Theorem 0.1. *Let M be an irreducible manifold which need not to be compact. Let F be an incompressible (compact, closed) boundary component of M. in $\partial M - F$, let F' be an incompressible surface which need neither be closed or compact. Suppose: if k is any closed curves in F, then some non-null multiple of k is homotopic to a curve in F'. Then M is homeomorphic to $F \times I$.*

Warm-up. Suppose I am a student in intro to abstract math. I'm going to use the signature proof strategy: implicitly assume $M = F \times I$, show that it satisfy the assumptions given, and then prove $M \cong F \times I$. It is like we are doing a surgery on a lab rat to figure out what can work, and then try to apply to the more complicated case and try to resolve what don't work.

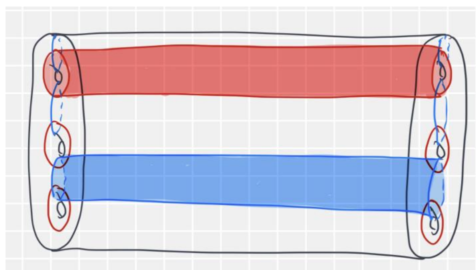

FIGURE 1. (boring) cylinders in $M = F \times I$

Suppose F has genus g, then we can construct a system of essential simple closed curves $k_1, \ldots, k_{2g}$ such that k_i and k_j intersect once traversely if $i = j \pm 1$, and they are disjoint otherwise. If you have trouble finding such system, email me at `mujie.wang@bc.edu` for more assistance. For each k_j, there is an cylinder $G_j = k_j \times I$ embedded in M. A pair of cylinders G_i and G_j intersect if and only if k_i and k_j intersect, and the intersection $G_i \cap G_j$ is exactly $(k_i \cap k_j) \times I$. See 1.

Here comes the fun part! We want to show that if we cut out the regular neighborhood of two boundary components $F_0, F_1 \cong F$ and all the cylinders, what's left is a ball.

We have the shell-bone (regular neighborhood of a bunch of things) and a ball, now what? I don't know. Time to sweep those under a rug and forget about them forever. :-)

Proof. actual proof of the lemma + figure out why the lemma follows. maybe using cell complexes? :-)

- M hyperbolic, pi1M = pi1S, want to show M is S x R with convex core one of 3 kinds. first show it is S x R using tameness theorem (???)