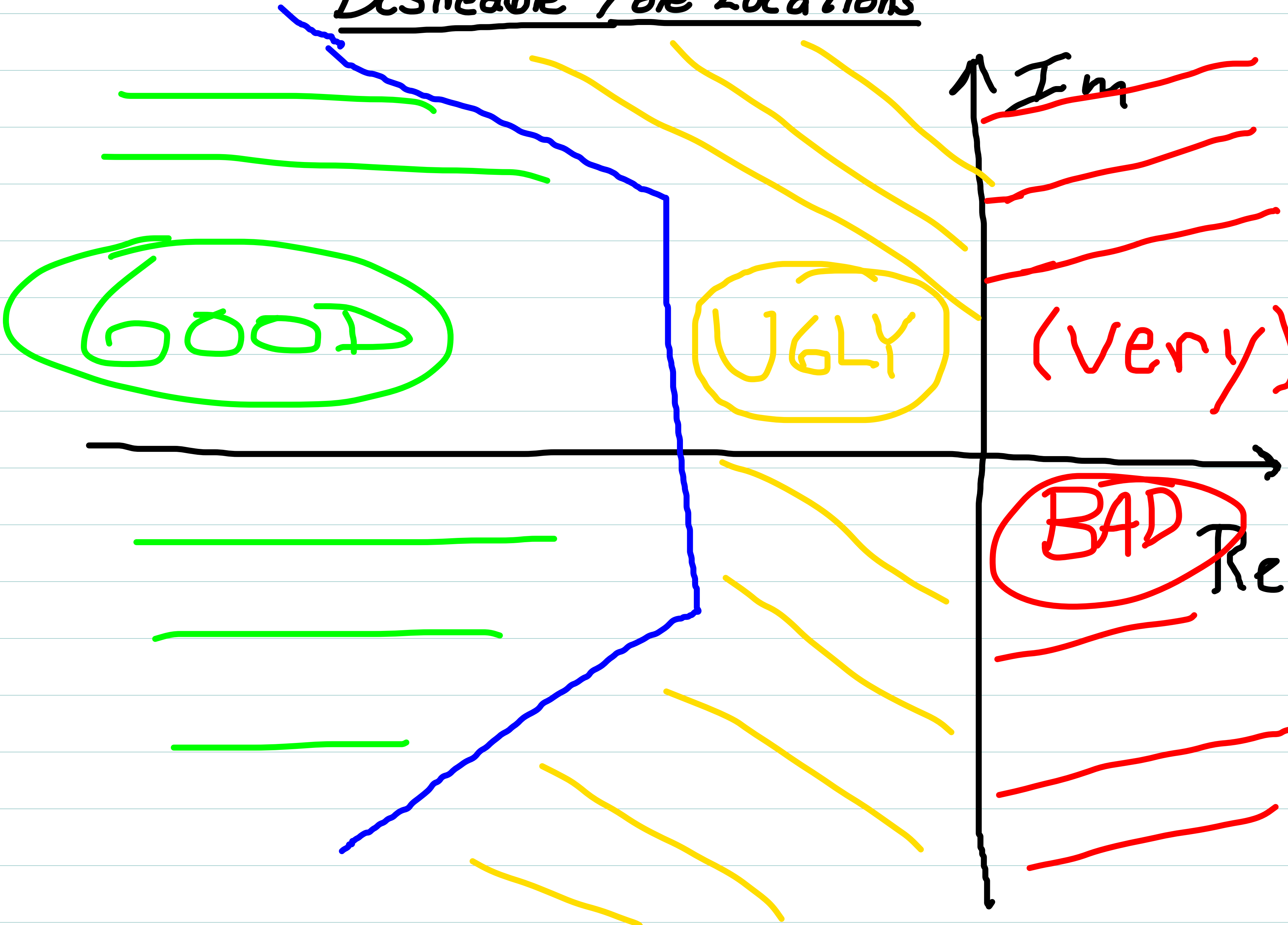


$\Rightarrow$ Upper bound on t_s gives lower bound on $|\mathrm{Re}\{p_i\}|$

$\Rightarrow$ Upper bound on M_p gives lower bound on ζ



Desireable Pole Locations



⇒ "Good" poles satisfy all transient performance constraints (upper bounds on t_s, M_p)

⇒ "Bad" poles are unstable

⇒ "Ugly" poles are stable, but have too much overshoot or take too long to settle.

⇒ Most aerospace system have natural dynamics which are "bad" or "ugly"

⇒ Goal of control is to make these systems "good"

Feedback "moves" poles

⇒ Already seen this on previous homeworks.

⇒ But it can be tricky!

Suppose $u(t) = K(y_d(t) - y(t))$

If system is modeled with $Y(s) = G(s)U(s)$

where $$G(s) = \frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$$

Then poles are moved to roots of

$$r_{CL}(s) = s^2 + (\alpha_1 + K\beta_1)s + (\alpha_0 + K\beta_0)$$

⇒ Tricky to predict movement of poles for all possible values of $K, \alpha_0, \alpha_1, \beta_0, \beta_1$

⇒ Even more complicated for $G(s)$ with additional poles and/or zeros

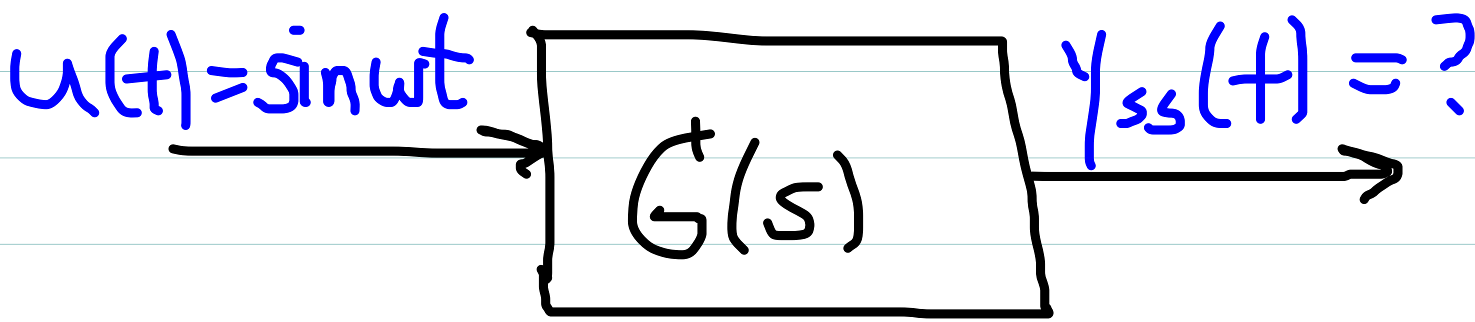
⇒ Need a more systematic tool to predict effectiveness of a control strategy.

⇒ One approach is BASED on a more careful analysis of the behavior of $G(j\omega)$.

Sinusoidal Response

Here we wish to understand the properties of the steady-state response of a $\boxed{\text{stable}}$ system when $\boxed{u(t) = \sin \omega t.}$

Note: our focus is shifting (temporarily) away from the transient response



Of course, we've already solved this problem:

$$u(t) = \sin\omega t = \operatorname{Im}\{e^{j\omega t}\}$$

$$\Rightarrow y_f(t) = \operatorname{Im}\{G(j\omega)e^{j\omega t}\} = |G(j\omega)|\sin(\omega t + \measuredangle G(j\omega))$$

Then $y(t) = y_f(t) + y_h(t)$

But if system is stable, $y_h(t) \to \phi$ as $t \to \infty$ for any set of initial cond'ns.

Hence $y_{tr}(t) = y_h(t)$ leaving us with

$$y_{ss}(t) = |G(j\omega)|\sin(\omega t + \measuredangle G(j\omega))$$

So:

$$u(t) = \sin \omega t \implies y_{ss}(t) = |G(j\omega)| \sin(\omega t + \measuredangle G(j\omega))$$

<u>Note:</u>

$y_{ss}(t)$ is sinusoidal at same frequency as $u(t)$

<u>But:</u>

Amplitude and phase of $y_{ss}(t)$ different.

Now, more generally suppose:

$$u(t) = B \sin(\omega t + \psi) = \mathrm{Im}\{U e^{j\omega t}\}, \quad U = B e^{j\psi}$$

then $$y_{ss}(t) = \mathrm{Im}\{G(j\omega) U e^{j\omega t}\}$$

$$= |G(j\omega)| \cdot |U| \sin(\omega t + \measuredangle G(j\omega) + \measuredangle U)$$

or $$y_{ss}(t) = |G(j\omega)| B \sin(\omega t + \measuredangle G(j\omega) + \psi)$$

Thus generally:

$$u(t) = B\sin(\omega t + \psi) \Longrightarrow y_{ss}(t) = A\sin(\omega t + \varphi)$$

where: $A = |G(j\omega)|B$

$$\varphi = \measuredangle G(j\omega) + \psi$$

<u>Define:</u>

<u>Amplitude ratio:</u> A/B (ratio of output ampl. to input ampl.)

<u>Phase shift:</u> $\varphi - \psi$ (Diff. between output and input phase)

<u>Then note:</u>

$$\boxed{\begin{aligned} A/B &= |G(j\omega)| \\ \varphi - \psi &= \measuredangle G(j\omega) \end{aligned}}$$

So generally

$|G(j\omega)|$ quantifies the ratio between output and input amplitude

$\angle G(j\omega)$ quantifies the change in phase of output compared to input

Note: these are frequency dependent

i.e. the amplitude ratio and phase shift depend on frequency of input.

Very useful to quantify this dependence!

Example

$$G(s) = \frac{3}{s+2}$$

$$|G(j\omega)| = \frac{3}{\sqrt{\omega^2+4}} \qquad \measuredangle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\omega = 1/2 \Rightarrow |G(j/2)| = 3/\sqrt{4.25} \approx 1.46$$

$$\measuredangle G(j/2) = -\tan^{-1}(1/4) = -.245 \text{ rad or } -14.04^\circ$$

$$\omega = 2 \Rightarrow |G(2j)| = 3/\sqrt{8} \approx 1.06$$

$$\measuredangle G(2j) = -\tan^{-1}(1) = -\pi/4 = -45^\circ$$

$$\omega = 20 \Rightarrow |G(20j)| = 3/\sqrt{404} = 0.15$$

$$\measuredangle G(20j) = -\tan^{-1}(10) = -1.47 \approx -84.3^\circ$$

$\Rightarrow$ Want to learn to <u>predict</u> these changes based on ZPK structure of $G(s)$

$\Rightarrow$ Useful also to visualize <u>graphically</u>

$\Rightarrow$ Three methods

1. Plot $|G(j\omega)|$ and $\measuredangle G(j\omega)$ vs. $\omega \geq \phi$ (2 plots)

2. Plot $G(j\omega)$ as ω varies from ϕ to ∞ as points in complex plane.

3. Plot $|G(j\omega)|$ vs. $\measuredangle G(j\omega)$ for $\phi \leq \omega < \infty$

$\Rightarrow$ Want to learn to predict these changes based on ZPK structure of $G(s)$

$\Rightarrow$ Useful also to visualize graphically

$\Rightarrow$ Three methods

1. Plot $|G(j\omega)|$ and $\measuredangle G(j\omega)$ vs. $\omega \geq 0$ (2 plots) "Bode diagrams"

2. Plot $G(j\omega)$ as ω varies from 0 to ∞ as points in complex plane. "Polar diagram"

3. Plot $|G(j\omega)|$ vs. $\measuredangle G(j\omega)$ for $0 \leq \omega < \infty$ "Nichols chart"

Bode is most fundamental, start there

$\Rightarrow$ want to see behavior for large range of $\omega \geq \emptyset$

$\Rightarrow |G(j\omega)|$ will vary enormously in size

$\Rightarrow$ Use logarithmic scales for plots.

$\Rightarrow$ Horizontal Axis on Bode diagram is freq on a log scale

$\Rightarrow$ equally spaced divisions on this scale are factors of 10 apart.

$\Rightarrow$ We call one of these divisions a "decade"

$\left.\begin{array}{l} 1/10 \rightarrow 1 \\ 2 \rightarrow 20 \end{array}\right\}$ one decade

$\left.\begin{array}{l} 1/10 \rightarrow 10 \\ 2 \rightarrow 200 \end{array}\right\}$ two decades

Decibels

$|G(j\omega)|$ is shown on Bode diagrams in special units called decibels.

Def'n: For any real number $X \geq 0$

$$X_{db} = 20 \log X$$

Conversely

$$X = 10^{(X_{db}/20)}$$

Example (from above): $X = 1.46 \Rightarrow X_{db} = 3.25$

$$X = 1.06 \Rightarrow X_{dB} = 0.51$$

$$X = 0.15 \Rightarrow X_{dB} = -16.5$$

Common Shorthand

$X = 0.15 = -16.5\,dB$

Note common conversions

X	X (dB)
.01	-40
.1	-20
1	0
10	20
100	40

Important ⟹

Zero on dB axis means magnitude of 1 !!

Bode diagrams show

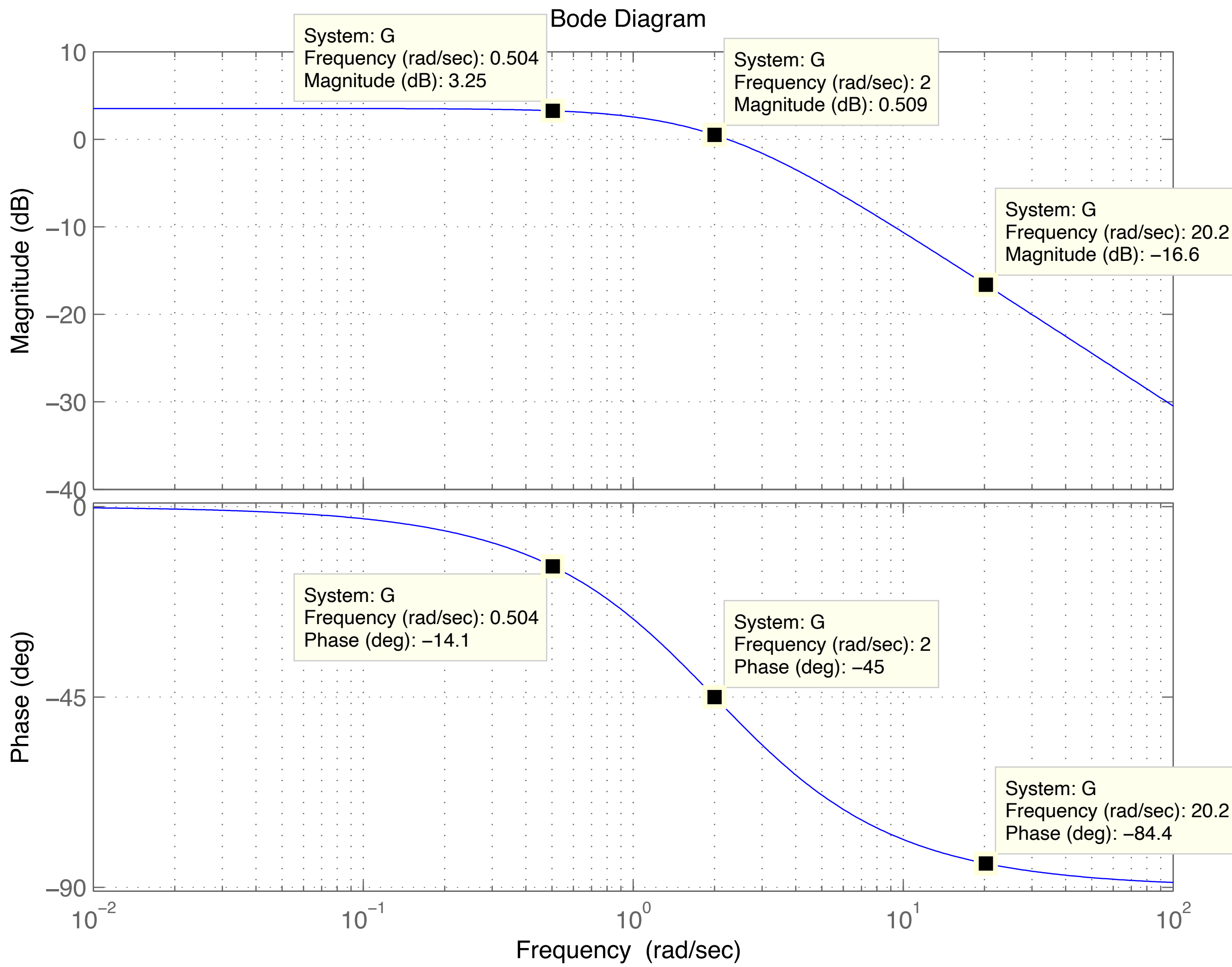
(1.) $|G(j\omega)|$ in dB vs ω on a log scale

(2.) $\measuredangle G(j\omega)$ in deg " " "

See example

Note: there are no negative frequencies on a Bode diagram!

The left limit of the horizontal scale corresponds to $\omega \to \emptyset$!



Bode Diagram
System: G
Frequency (rad/sec): 0.504
Magnitude (dB): 3.25
System: G
Frequency (rad/sec): 2
Magnitude (dB): 0.509
System: G
Frequency (rad/sec): 20.2
Magnitude (dB): −16.6
Magnitude (dB)
10
0
−10
−20
−30
−40
System: G
Frequency (rad/sec): 0.504
Phase (deg): −14.1
System: G
Frequency (rad/sec): 2
Phase (deg): −45
System: G
Frequency (rad/sec): 20.2
Phase (deg): −84.4
Phase (deg)
0
−45
−90
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)