Theorem 0.1. *Let M be an irreducible manifold which need not to be compact. Let F be an incompressible (compact, closed) boundary component of M. in $\partial M - F$, let F' be an incompressible surface which need neither be closed or compact. Suppose: if k is any closed curves in F, then some non-null multiple of k is homotopic to a curve in F'. Then M is homeomorphic to $F \times I$.*

Warm-up. Suppose I am a student in intro to abstract math. I'm going to use the signature proof strategy: implicitly assume $M = F \times I$, show that it satisfy the assumptions given, and then prove $M \cong F \times I$. It is like we are doing a surgery on a lab rat to figure out what can work, and then try to apply to the more complicated case and try to resolve what don't work.

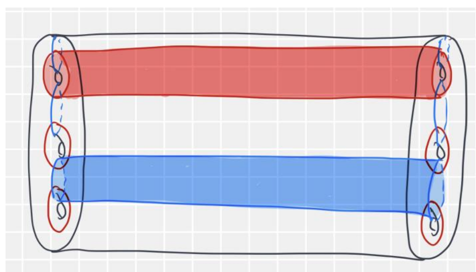

FIGURE 1. (boring) mapping cylinders in $M = F \times I$

Suppose F has genus g, then we can construct a system of essential simple closed curves $k_1, \ldots, k_{2g}$ such that k_i and k_j intersect once traversely if $i = j \pm 1$, and they are disjoint otherwise. If you have trouble finding such system, email me at `mujie.wang@bc.edu` for more assistance. For each k_j, there is an cylinder $G_j = k_j \times I$ embedded in M. A pair of cylinders G_i and G_j intersect if and only if k_i and k_j intersect, and the intersection $G_i \cap G_j$ is exactly $(k_i \cap k_j) \times I$.

Here comes the second hardest part of this fake proof.

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