University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #9

Issued: 20 Apr. 2019
Due By: 26 Apr. 2019

Question 1:

A PID compensator has the form

$$u(t) = K_p e(t) + K_d \dot{e}(t) + K_I \int_0^t e(\tau) d\tau$$

or equivalently

$$\begin{aligned} u(t) &= K_p e(t) + K_d \dot{e}(t) + K_I x_1(t) \\ \dot{x}_1(t) &= e(t) \end{aligned}$$

This is one of the most commonly used controllers, at least for systems with relatively simple dynamics and when derivative measurements are available. Logic to implement this calculation is often directly implemented in the circuitry of standard embedded motion control chips or off-the-shelf drone autopilots. Since it is so popular, there are many *ad hoc* "tuning rules" for the three gains, so that people who really don't know anything about controls, or even the exact dynamics of their system, can still get a feedback loop up and running using these controllers.

The most common of these tuning rules are the Ziegler-Nichols (ZN) tuning rules, which can be summarized as follows:

- Set K_d and K_I to zero. Increase K_p until the controlled system begins to exhibit persistent oscillations (this can be done experimentally, if needed). Let K_u be this value of K_p, and let T_u be the observed period of the oscillations.
- Choose the values: $K_p = 3K_u/5$, $K_I = 2K_p/T_u$, and $K_d = K_p T_u/8$.

Pretty cookbook, no?

Let's try this for a system whose dynamics are known to be

$$G(s) = \frac{4}{s(s+2)^2}$$

Since we actually have a model of the system here, we can use Bode methods to directly identify the numerical values of K_u and T_u, and thus the numerical values of the three PID gains. To this end, note that the first step is equivalent to setting $H(s) = K$, and K_u is then the value of K for which the phase margin and gain margins are zero. As we have seen previously, in such a situation the resulting closed-loop system will then have poles on the imaginary axis at $\pm j\omega_\gamma$, and the period of the corresponding oscillations is thus $T_u = 2\pi/\omega_\gamma$.

Using these observations, determine the $L(s)$ which results from Ziegler-Nichols tuning of a PID controller for this $G(s)$. (Note that it should be possible to determine K_u and T_u *by inspection* of this system!) Quantify the resulting crossovers and margins, compute the resulting closed-loop poles, and quantify the significant features of a unit step response for $T(s)$. In your judgement, is this a particularly good feedback loop design?

Question 2:

Let's use our deeper understanding of controls to better tune the PID controller in Question #1. Specifically, let's tune the compensator so the loop has a phase margin of $50°$ at a crossover of 2 rad/sec. Determine the equivalent PID gains which will achieve this target, generate a step response of the resulting $T(s)$, and compare with Question #1.

NOTE: You still have some freedom in the placement of the zeros of $H(s)$. The simplest design will assume these zeros are repeated, but this is in no way required nor necessarily optimal. Time permitting, you might explore the extent to which you can improve the metrics for the step response of $T(s)$, while maintaining the same phase margin and crossover, by varying the zero locations in $H(s)$.

Question 3:

For your design in Question #2:

a.) What is the delay margin, in seconds? What is the minimum acceptable loop update rate for the controller calculations, in Hz (this is just the inverse of the delay margin)?

b.) If the loop rate is fixed by the available electronics at 10 Hz, what is the effective phase margin of your feedback loop? What is the effective gain margin? Qualitatively, how do you think these reductions would affect the step response properties of $T(s)$?

c.) If $y_d(t) = a_0 + a_1 t + a_2 t^2$, determine the amount of steady-state tracking error your design would exhibit, as a function of the polynomial coefficients.

d.) If your feedback loop is subjected to an external disturbance of the form $d(t) = b_0 + b_1 t$, determine the amount of additional steady-state tracking error the disturbance would cause, as a function of the polynomial coefficients.

e.) If instead the disturbance is sinusoidal, determine the range of disturbance frequencies for which the induced additional tracking steady-state tracking error will be no more than 1% of the amplitude of the disturbance. Determine also the frequency for the disturbance which would result in the largest additional tracking error.

f.) Find the minimum distance from the Nyquist plot to the -1 point for the open-loop dynamics corresponding to your design (neglecting delay effects).

Question 4:

On Problem Set #7, we showed that the transfer function

$$H(s) = \frac{20(s+2)^3}{s(s+6)(s+12)}$$

could be implemented as

$$\begin{aligned} u(t) &= 20e(t) + 2.22x_1(t) + 35.56x_2(t) - 277.78x_3(t) \\ \dot{x}_1(t) &= e(t) \\ \dot{x}_2(t) &= -6x_2(t) + e(t) \\ \dot{x}_3(t) &= -12x_3(t) + e(t) \end{aligned}$$

However, this implementation is not unique. Use Laplace algebra to show that the following implementation equations also correspond to the transfer function $H(s)$ above

$$\begin{aligned} u(t) &= 20e(t) + 2x_1(t) - 15x_2(t) - 3x_3(t) \\ \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= -72x_2(t) - 18x_3(t) + 80e(t) \end{aligned}$$

as does

$$\begin{aligned} u(t) &= 20e(t) + 2.5x_1(t) - 15.68x_2(t) + 1.92x_3(t) \\ \dot{x}_1(t) &= 8e(t)/9 \\ \dot{x}_2(t) &= -13.61x_2(t) + 3.5x_3(t) + 15.68e(t) \\ \dot{x}_3(t) &= -3.5x_2(t) - 4.39x_3(t) + 1.92e(t) \end{aligned}$$

(some small round-off error may occur here).

In fact, there are an infinite number of possible derivative-free "realizations" for a given transfer function in these general forms. The original one, at top, is the simplest and easiest to "discretize" for computer implementation, but there are occasions where alternative forms such as those above are useful, and indeed necessary.