GEOMETRY AND RANK OF FIBERED HYPERBOLIC 3-MANIFOLDS (EXTENDED VERSION)

Contents

1. The name of section 1

Let Σ_g be a closed orientable surface of genus g and $\phi : \Sigma_g \to \Sigma_g$ a homeomorphism. We can construct a mapping torus M_ϕ. Thurston has proven that if ϕ is pseudo-anosov, then M_ϕ can be given a hyperbolic metric.

The fundamental group of M_ϕ is given by an HNN-extension

$$1 \to \pi_1(\Sigma_g) \to \pi_1(M_\phi) \to \mathbb{Z} \to 1.$$

Suppose $\pi_1(\Sigma_g)$ is a group with presentation $\langle S \mid R\rangle$, then $\pi_1(M_\phi) = \langle S, t \mid R, t^{-1}st = \phi_*(s),\ \forall s \in S\rangle$, where t is the generator of the longtitude of the mapping torus. Here $\pi_1(M_\phi)$ is called the HNN-extension of $\pi_1(\Sigma_g)$ relative to ϕ_*. Since $\text{rank}(\pi_1(\Sigma_g)) = 2g$, $\text{rank}(\pi_1(M_\phi)) \leq 2g+1$. The equality holds when the map ϕ is the identity map. The equality also holds when the map is complicated enough, like ϕ^n for some large n and pseudo-anosov ϕ due to J. Souto. It's probably not hard to construct a map ϕ such that $\pi_1(M_\phi)$ has rank 2 to $2g$, but I'm still trying to figure out how to construct map ϕ such that M_ϕ is hyperbolic while having π_1 with those ranks.