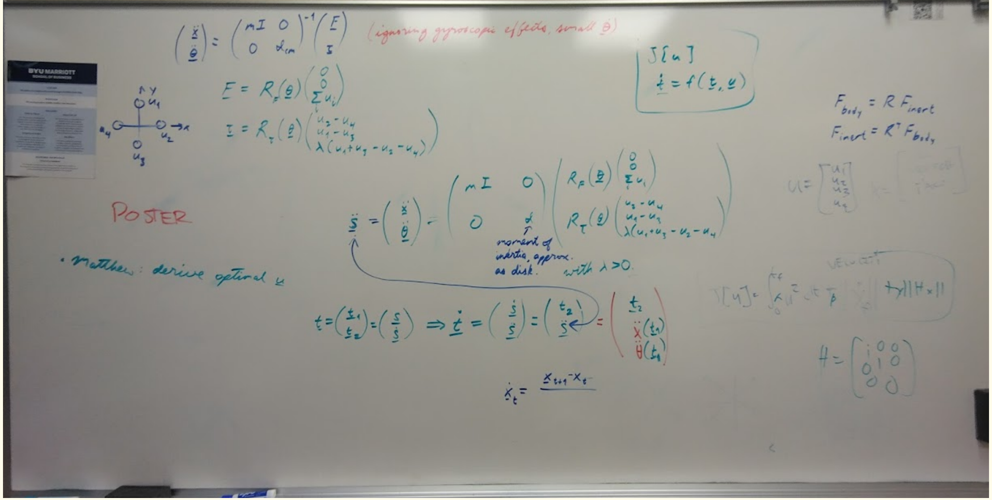
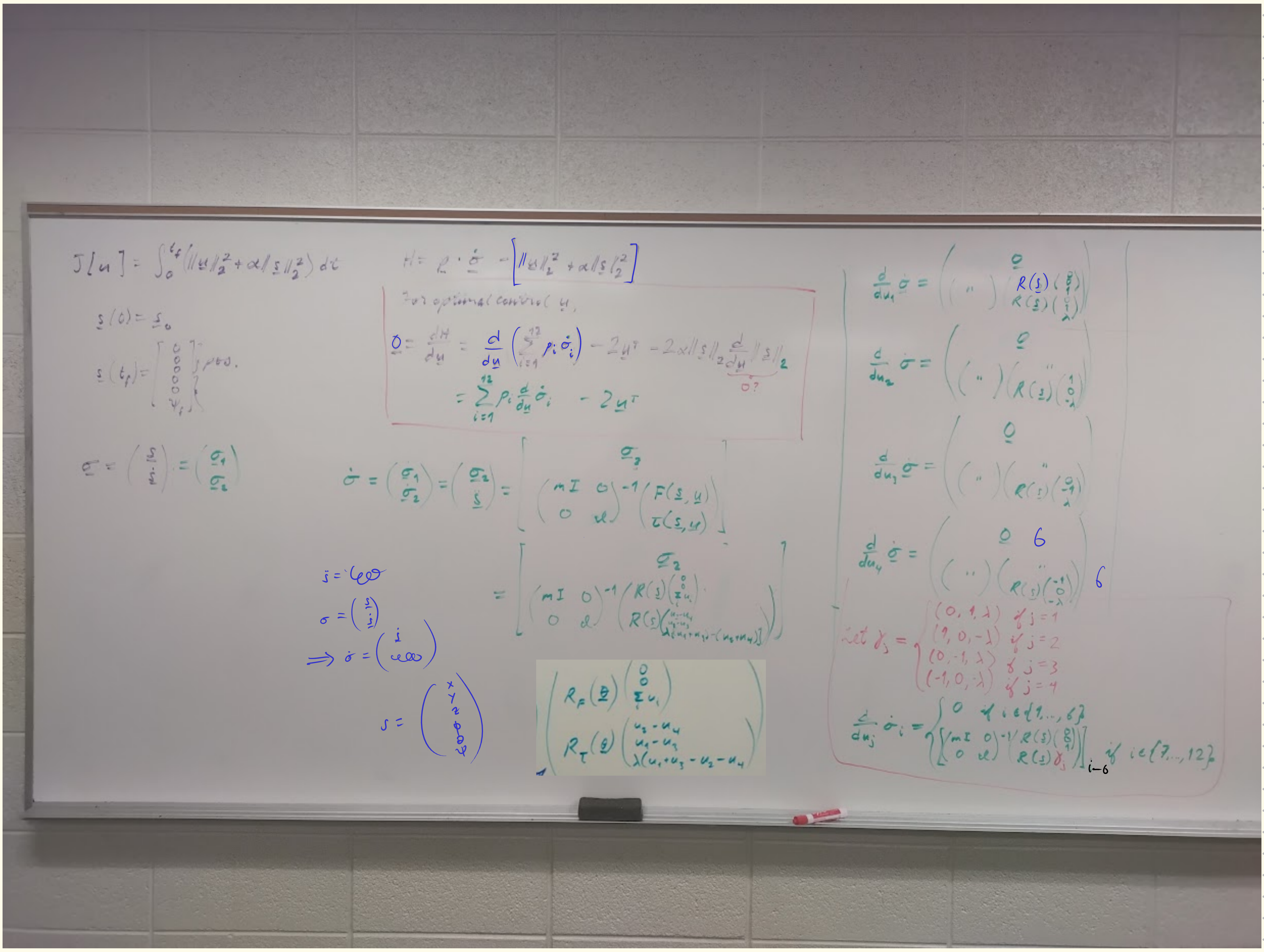


$$\ddot{\underline{s}} = \begin{pmatrix} mI & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} R_F(\underline{\theta}) \begin{pmatrix} 0 \\ 0 \\ \sum_i u_i \end{pmatrix} \\ R_\tau(\underline{\theta}) \begin{pmatrix} u_2 - u_4 \\ u_1 - u_3 \\ \lambda(u_1 + u_3 - u_2 - u_4) \end{pmatrix} \end{pmatrix}$$

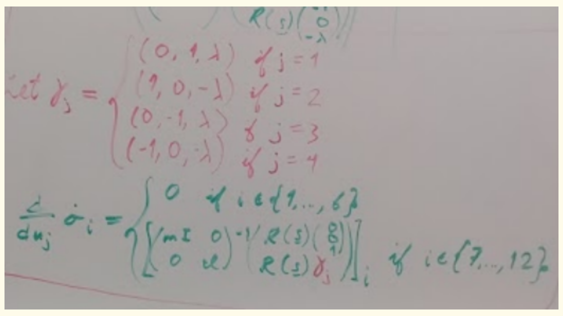
R_F and R_τ depend on $\underline{s}$. Not linear

Start with small angle approximations on rotation matrices

"Convert between inertial and non-inertial reference frames



$$2\underline{u}^T = \sum_{i=1}^{12} p_i \frac{d}{d\underline{u}} \dot{\sigma}_i$$



$$\Rightarrow \begin{cases}
u_1 = \frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1} \begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix} \\ R(\underline{s})\begin{pmatrix}0\\1\\\lambda\end{pmatrix}\end{pmatrix}\right]_{i-6} \\
u_2 = \frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1} \begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix} \\ R(\underline{s})\begin{pmatrix}1\\0\\-\lambda\end{pmatrix}\end{pmatrix}\right]_{i-6} \\
u_3 = \frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1} \begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix} \\ R(\underline{s})\begin{pmatrix}0\\-1\\\lambda\end{pmatrix}\end{pmatrix}\right]_{i-6} \\
u_4 = \frac{1}{2}\sum_{i=7}^{12} \underset{\uparrow\ \text{costate}}{p_i} \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1} \begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix} \\ R(\underline{s})\begin{pmatrix}-1\\0\\-\lambda\end{pmatrix}\end{pmatrix}\right]_{i-6}
\end{cases}$$

$$\underset{\uparrow\ \text{state}}{\underline{\sigma}} = \begin{pmatrix}\underline{s}\\ \dot{\underline{s}}\end{pmatrix}$$

Now I have $u(\underline{\sigma}, \underline{p})$.

Need evolution of costate.

$$H = \underline{p} \cdot \dot{\underline{\sigma}} - \left[\| \underline{u} \|_2^2 + \alpha \| \underline{s} \|_2^2 \right]$$

$$H = \underline{p} \cdot \dot{\underline{\sigma}} - \left[\| \underline{u} \|_2^2 + \alpha \| \underline{s} \|_2^2 \right]$$

$$\underline{p}' = - \frac{DH}{D\underline{\sigma}}$$

$$\sigma = \begin{pmatrix} \underline{s} \\ \dot{\underline{s}} \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \\ \text{der.} \\ \\ \end{pmatrix}$$

$$\frac{DH}{D\sigma_i} = \frac{D}{D\sigma_i} \left(- \| \underline{u} \|_2^2 - \alpha \| \underline{s} \|_2^2 \right)$$

$$= \begin{cases} 0 & \text{if } i \geq 7 \\ \text{something} & \text{otherwise} \end{cases}$$

So let's say $k < 7$. k instead of i

$$\textcircled{1} \quad \frac{\partial}{\partial \underline{\sigma}_k}\left(-\|\underline{u}\|_2^2\right) \qquad \underline{\sigma}_k = \begin{pmatrix} \underline{s} \\ \dot{\underline{s}} \end{pmatrix} \qquad \underline{s} = \begin{pmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{pmatrix}$$

$$= -\frac{\partial}{\partial \sigma_k}\left(\|\underline{u}\|_2^2\right)$$

$$= -\frac{\partial}{\partial s_k}\Bigg[\left(\frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ R(\underline{s})\begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix}\end{pmatrix}\right]_{i-6}\right)^2$$

$$+\left(\frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ R(\underline{s})\begin{pmatrix} 1 \\ 0 \\ -\lambda \end{pmatrix}\end{pmatrix}\right]_{i-6}\right)^2$$

$$+\left(\frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \\ R(\underline{s})\begin{pmatrix} 0 \\ -1 \\ \lambda \end{pmatrix}\end{pmatrix}\right]_{i-6}\right)^2$$

$$+\left(\frac{1}{2}\sum_{i=7}^{12} p_i \left[\begin{pmatrix} mI & 0 \\ 0 & \mathcal{I} \end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ R(\underline{s})\begin{pmatrix} -1 \\ 0 \\ -\lambda \end{pmatrix}\end{pmatrix}\right]_{i-6}\right)^2\Bigg]$$

$$= -\frac{\partial}{\partial s_k}\sum_{j=1}^{4}\left(\frac{1}{2}\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right)^2$$

$$= -\sum_{j=1}^{4}\frac{\partial}{\partial s_k}\left(\frac{1}{2}\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right)^2$$

$$= -\sum_{j=1}^{4}\left(\left[\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right]\frac{\partial}{\partial s_k}\left(\frac{1}{2}\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right)\right)$$

$$= -\sum_{j=1}^{4}\left(\left[\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right]\left(\frac{1}{2}\sum_{i=7}^{12} p_i\left[\begin{pmatrix} mI & 0\\ 0 & \ell\end{pmatrix}^{-1}\begin{pmatrix} \frac{\partial}{\partial s_k}R(\underline{s})\begin{pmatrix}0\\0\\1\end{pmatrix}\\ \frac{\partial}{\partial s_k}R(\underline{s})\,\gamma_j\end{pmatrix}\right]_{i-6}\right)\right)$$

$$\frac{d}{ds_k} R(\underline{s}) = \frac{d}{ds_k} \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}$$

$=$

$\underline{0}$ if $k \in \{1, 2, 3\}$

```
In[5]:= D[rotation[ϕ, θ, ψ], ϕ] // MatrixForm
Out[5]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 \\ \mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta] + \mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\phi] + \mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\psi] & \mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi] \\ -2\,\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi] + \mathrm{Cos}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi] - \mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\phi] \end{pmatrix}$$

if $k = 4$

```
In[6]:= D[rotation[ϕ, θ, ψ], θ] // MatrixForm
Out[6]//MatrixForm=
```

$$\begin{pmatrix} -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta] & -\mathrm{Sin}[\theta]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\theta] \\ \mathrm{Cos}[\theta]\,\mathrm{Cos}[\psi]\,\mathrm{Sin}[\phi] & \mathrm{Cos}[\theta]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi] \\ \mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]^2 & \mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta] \end{pmatrix}$$

if $k = 5$

```
In[7]:= D[rotation[ϕ, θ, ψ], ψ] // MatrixForm
Out[7]//MatrixForm=
```

$$\begin{pmatrix} -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\psi] & \mathrm{Cos}[\theta]\,\mathrm{Cos}[\psi] & 0 \\ -\mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi] - \mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & \mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi] - \mathrm{Cos}[\phi]\,\mathrm{Sin}[\psi] & 0 \\ \mathrm{Cos}[\psi]\,\mathrm{Sin}[\phi] & \mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta] + \mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & 0 \end{pmatrix}$$

if $k = 6$

$$\frac{\partial}{\partial \sigma_k}\left(-\alpha \|\underline{s}\|_2^2\right) \qquad 2$$

$$= \frac{\partial}{\partial \sigma_k} - \alpha\left(\sum_{i=1}^{6} s_i^2\right)$$

$$= -\alpha \sum_{i=1}^{6} \frac{\partial}{\partial \sigma_k} s_i^2 \qquad \underline{\sigma} = \left(\underline{s} : \underline{s}\right)$$

$$= -\alpha \sum_{i=1}^{6} \frac{\partial}{\partial \sigma_k} \sigma_i^2$$

$$= \begin{cases} -2\alpha\sigma_k & \text{if } k < 7 \\ 0 & \text{if } k \geq 7 \end{cases}$$

So

$$\underline{p}' = -\frac{DH}{D\underline{\sigma}} = -\frac{D}{D\sigma_i}\left(-\|\underline{u}\|_2^2 - \alpha\|\underline{s}\|_2^2\right)$$

depends only on $\underline{\sigma}$, $\underline{p}$.

solve 6vp