1 Chabauty space of $\mathbf{PSL}_2(\mathbb{R})$

Let $G = \mathrm{PSL}_2(\mathbb{R})$. The group G acts on $\mathbb{H}^2$ by fractional linear transformation on upper half plane. An elemenatary subgroup of G is a subgroup of G that has a finite orbit in $\mathbb{H}^2 \cup \partial\mathbb{H}^2$.

Let the set A be a finite orbit of an elementary subgroup $H \leq G$. If H contains an element that is parabolic or hyperbolic, then A does not contain any element in $\mathbb{H}^2$. So in the case where A is contained in $\mathbb{H}^2$, we know that H can only have elliptic elements. The following fact is from Katok, where it is proved algebraically.

Fact 1. *If H contains only elliptic elements, then it has a fixed point in $\mathbb{H}^2$.*