The transition probability between two states is related to the off-diagonal matrix elements of the perturbation Hamiltonian. Since our perturbation Hamiltonian is proportial to the identity operator, there are no off-diagonal matrix elements, making the transition probability from an eigenstate of $\hat{H}_0$ to some other state $\left|m^{(0)}\right\rangle$ zero.

Using the interaction picture, the time evolution for a state is given by

$$\begin{aligned}
i\hbar\frac{d}{dt}|\Psi\rangle_I &= i\hbar\frac{d}{dt}\left(\hat{U}_0^\dagger|\Psi\rangle_S\right) \\
&= i\hbar\frac{i}{\hbar}\hat{H}_0\hat{U}_0^\dagger|\Psi\rangle_S + \hat{U}_0^\dagger\hat{H}|\Psi\rangle_S \\
&= \left(-\hat{H}_0 + \hat{U}_0^\dagger\hat{H}\hat{U}_0\right)\hat{U}_0^\dagger|\Psi\rangle_S \\
&= \hat{U}_0^\dagger\left(-\hat{H}_0 + \hat{H}\right)\hat{U}_0|\Psi\rangle_i \\
&= \hat{U}_0^\dagger\hat{V}\hat{U}_0|\Psi\rangle_I
\end{aligned}$$

where we use the following definitions:

$$|\Psi\rangle_I = \hat{U}_0^\dagger|\Psi\rangle_S,$$

$$\hat{O}_I(t) = \hat{U}_0^\dagger(t,t_0)\hat{O}_S\hat{U}_0(t,t_0)$$

$$\hat{U}_0 = e^{-\frac{i(t-t_0)}{\hbar}\hat{H}_0}$$

Just for fun:

$$\frac{\sqrt{\hat{a}^2+\hat{b}^2}+\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n!}\right)\ln\left(\frac{n^2+\hat{\pi}}{e^{\hat{\gamma}}}\right)}{\int_0^{\hat{\aleph}_9+42}x^{\hat{a}-1}(1-\hat{x})^{\hat{\beta}-1}dx}+\prod_{k=1}^{\infty}\left(1-\frac{\hat{\Xi}}{k^2\hat{\xi}}\right)$$

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