Space of Projectivised $\mathbb{Z}^2$ Lattices in $\mathbb{C}$

A $\mathbb{Z}^2$-lattice in $\mathbb{C}$ is a closed subgroup in $\mathbb{C}$ that is isomorphic to $\mathbb{Z}^2$. We are investigating the shape of the space L of such lattices with Chabauty topology. The object is not hard to understand — it is just lattices in $\mathbb{R}^2$, even my Calculus 1 students can draw a lattice in $\mathbb{R}^2$. What makes this problem hard is that there are so many ways to build up the same $\mathbb{Z}^2$: think about $\langle(1,0),(0,1)\rangle \cong \langle(1,0),(1,1)\rangle$.

Figure 1: My first impression on this problem

Our first quest is to uniquely find two small equivalence classes of vectors such that we can choose one vector from each class and reconstruct the lattice. Figure

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