$$\sigma = (p, q, r, \psi, \theta, \phi, u, v, w, x, y, z)$$

$$\dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{L}{I_x}(F_3 + F_4 - F_1 - F_2)$$

$$\dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{L}{I_y}(F_1 + F_3 - F_2 - F_4)$$

$$\dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{1}{I_z}(\tau_2 + \tau_3 - \tau_1 - \tau_4)$$

$$\dot{\psi} = \frac{1}{c\theta}(q s\phi + r c\phi)$$

$$\dot{\theta} = q c\phi - r s\phi$$

$$\dot{\phi} = \frac{1}{c\theta}(q s\theta s\phi + r s\theta c\phi) + p$$

$$\dot{u} = rv - qw - g \sin\theta$$

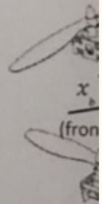
$$\dot{v} = pw - ru + g \cos\theta \sin\phi$$

$$\dot{w} = qu - pv + g \cos\theta \cos\phi - (F_1 + F_2 + F_3 + F_4)/m$$

$$\dot{x} \quad \dot{p}_n = u c\phi c\psi + v(s\phi s\theta c\psi - c\phi s\psi) + w(c\phi s\theta c\psi + s\phi s\psi)$$

$$\dot{y} \quad \dot{p}_e = u c\theta s\psi + v(s\phi s\theta s\psi + c\phi c\psi) + w(c\phi s\theta s\psi - s\phi c\psi)$$

$$-\dot{z} \quad -\dot{h} = -u s\theta + v s\phi c\theta + w c\phi c\theta$$



$$F_i = u_i, \quad \tau_i = \lambda u_i \quad \lambda > 0$$

Let $\underset{\sim}{s} = (\psi, \theta, \phi, x, y, z) \subset \underset{\sim}{\sigma}$

$$J[u] = \int_0^{t_f} \left[\|\underset{\sim}{u}\|_2^2 + \alpha \|\underset{\sim}{s}\|_2^2 \right] dt$$

$$H = \underset{\sim}{p} \cdot \dot{\sigma} - \left(\|\underset{\sim}{u}\|_2^2 + \alpha \|\underset{\sim}{s}\|_2^2 \right)$$

$$\frac{d}{du_1} \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{bmatrix} \frac{L}{I_x}(-1) \\ \frac{L}{I_y}(1) \\ \frac{1}{I_z}(-\lambda) \end{bmatrix} \qquad \frac{d}{du_1} \begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \underset{\sim}{0}$$

$$\frac{d}{du_1} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{m} \end{pmatrix} \qquad \frac{d}{du_1} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \underset{\sim}{0}$$

Let $$\gamma_i = \begin{cases} (-1, \ 1, \ -\lambda) & \text{if } i=1 \\ (-1, \ -1, \ \lambda) & \text{if } i=2 \\ (1, \ 1, \ \lambda) & \text{if } i=3 \\ (1, \ -1, \ -\lambda) & \text{if } i=4 \end{cases}$$

So

$$\frac{d}{du_i}\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{bmatrix} \frac{L}{I_x} \\ \frac{L}{I_y} \\ \frac{1}{I_z} \end{bmatrix} \odot \gamma_i \qquad \frac{d}{du_i}\begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \underline{0}$$

$$\frac{d}{du_i}\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{m} \end{pmatrix} \qquad \frac{d}{du_i}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \underline{0}$$

$$\text{and} \quad \frac{d}{du_i}\dot{\sigma} = \begin{pmatrix} \begin{bmatrix} \frac{L}{I_x} \\ \frac{L}{I_y} \\ \frac{1}{I_z} \end{bmatrix} \odot \gamma_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{m} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

1

For the optimal control $\underline{u}$,

$$\frac{dH}{d\underline{u}} = \underline{p}\cdot\frac{d}{d\underline{u}}\dot{\sigma} - 2\underline{u} = 0,$$

so $$\underline{u} = \frac{1}{2}\, p\cdot\underbrace{\frac{d}{d\underline{u}}\dot{\sigma}}_{1}$$

$$H = \underline{p}\cdot\dot{\sigma} - \left(\|\underline{u}\|_2^2 + \|\underline{s}\|_2^2\right)$$

Now deriving the costate equation with Pontryagin's Maximum Principle,

$$\dot{\underline{p}} = -\frac{DH}{D\underline{\sigma}}.$$

$$\frac{DH}{Dp} = p_2 \frac{I_z - I_x}{I_y} r + p_3 \frac{I_x - I_y}{I_z} q$$

$$+ p_6$$

$$+ p_8 w - p_9 v$$

$$- 0$$

$$\frac{DH}{Dq} = p_1 \frac{I_y - I_z}{I_x} r + p_3 \frac{I_x - I_y}{I_z} p$$

$$+ p_4 \frac{s\phi}{c\theta} + p_5 c\phi + p_6 \frac{s\theta s\phi}{c\theta}$$

1 $\dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{L}{I_x}(F_3 + F_4 - F_1 - F_2)$

2 $\dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{L}{I_y}(F_1 + F_3 - F_2 - F_4)$

3 $\dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{1}{I_z}(\tau_2 + \tau_3 - \tau_1 - \tau_4)$

4 $\dot{\psi} = \frac{1}{c\theta}(q s\phi + r c\phi)$

5 $\dot{\theta} = q c\phi - r s\phi$

6 $\dot{\phi} = \frac{1}{c\theta}(q s\theta s\phi + r s\theta c\phi) + p$

7 $\dot{u} = rv - qw - g \sin\theta$

8 $\dot{v} = pw - ru + g \cos\theta \sin\phi$

9 $\dot{w} = qu - pv + g \cos\theta \cos\phi - (F_1 + F_2 + F_3 + F_4)/m$

10 $\dot{p}_n = u c\phi c\psi + v(s\phi s\theta c\psi - c\phi s\psi) + w(c\phi s\theta c\psi + s\phi s\psi)$

11 $\dot{p}_e = u c\theta s\psi + v(s\phi s\theta s\psi + c\phi c\psi) + w(c\phi s\theta s\psi - s\phi c\psi)$

12 $-\dot{h} = -u s\theta + v s\phi c\theta + w c\phi c\theta$