Why can't I make the title look nice

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Abstract

Future me will put some abstract here

1 Introduction

2 Chab Chab

Let G be a Lie group, and A_n and A be its closed subgroups. We want to find a notion of convergence for closed subgroups. One such notion is Chabauty topology.

Definition 2.1. A sequence of subgroups (A_n) converge to A if

1. Any convergent sequence (x_n) where $x_n \in A_n$ will converge to some element in $x \in A$.
2. If $x \in A$, then there exists some subsequence (x_{n_k}) where $x_{n_k} \in A_{n_k}$ such that it converge to x. In other words, x is an accumulation point.

For this section, we let $G = \mathrm{PSL}_2(\mathbb{R})$. The group G is the group of isometries acting on the upper half plane model of $\mathbb{H}^2$ by fractional linear transformation. Using Chabauty topology, we are able to describe the shape of the collection of all closed subgroups of G, denoted by $S(G)$. To do so, We want to break $S(G)$ into pieces that we know the shape of, and then try to glue it back together.

A sequence of cyclic subgroup generated by a hyperbolic isometry can converge to a cyclic subgroup generated by a parabolic isometry.

Example 2.2. Let $\xi_n = 1 + 1/n$, Consider

$$H_n = \frac{1}{2}\begin{bmatrix} \xi_n + \xi_n^{-1} & n(-\xi_n + \xi_n^{-1}) \\ \frac{1}{n}(-\xi_n + \xi_n^{-1}) & \xi_n + \xi_n^{-1} \end{bmatrix}, \quad P = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

The sequence of subgroups $(\langle H_n \rangle)$ converges to the subgroup $\langle P \rangle$.

Here, H_n is a hyperbolic isometry that fixes $n, -n \in \partial\mathbb{H}^2$, leaves geodesic connecting $n, -n$ invariant, and travels along the geodesic by $2\ln(\xi_n)$; P is a parabolic isometry that fixes $\infty \in \partial\mathbb{H}^2$, and maps $z \mapsto z - 1$.

It is not hard to show $H_n \to P$ as $n \to \infty$ (I promise). However, as a noob in Chabauty topology, one might (I did) think we are done with this proof by showing that the generators converges to the generator.

[MAYBE A PICTURE HERE]

3 The plot thickens

Jorgenson first discovered the following interesting phenomenon.

Fact 3.1. *There exists a sequence of infinite cyclic groups $\langle H_n \rangle$, where H_n is an isometry of hyperbolic type, such that the sequence converges to a subgroup isomorphic to $\mathbb{Z}^2$, whose generators are both parabolic isometries.*

In this section, we will construct one such sequence explicitly. We will use the upper half-plane model $\mathbb{C} \times \mathbb{R}^+$ for $\mathbb{H}^3$. The isometry group $\mathcal{I}^+(\mathbb{H}^3) \cong \mathrm{PSL}_2(\mathbb{C})$ acts on the boundary $\overline{\mathbb{C}}$ by fractional linear transformation.

4 Quotient things up

5 Convergence of the quotient

6 Closet of necessary backgrounds

References

[PH] Author names. Article Titles. *Journal Title*, ??? some mysterious numbers.