PHYS 131: MECHANICS I

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.

-Rigid-body Mechanics

-Statics

-Dynamics

-Deformable-body mechanics, and

-Fluid Mechanics

Statics: deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).

Dynamics: deals with motion of bodies (accelerated motion)

Fundamental Concepts

Length (Space): needed to locate position of a point in space, & describe size of the physical system Distances, Geometric Properties

Time: measure of succession of events basic quantity in Dynamics

Mass: quantity of matter in a body measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another characterized by its magnitude, direction of its action, and its point of application Force is a Vector quantity.

Newtonian Mechanics Length, Time, and Mass are absolute concepts independent of each other Force is a derived concept not independent of the other fundamental concepts. Force acting on a body is related to the mass of the body and the variation of its velocity with time. Force can also occur between bodies that are physically separated (Ex: gravitational, electrical, and magnetic forces)

Mass is a property of matter that does not change from one location to another. • Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located • Weight of a body is the gravitational force acting on it.

Rigid Body: A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load. Material properties of a rigid body are not required to be considered when analyzing the forces acting on the body. In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis Mechanics: Idealizations Concentrated Force: Effect of a loading which is assumed to act at a point (CG) on a body. • Provided the area over which the load is applied is very small compared to the overall size of the body.

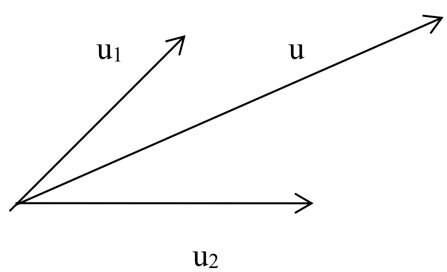
SCALARS AND VECTORS

Scalars: only magnitude is associated.

Ex: time, volume, density, speed, energy, mass

Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law). e.g displacement, velocity, acceleration, force, moment, momentum

Speed is the magnitude of velocity.



A Vector V can be written as: V = Vn

V = magnitude of V

n = unit vector whose magnitude is one and whose direction coincides with that of V. Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

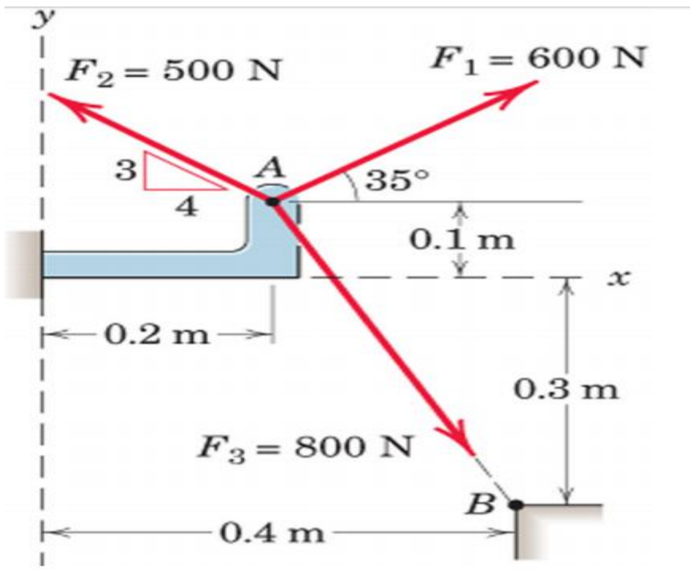
Vectors represented by Bold and Non-Italic letters (V). Magnitude of vectors represented by Non-Bold, Italic letters (V)

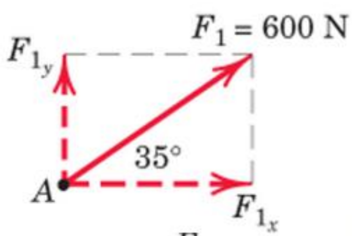
Components and Projections of Force

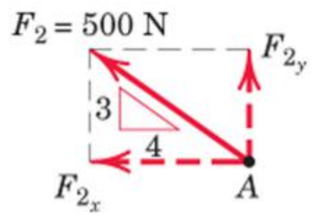
Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

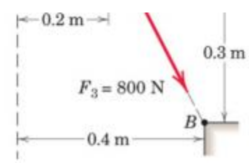
Example 1:

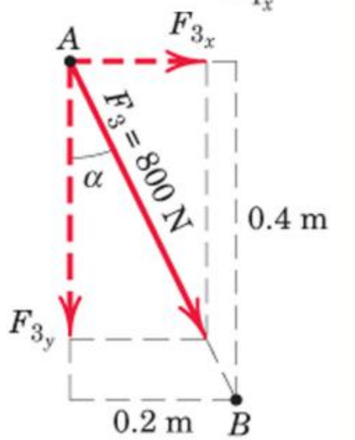
Determine the x and y scalar components of F 1, F2, and F3 acting at point A of the bracket











$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

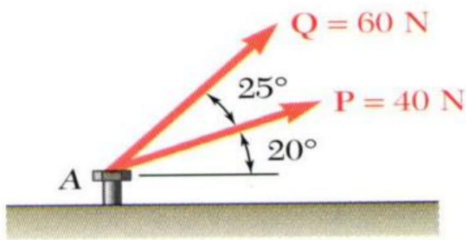
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

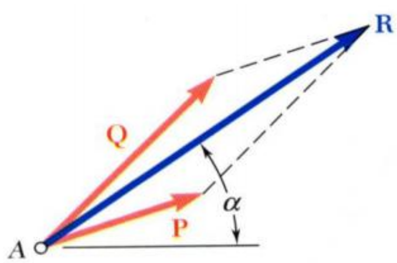
Example 2: The two forces act on a bolt at A. Determine their resultant.



Graphical solution - construct a parallelogram with sides in the same direction as P and Q and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

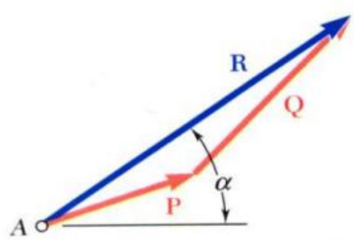
Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Solution:



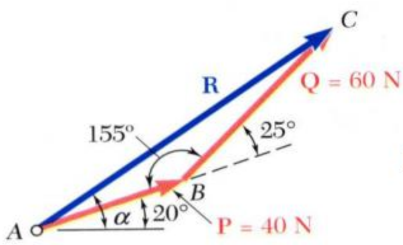
- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\boxed{\mathbf{R} = 98\text{ N} \quad \alpha = 35°}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\text{ N} \quad \alpha = 35°}$$



From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\,\mathrm{N})^2 + (60\,\mathrm{N})^2 - 2(40\,\mathrm{N})(60\,\mathrm{N})\cos 155°$$

$$\boxed{R = 97.73\,\mathrm{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin A = \sin B\,\frac{Q}{R}$$
$$= \sin 155°\,\frac{60\,\mathrm{N}}{97.73\,\mathrm{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$$

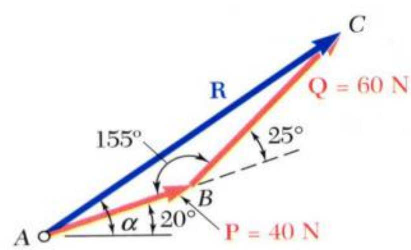
$$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$$
$$= 37.58\mathbf{i} + 13.68\mathbf{j}$$

$$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$$
$$= 42.43\mathbf{i} + 42.43\mathbf{j}$$

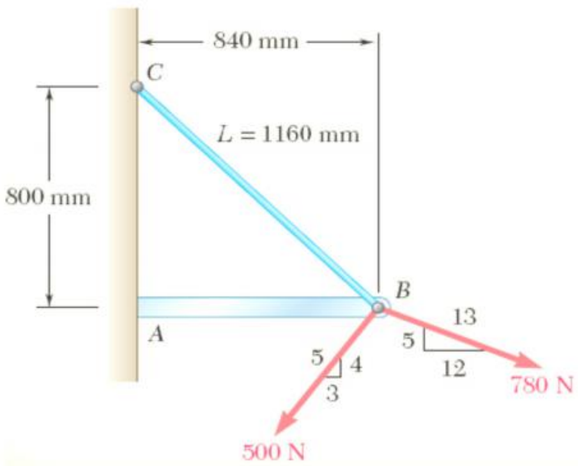
$$\boldsymbol{R} = 80.01\boldsymbol{i} + 56.10\boldsymbol{j}$$

$$R = 97.72$$

$$\alpha = 35.03^o$$

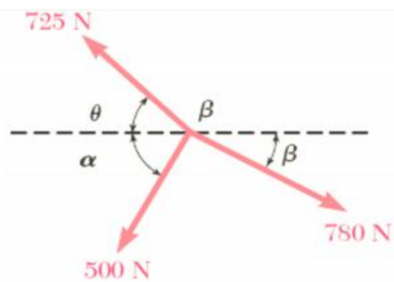


Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.

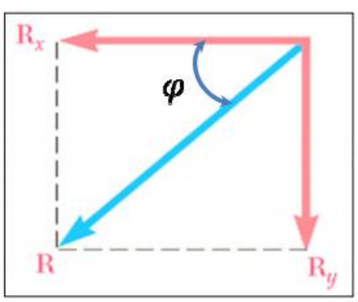


Solution:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.



Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



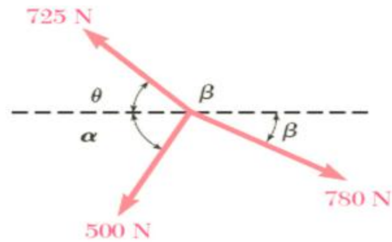
$$\mathbf{R} = R_x i + R_y j \qquad \mathbf{R} = (-105)i + (-200)j$$

Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62.3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Components of Force



Alternate solution

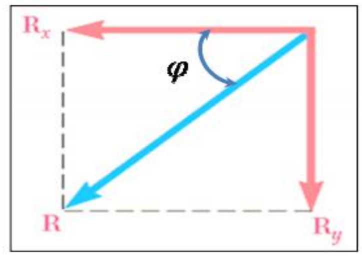
$$\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$\boldsymbol{F_1} = 725[-0.724\boldsymbol{i} + 0.689\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 500[-0.6\boldsymbol{i} - 0.8\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 780[0.923\boldsymbol{i} - 0.384\boldsymbol{j}]$$

$$\boldsymbol{R} = -105\boldsymbol{i} - 200\boldsymbol{j}$$



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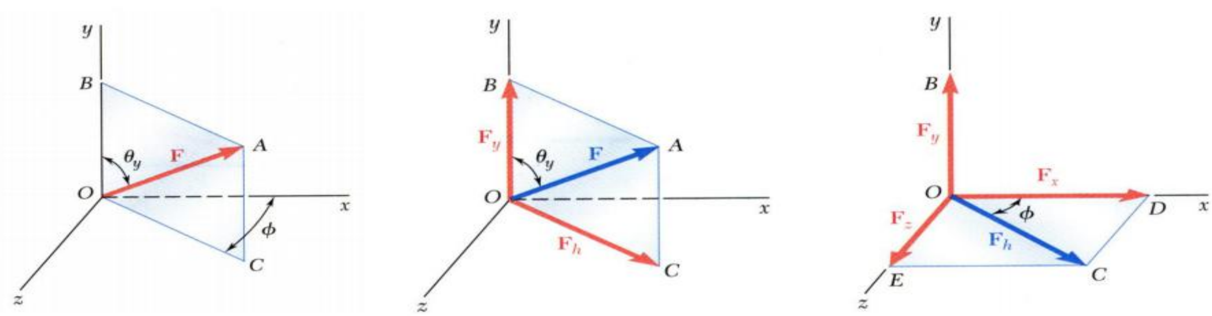
Rectangular Components in Space



- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

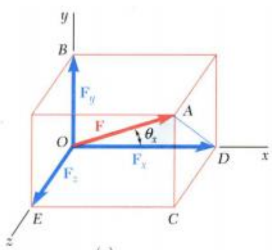
$$F_y = F\cos\theta_y$$

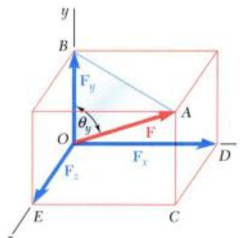
$$F_h = F\sin\theta_y$$

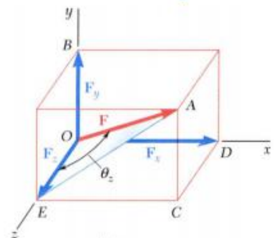
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$







$F_x = F cos\theta_x$ $\quad$ $F_y = F cos\theta_y$ $\quad$ $F_z = F cos\theta_z$

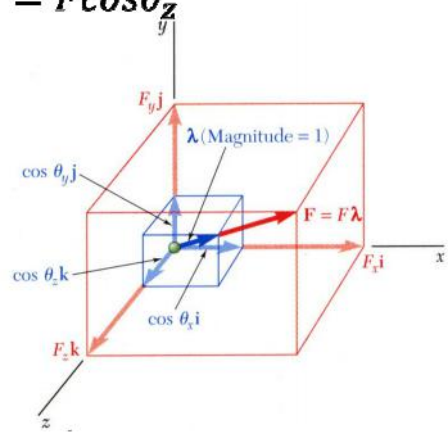
$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$

$\boldsymbol{F} = F cos\theta_x\boldsymbol{i} + F cos\theta_y\boldsymbol{j} + F cos\theta_z\boldsymbol{k}$

$\boldsymbol{F} = F\left(cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}\right)$

$\boldsymbol{F} = F\boldsymbol{\lambda}$

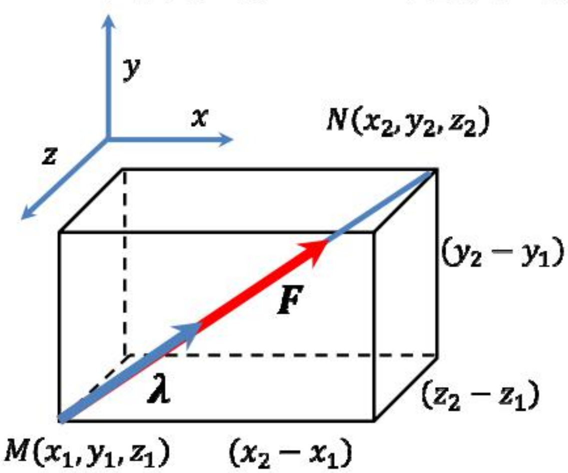
Where $\boldsymbol{\lambda} = cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}$



$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $cos\theta_x$, $cos\theta_y$ and $cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$

$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$

$d_z = (z_2 - z_1)$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

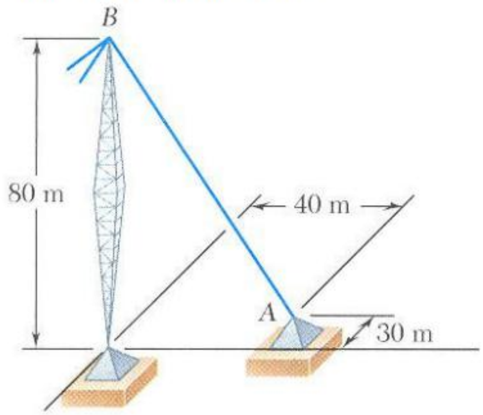
$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Example: The tension in the guy wire is 2500 N. Determine:

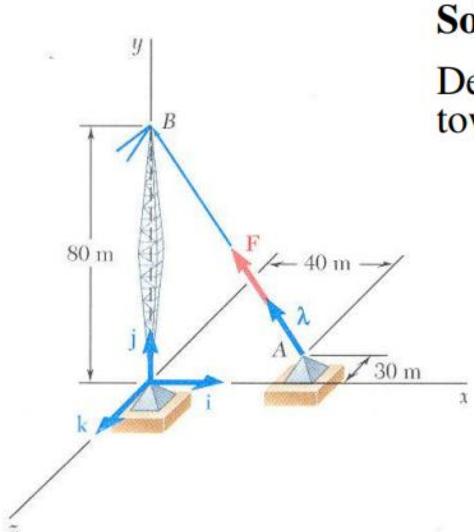
a) components F_x, F_y, F_z of the force acting on the bolt at A,

b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.



Solution

Determine the unit vector pointing from A towards B.

$$\boldsymbol{AB} = -40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}}{94.3}$$

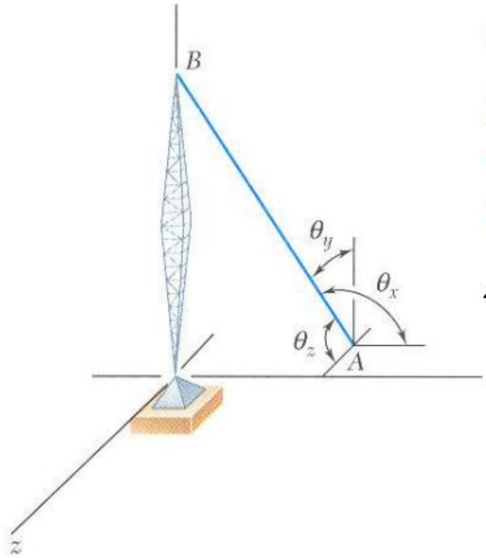
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \quad = 2500(-0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$



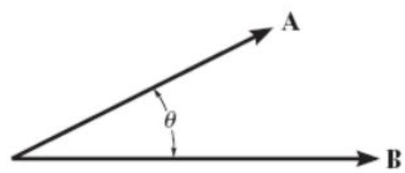
Solution

Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned}\theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ\end{aligned}}$$

Vector Products



Dot Product $\boldsymbol{A}.\boldsymbol{B} = ABcos\theta$

Applications:

to determine the angle between two vectors
to determine the projection of a vector in a specified direction

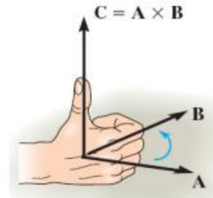
A.B = ***B.A*** (commutative)
A.(B+C) = ***A.B+A.C*** (distributive operation)

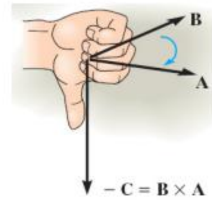
$$\boldsymbol{A}.\boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}).(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}) \qquad \boldsymbol{i}.\boldsymbol{i} = 1$$

$$= A_xB_x + A_yB_y + A_zB_z \qquad \boldsymbol{i}.\boldsymbol{j} = 0$$

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = ABsin\theta$

$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$

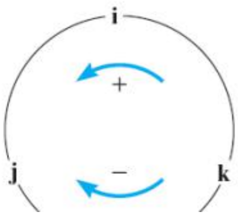


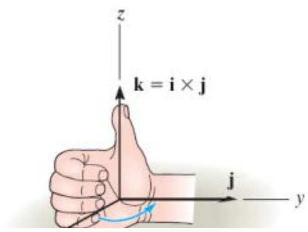


$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}) \times (B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} (A_yB_z - A_zB_y)\boldsymbol{i} + (A_zB_x - A_xB_z)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}$$

Cartesian Vector



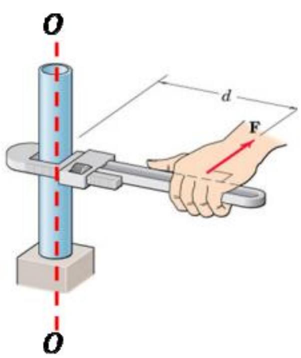


$\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$

$\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\quad \mathbf{j} \times \mathbf{i} = -\mathbf{k}$ $\quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$

$\mathbf{k} \times \mathbf{i} = \mathbf{j}$ $\quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$ $\quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$

Moment of a Force (Torque)

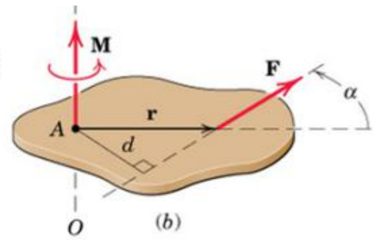


Moment about axis O-O is $\boldsymbol{M_o} = Fd$

Magnitude of $\boldsymbol{M_o}$ measures tendency of **F** to cause rotation of the body about an axis along $\boldsymbol{M_o}$.

O

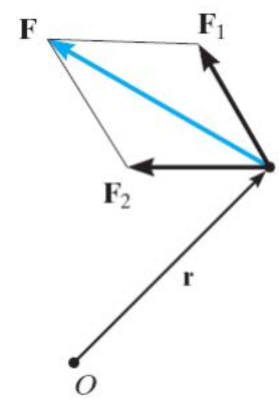
Moment about axis O-O is $\boldsymbol{M_o} = Fr\sin\alpha$



$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

Sense of the moment may be determined by the right-hand rule

produces the same moment.



Varignon's Theorem
(Principle of Moments)

Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

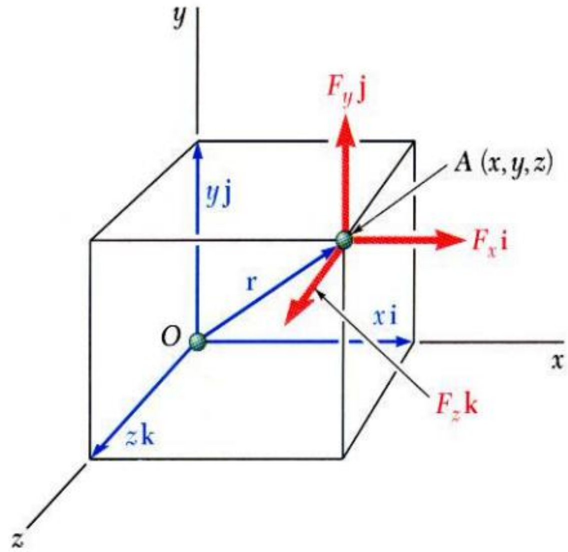
Rectangular components of moments

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$



$$\boldsymbol{M_o} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

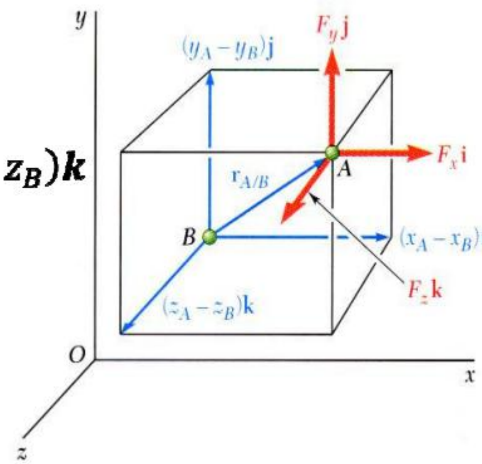
$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M_B} = \boldsymbol{r_{AB}} \times \boldsymbol{F}$$

$$\boldsymbol{r_{AB}} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

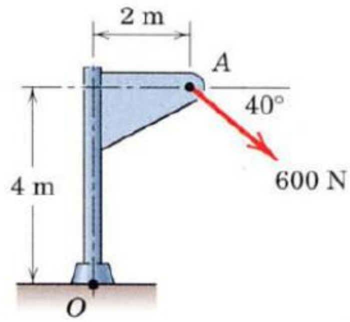


$$\boldsymbol{M_B} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

Moment: Example

Calculate the magnitude of the moment about the base point O of the 600 N force in different ways

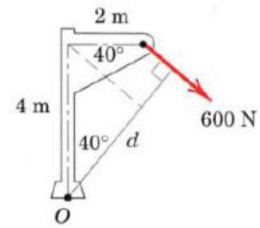


Solution 1.

Moment about O is

$$M_o = dF \quad d = 4cos40^o + 2sin40^o = 4.35m$$

$$M_o = 600(4.35) = 2610\, N.m \ \textbf{Ans}$$

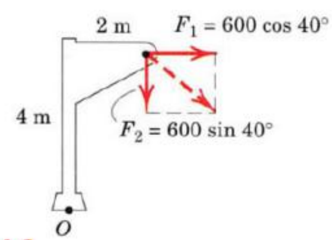


Solution 2.

$$F_x = 600cos40^o = 460\ N$$

$$F_x = 600sin40^o = 386\ N$$

$$M_o = 460(4.00) + 386(2.00) = 2610\, N.m \ \textbf{Ans}$$

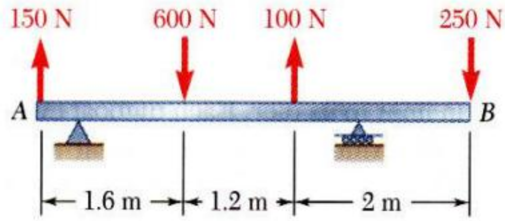


Equivalent Systems: Resultants

Equilibrium

Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

Equivalent Systems: Example



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

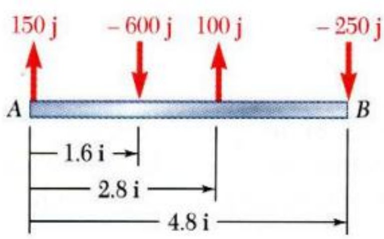
Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

Solution:

a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.



(a) Compute the resultant force and the resultant couple at *A*.

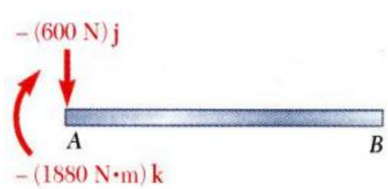
$$\boldsymbol{R} = \sum \boldsymbol{F} = 150\boldsymbol{j} - 600\boldsymbol{j} + 100\boldsymbol{j} - 250\boldsymbol{j}$$

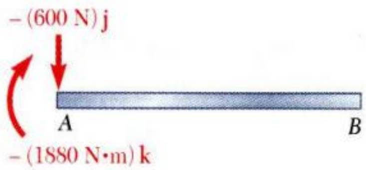
$$\boldsymbol{R} = -(600N)\boldsymbol{j}$$

$$\boldsymbol{M}_A^R = \sum \boldsymbol{r} \times \boldsymbol{F}$$

$$= 1.6\boldsymbol{i} \times (-600\boldsymbol{j}) + 2.8\boldsymbol{i} \times (100\boldsymbol{j}) + 4.8\boldsymbol{i} \times (-250\boldsymbol{j})$$

$$\boldsymbol{M}_A^R = -(1880\ N.m)\boldsymbol{k}$$



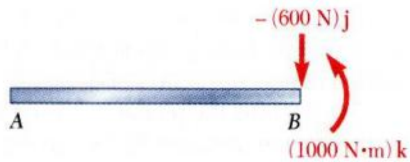


b) Find an equivalent force-couple system at B based on the force-couple system at A.

The force is unchanged by the movement of the force-couple system from A to B.

$$\boldsymbol{R} = -(600N)\boldsymbol{j}$$

The couple at B is equal to the moment about B of the force-couple system found at A.



$$\boldsymbol{M}_{\boldsymbol{B}}^{\boldsymbol{R}} = \boldsymbol{M}_{\boldsymbol{A}}^{\boldsymbol{R}} + \boldsymbol{r}_{\boldsymbol{BA}} \times \boldsymbol{R}$$

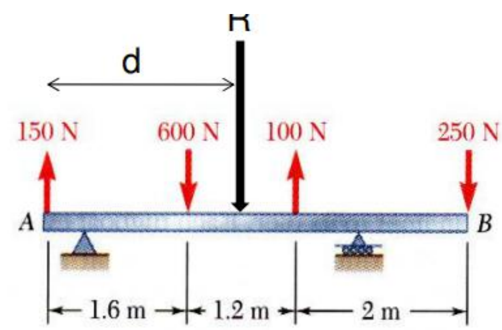
$$= -1800\boldsymbol{k} + (-4.8\boldsymbol{i}) \times (-600\boldsymbol{j})$$

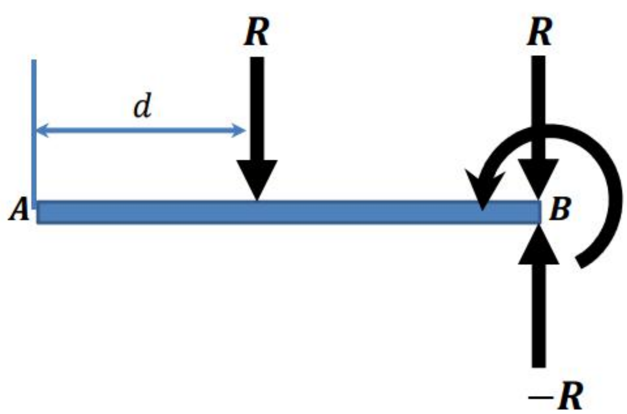
$$= (1000N.m)\boldsymbol{k}$$

$$R = 150 - 600 + 100 - 250 = -600\ N$$

$$Rd = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$$

$$d = 3.13\ m$$





MOTION IN ONE DIMENSION

The part of mechanics that describes motion without regard to its causes is called kinematics.

Here we will focus on one dimensional motion.

Position

To describe the motion of an object, one must be able to specify its position at all time using some convinient coordinate system.

For example, a particle might be located at x=+5 m, which means that it is 5 m in the positive direction from the origin. If it were at x=-5 m, it would be just as far from the origin but in the opposite direction.

Position is an example of a vector quantity, i.e. the physical quantity that requires the specification of both direction and magnitude. By contrast, a scalar is quantity that has magnitude and no direction.

Displacement

A change from one position to another position is called displacement

For examples, if the particles moves from +5 m to +12 m, then the + sign indicates that the motion is in the positive direction. The plus sign for vector quantity need to be shown, but a minus sign must always be shown.

The displacement is a vector quantity

Several quantities are associated with the phrase "how fast". One of them is the average velocity which is the ratio of the displacement that occurs during a particular time interval to that interval:

$$V = \frac{\Delta x}{\Delta t}$$

A common unit of velocity is the meter per second (m/s)

Example 1

- A renger in a national park is driving at 60 km/h when a deer jumps into the road 50 m ahead of the vehicle. After a reaction time of t_1, the ranger applies the brakes to produce an acceleration of $a = -3\ \text{m/s}^2$. What is the maximum reaction time allowed if she is to avoid hitting the deer?

  $\vdash$

  $$\begin{array}{l} v_0 = +60\ \text{km/h} = +16.7\ \text{m/s} \\ l = 50\ \text{m} \\ a = -3\ \text{m/s}^2 \\ \hline t_1 = ? \end{array}$$

  $$\begin{aligned} t_2 &= \frac{\Delta v}{a} = \frac{-v_0}{a} \\ &= 5.56\ \text{s} \\ l_1 &= v_0 t_1 \\ l_2 &= v_0 t_2 + \frac{1}{2} a t_2^2 \\ l &= l_1 + l_2 = v_0(t_1 + t_2) + \frac{1}{2} a t_2^2 \\ t_1 &= \frac{l - \frac{1}{2} a t_2^2 - v_0 t_2}{v_0} \\ &= 0.22\ \text{s} \end{aligned}$$

Example 2

- A peregrine falcon dives at a pigeon. The falcon starts downward from rest and falls with free-fall acceleration. If the pigeon is 76 m below the initial position of the falcon, how long does it take the falcon to reach the pigeon? Assume that the pigeon remains at rest.

  $\vdash$

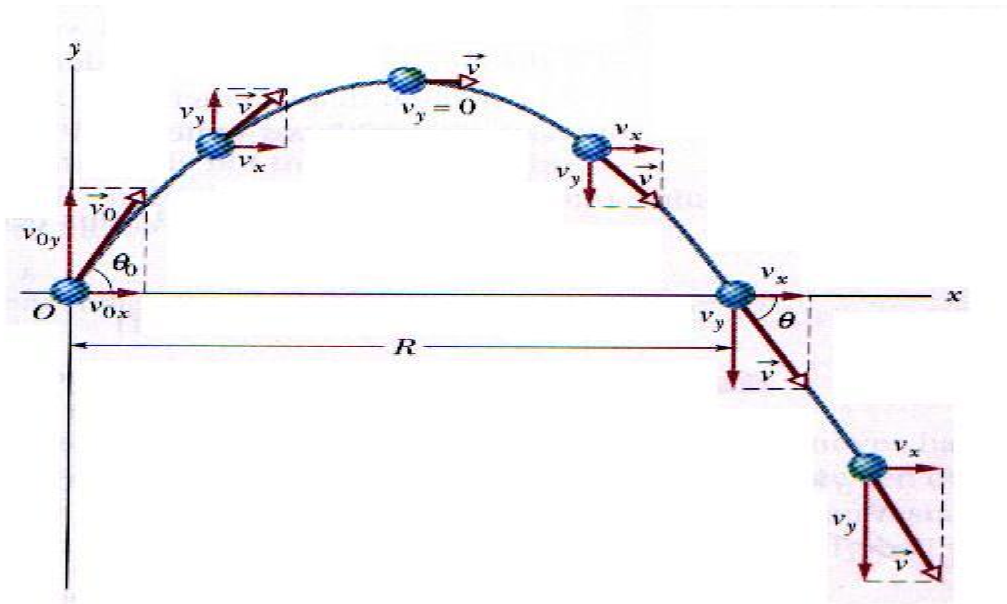
  $$\begin{array}{l} l = 76\ \text{m} \\ \hline t = ? \end{array}$$

  $$\begin{aligned} l &= \frac{1}{2} g t^2 \\ t &= \sqrt{\frac{2l}{g}} = \sqrt{\frac{2 \cdot 76\ \text{m}}{9.8\ \text{m/s}^2}} = 3.9\ \text{s} \end{aligned}$$

- In one-dimensional motion the vector nature of some physical quantities was taken into account through the use of positive (+) and negative (-) signs.
- In two-dimensional motion there are an infinity possibilities for the vector directions. So, we must make use of vectors.
- Position: $\vec{r} = x\hat{i} + y\hat{i} + z\hat{i}$
- Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$.
- Average velocity: $\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = v_x\hat{i} + v_y\hat{i} + v_z\hat{i}$, where $v_x = \frac{\Delta x}{\Delta t}$, $v_y = \frac{\Delta y}{\Delta t}$, $v_z = \frac{\Delta z}{\Delta t}$.
- Average acceleration: $\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} = a_x\hat{i} + a_y\hat{i} + a_z\hat{i}$, where $a_x = \frac{\Delta v_x}{\Delta t}$, $a_y = \frac{\Delta v_y}{\Delta t}$, $a_z = \frac{\Delta v_z}{\Delta t}$.
- Instantaneous velocity (instantaneous acceleration) is definied as the limit of the average velocity (average acceleration) when Δt goes to zero.

- We next consider a special case of two-dimensional motion: a particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the free-fall acceleration, which is downword. Such motion is called projectile motion. (It might be a golf ball in flight.)
- Initial velocity can be written as

$$\begin{aligned}\vec{v}_0 &= v_{x0}\hat{i} + v_{y0}\hat{j} \\ &= v_0\cos\theta_0\hat{i} + v_0\sin\theta_0\hat{j}\end{aligned}$$



- The horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.
- The acceleration in the x direction is 0 (air resistance is neglected), so v_{0x} remains constant, and horizontal position of the projectile is:

$$x = x_0 + v_{x0}t.$$

- The acceleration in the y direction is $-g$ and we have

$$\begin{aligned} y &= y_0 + v_{y0}t - \frac{1}{2}gt^2 \\ v_y &= v_{y0} - gt. \end{aligned}$$

- We can find the equation of the projectile's path (its trajectory) by eliminating t

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

For simplicity we let $x_0 = 0$ and $y_0 = 0$.

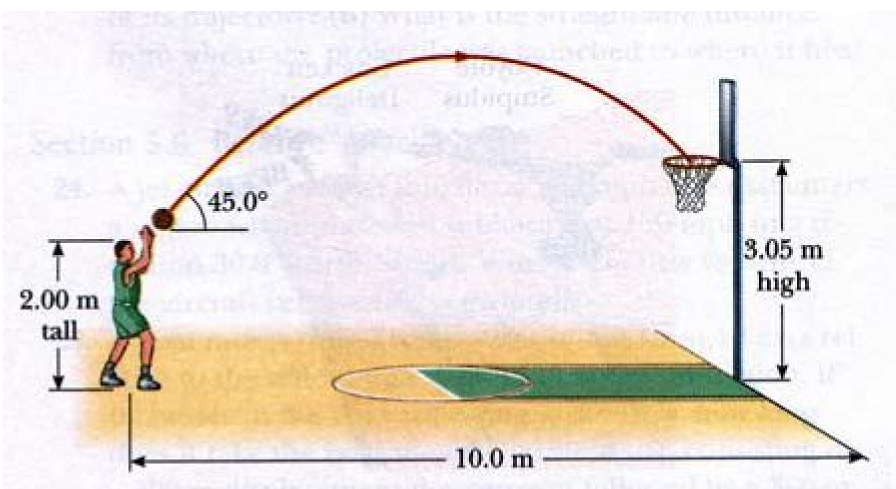
Example 3

- A 2.00-m-tall basketball player wants to make a goal from 10 m from the basket, as in Figure. If he shots the ball at a 45^0 angle, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard?

$\vdash$

$$\begin{aligned} x &= 10 \text{ m} \\ y &= 3.05 \text{ m} - 2 \text{ m} = 1.05 \text{ m} \\ \theta_0 &= 45^0 \\ \hline v_0 &= ? \end{aligned}$$

$$\begin{aligned} y &= (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2} \\ v_0 &= \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}} \\ &= 10 \text{ m/s} \end{aligned}$$



- A person walks first at a constant speed of 5 m/s along a stright line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s. What is her average speed over the entire trip and what is her average velocity over the entire trip?
  (A: 3.75 m/s; 0 m/s)
- A ball thrown vertically upward is caught by the thrower after 2 s. Find the initial velocity of the ball and the maximum height it reaches.
  (A: 9.8 m/s; 4.9 m)
- A parachutist with a camera, both descending at a speed of 10 m/s, releases that camera at an altitude of 50 m. How long does it take the camera to reach the ground, and what is the velocity of the camera just before hits the ground?
  (A: 2.33 s; -32.9 m/s)

Newton's Three Laws of Motion

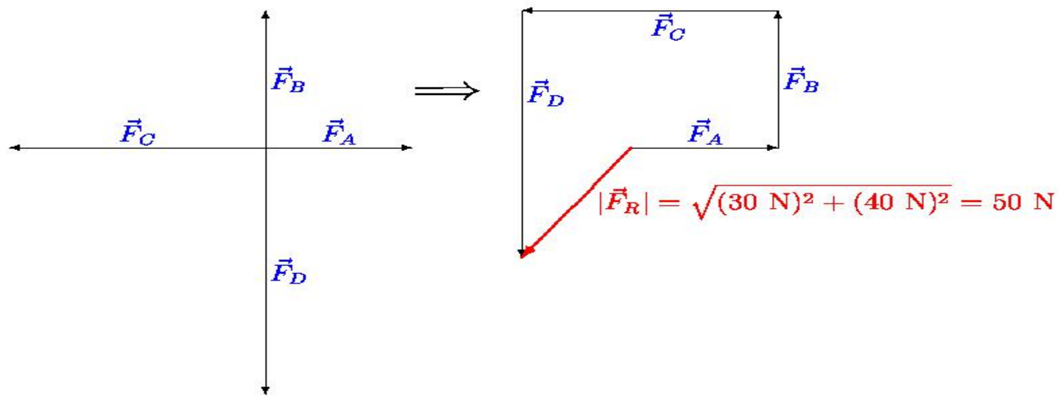
First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force. First law contains the principle of the equilibrium of forces main topic of concern in Statics Basis of formulation of rigid body mechanics.

Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force. m F = ma Second Law forms the basis for most of the analysis in Dynamics Mechanics:

Third law is basic to our understanding of Force Forces always occur in pairs of equal and opposite forces. Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.

Example 1

- Four forces act on an object, given by $\vec{F}_A = 40$ N east, $\vec{F}_B = 50$ N north, $\vec{F}_C = 70$ N west, and $\vec{F}_D = 90$ N south. What is the magnitude of the net force on the object?



Example 2

- A child holds a sled at rest on a frictionless, snow-covered hill. If sled weighs 77 N, find the force exerted on the rope by child and the force exerted on the sled by the hill.
  Applying the condition for equilibrium ($\vec{a} = 0$) to the sled, we find that

$$\begin{aligned} \sum F_x &= F_T - F_g \sin 30^0 = 0 \\ F_T &= F_g \sin 30^0 = (77\text{ N}) \sin 30^0 = 38.5\text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_y &= F_N - F_g \cos 30^0 = 0 \\ F_N &= F_g \cos 30^0 = 66.7\text{ N}. \end{aligned}$$

Example 3

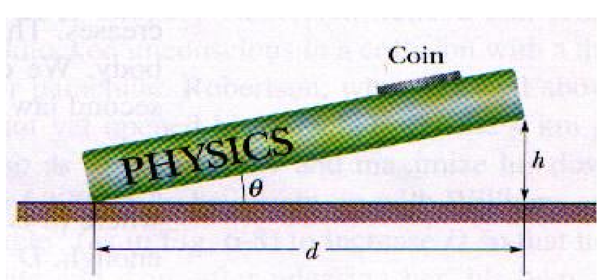
The force of the wind on the sails of a sailboat is 390 N north. The water exerts force of 180 N east. If the boat has a mass of 270 kg, what are the magnitude and direction of its acceleration?

$$
\begin{aligned}
F &= \sqrt{F_{wind}^2 + F_{water}^2} = \sqrt{(390\text{ N})^2 + (180\text{ N})^2} = 429.5\text{ N} \\
a &= \frac{F}{m} = 1.59\text{ m/s}^2 \\
\theta &= \arctan\frac{F_{wind}}{F_{water}} = 65.2^0\text{ north of east}
\end{aligned}
$$

Example 4

- A coin of mass m at rest on a book that has been tilted at an angle θ with the horizontal. By experiment, when θ is increased to 15^0, the coin is on the verge of sliding down the book, which means that even a slight increase beyond 15^0 produces sliding. What is the coefficient of static friction μ_s between the coin and the book?

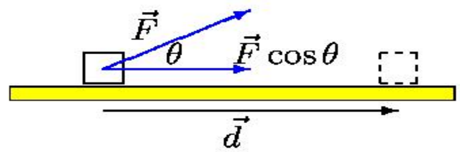
$$
\begin{aligned}
F_{s,max} &= \mu_s F_N \\
F_N &= mg\cos\theta \\
F_{s,max} &= mg\sin\theta \\
\mu_s mg\cos\theta &= mg\sin\theta \\
\mu_s &= \frac{mg\sin\theta}{mg\cos\theta} \\
&= \tan\theta = \tan 15^0 = 0.27
\end{aligned}
$$



WORK

The concept of energy is one of the most important in the world of science. In everyday use, the term energy has to do with the cost of fuel for transportation and heating, electricity for lights and appliances, and the foods we consume. Energy is present in the Universe in a variety of forms, including mechanical energy, chemical energy, electromagnetic energy, nuclear energy, and many others. Here we are concerned only with mechanical energy, and begin by defining work.

We see an object that undergoes a displacement of $\vec{d}$ along a stright line while acted on by a constant force, $\vec{F}$, that makes an angle of θ with $\vec{d}$.



The work W done on an object by a constant force $\vec{F}$ during a displacement is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement.

$$W = (F\cos\theta)d$$

As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy book at arm's length. After 5 minutes, your tired arms may lead yout to think that you have done a considerable amout of work. According to our definition, however, you have done no work on the book. Your muscles are continuosly contracting and relaxing while the book is being supported. Thus, work is being done on your body, but not on the book.

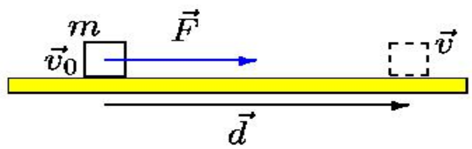
A force does no work on a object if the object does not move. The sign of the work depends on the angle θ between the force and displacement. Work is a scalar quantity, and its units is joule (1 J = 1 Nm)

For example, a man cleaning his apartment pulls the canister of a vacuum cleaner with a force of magnitude 50 N at an angle 30°. He moves the vacuum cleaner a distance of 3 m. Calculate the work done by the force.

$$\begin{aligned} W &= F\cos\theta\, d = (50\,\mathrm{N})(\cos 30^0)(3\,\mathrm{m}) \\ &= 130\,\mathrm{J} \end{aligned}$$

KINETIC ENERGY

Figure shows an object of mass m moving to the right under the action of a constant net force, $\vec{F}$.



The work done by $\vec{F}$ is

$$W = Fd = (ma)d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The quantity $\frac{1}{2}mv^2$ has a special name in physics: kinetic energy. Any object of mass m and speed v is defined to have a kinetic energy E_k, of

$$E_k = \frac{1}{2}mv^2$$

We see that it is possible to write W as $W = E_{k,2} - E_{k,1}$.

Example 1

- A car with mass of 1400 kg has a net forward force of 4500 N applied to it. The car starts from rest and travels down a horizontal highway. What are its kinetic energy and speed after it has traveled 100 m? (Ignore friction and air resistance.)

  The work done by the net force on the car is

$$W = Fd = (4500\ \mathrm{N})(100\ \mathrm{m}) = 4.5 \cdot 10^5\ \mathrm{J}$$

  This work all goes into changing the kinetic energy of the car, thus the final of the kinetic energy is also $E_k = 4.5 \cdot 10^5$ J.

  The speed of the car can be found from

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(4.5 \cdot 10^5\ \mathrm{J})}{1400\ \mathrm{kg}}} \\ &= 25.4\ \mathrm{m/s} \end{aligned}$$

CONSERVATIVE AND NONCONSERVATIVE FORCES

Conservative forces

A force is conservative if the work it does on an object moving between two points is independent of the path the object takes between the points. In other words, the work done on an object by a conservative force depends only on the initial and final positions of the object.

The gravitational force is conservative.

Nonconservative forces

A force is nonconservative if it leads to a dissipation of mechancal energy. If you moved an object on a horizontal surface, returning it to the same location and same state of motion, but found it necessary to do net work on the object, then something must have dissipated the energy transferred to the object. That dissipative force is recognized as friction between object and surface.

Friction force is a nonconservative force.

Newton's Law of Gravitational Attraction

$$F = \frac{Gm_1 m_2}{r^2}$$

F = mutual force of attraction between two particles

G = universal constant of gravitation Experiments G = 6.673x10^{-11} m^3/(kg.s^2). Rotation of Earth is not taken into account m^1, m^2 = masses of two particles, r = distance between two particles

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics. This law governs the gravitational attraction between any two particles. Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass m_1=m, assuming earth to be non rotating sphere of constant density

$$W = \frac{GM_e m}{r^2}$$

Let g = G Me /r ^{2} = acceleration due to gravity (9.81m/s^2)

$$W = mg$$

The Frictional Force

If we slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. The resistance is considered to be a single force called the frictional force

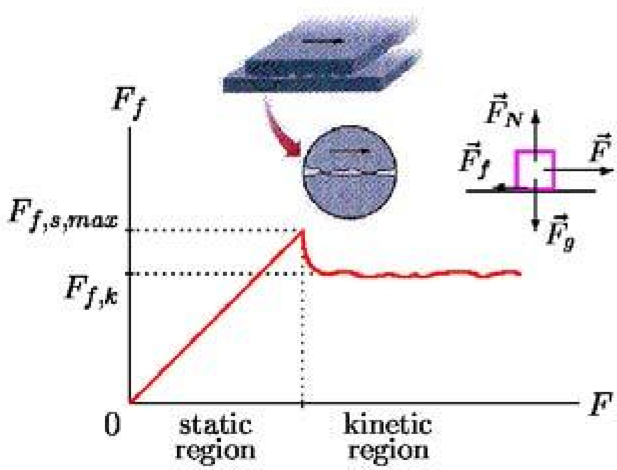
This force is very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

The frictional force is directed along the surface, opposite the direction of the intended motion.

For an object in motion the frictional force we call kinetic frictional force; otherwise static frictional force.

Both, kinetic and static frictional force are proportional to the normal force acting on the object where are coefficients of static and kinetic friction.

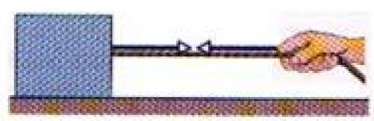
$F_{f,s,\max} = \mu_s F_N$ $F_{f,k} = \mu_k F_N$



The Tension Force

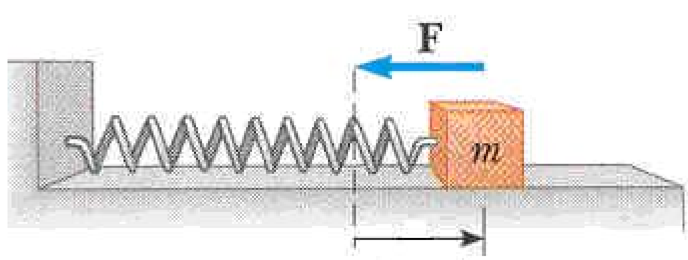
When a cord is attached to a object and pulled taut, the cord pulls on the object with the force called a tension force.

The force is directed away from the object and along the cord.



The Spring Force

A good approximation for many springs, the force from a spring is proportional to the displacement of the free end from its position when the spring is in the relaxed state



This is known as Hooke's law.

The minus sign indicates that the spring force is always opposite in direction from the displacement of the free end.

The constant k is called the spring constant

It is a measure of the stiffness of the spring. The larger k is, the stiffer the spring.

Note that a spring force is a variable force.

CONSERVATION OF MECHANICAL ENERGY

Conservative principles play a very important role in physics, and conservation of energy is one of the most important. Let us assume that the only force doing work on the system is conservative. In this case we have

$$W = E_{p1} - E_{p2} = E_{k2} - E_{k1}$$

$$E_{k1} + E_{p1} = E_{k2} + E_{p2}$$

The total mechanical energy in any isolated system of objects remains constant if the objects interact only through conservative forces. This is equivalent to saying that, if the kineic energy of a conservative system increases by some amount, the potential energy of the system must decrease by the some amount. If the gravitational force is the only force doing work on an object, then the total mechanical energy of the object remains constant

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

Example 1

- A 77-kg diver drops from a board 10 m above the water surface. Use conservation of mechanical energy to find his speed 5 m above the water surface.

$\vdash$

Conservation of mechanical energy gives

$$\begin{aligned}
\frac{1}{2}mv_1^2 + mgh_1 &= \frac{1}{2}mv_2^2 + mgh_2 \\
0 + mgh_1 &= \frac{1}{2}mv_2^2 + mgh_2 \\
v_2 &= \sqrt{\frac{2mg(h_1 - h_2)}{m}} \\
&= \sqrt{\frac{2(77\text{ kg})(9.8\text{ m/s}^2)(10\text{ m} - 5\text{ m})}{77\text{ kg}}} \\
&= 9.9\text{ m/s}^2
\end{aligned}$$

ELASTIC POTENTIAL ENERGY

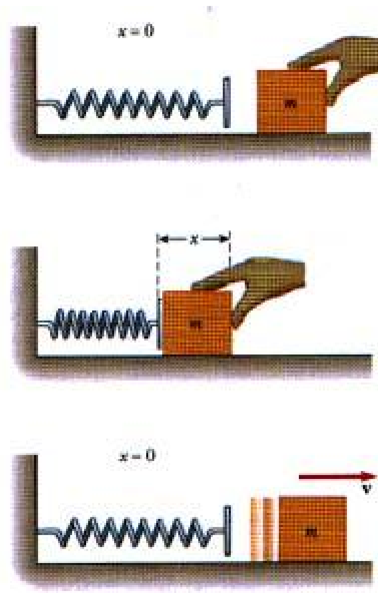
The concept of potential energy is of tremendous value in descriptions of certain types of mechanical energy. One of these is the motion of a mass attracted to a streched or compressed spring. In order to compress the spring, we must exert on the block a force of

$$\vec{F} = -k\vec{x}$$

where k is a constant for a particular spring called the spring constant. The force increases linearly with position. It is possible to find the work done by the applied force. This work is stored in the compressed spring as elastic potential energy.

$$E_{p,e} = \frac{1}{2}kx^2$$

The elastic potential energy stored in the spring is zero when the spring is in equilibrium (x=0). Note that energy is stored in the spring when it is stretched as well.



POWER

From the practical viewpoint, it is interesting to know not only the amount of energy transferred to or from a system, but also the rate at which the transfer occured.
Power is defined as the time rate of energy transfer.
If an external force is applied to an object and if the work done by this force is W in the time interval Δt, then the average power $\bar{P}$ during this time interval is defined as the ratio of the work to the time interval:

$$\bar{P} = \frac{W}{\Delta t}$$

The units of power in SI system are joules per second, which are also called watts (1 W).

Note, that a kilowatt-hour is a unit of energy, not power.
When you pay your electric bill, you are buying energy.
For example, an electric bulb rated at 100 W would "consume" 3.6×10^5 J of energy in 1 h, or 0.1 kWh (kilowatt-hour).

- Can the kinetic energy of an object be negative? $\vdash$ No.
- If the speed of a particle is doubled, what happens to its kinetic energy?

  $\vdash$

$$E_{k,2} = \frac{1}{2}mv_2^2 = \frac{1}{2}m(2v_1)^2 = 4\frac{1}{2}mv_1^2 = 4E_{k,1}$$

- Which has the greater kinetic energy, a 1000-kg car traveling at 50 km/h or a 500-kg car traveling at 100 km/h?

  $\vdash$

$$\begin{aligned}
v_1 &= 50 \text{ km/h} = 50 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 13.9 \text{ m/s} \\
E_{k,1} &= \frac{1}{2}m_1v_1^2 = 9.6 \cdot 10^4 \text{ J} \\
v_2 &= 100 \text{ km/h} = 100 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 27.8 \text{ m/s} \\
E_{k,2} &= \frac{1}{2}m_2v_2^2 = 1.9 \cdot 10^5 \text{ J}
\end{aligned}$$

Example 2

- Water flows over a section of Niagara Falls at the rate of $1.2 \cdot 10^6$ kg per second and falls 50 m. How much power is generated by the falling water?

  $\vdash$

$$\begin{aligned}
P &= \frac{W}{t} \\
&= \frac{mgh}{t} = 5.9 \cdot 10^8 \text{ W}
\end{aligned}$$

- The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there.
- **System of particles**

  If n particles are distributed in three dimensions, the center of mass must be identified by three coordinates. They are

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

  M is the total mass of the system

$$M = m_1 + m_2 + m_3 + \ldots + m_n = \sum_{i=1}^{n} m_i$$

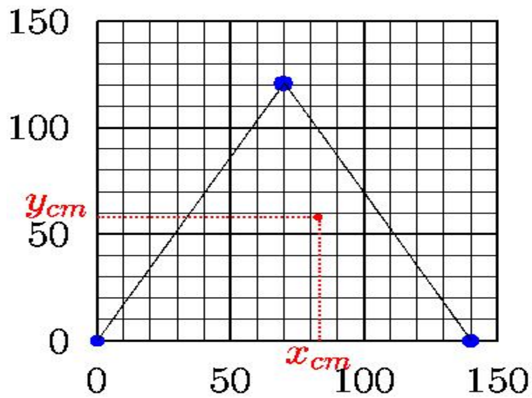
  and x_i, y_i, z_i are coordinates of i-th particle position.

Example 1

- Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg from an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this three-particle system?

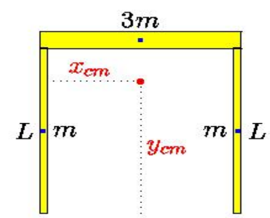
  The three particles have following coordinates: 1.2 kg: (0,0); 2.5 kg: (140 cm, 0); 3.4 kg: (70 cm, 121 cm). The total mass of the system is $M = m_1 + m_2 + m_3 = 7.1$ kg. The coordinates of the center of mass are

$$\begin{aligned} x_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i x_i \\ &= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}} \\ &= 83\text{ cm} \\ y_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i y_i \\ &= \frac{(1.2\text{ kg})0 + (2.5\text{ kg})(0) + (3.4\text{ kg})(121\text{ cm})}{7.1\text{ kg}} \\ &= 58\text{ cm} \end{aligned}$$



Example 2

- Three thin rods, each of length L, are arranged in an inverted U. The two rods on the arms of the U each have mass m; the third rod has mass $3m$. Where is the center of mass of the assembly?



$$
\begin{aligned}
x_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i x_i \\
&= \frac{(m)(0)+(3m)(L/2)+(m)(L)}{5m} = \frac{1}{2}L \\
y_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i y_i \\
&= \frac{(m)(L/2)+(3m)(L)+(m)(L/2)}{5m} = \frac{4}{5}L
\end{aligned}
$$

Linear momentum

- The linear momentum of an object of mass m moving with a velocity $\vec{v}$ is defined as the product of the mass and velocity

$$\vec{p} = m\vec{v}$$

Momentum is a vector quantity, with its direction matching that of the velocity.

- Often we will work with the components of momentum. For two-dimensional motion, these are

$$p_x = mv_x \qquad p_y = mv_y$$

- Newton didn't write the second law as $\vec{F} = m\vec{a}$ but as

$$\vec{F} = \frac{\text{change in momentum}}{\text{time interval}} = \frac{\Delta\vec{p}}{\Delta t}$$

where Δt is the time interval during which the momentum changes $\Delta\vec{p}$. This expression is equivalent to $\vec{F} = m\vec{a}$ for an object of constant mass.

- Newton's second law $\vec{F} = (\Delta\vec{p})/(\Delta t)$ can be written as

$$\vec{F}\Delta t = \Delta\vec{p}$$

The term $\vec{F}\Delta t$ is called the impulse of the force $\vec{F}$ for the time interval Δt. We see that the impulse of the force acting on an object equals the change in momentum of that object.

- To change the momentum of an object we shuld consider the impulse, that is, the amount of force and the time of contact.

- For example, think what you do when you jump from a high position to the ground. As you strike the ground, you bend you knees. If you were to land on the ground with your legs locked, you would receive a painful shock in your legs as well as along your spine. The lending is much less painful if you bend your knees. By bending your knees, the change in momentum occurs over a longer time interval than with the knees locked. Thus, the force on the body is less than with the knees locked.

- Now consider a system of n particles, each with its own mass, velocity, and linear momentum. The particle may interact with each other, and external force may act on them as well. The system as whole has a total linear momentum $\vec{p}$ as a sum of the individual particles' linear momentum

$$\begin{aligned}\vec{p} &= \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n \\ &= m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = \sum_{i=1}^{n} m_i\vec{v}_i \\ &= M\vec{v}_{cm},\end{aligned}$$

where $M = m_1 + \ldots + m_n$ is the total mass of the system, and

$$\vec{v}_{cm} = \frac{1}{M}(m_1\vec{v}_1 + \ldots + m_n\vec{v}_n)$$

the velocity of the center of mass.

Conservation of linear momentum

- Suppose that the net external force acting on a system of particle iz zero (the system is isolated) and that no particles leave or enter the system (the system is closed). From the previous we have

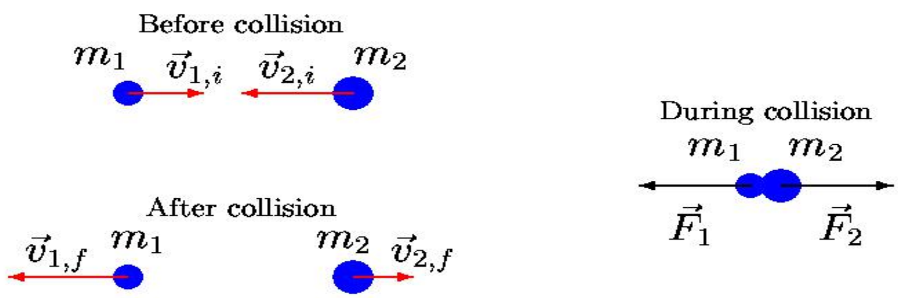
$$\Delta\vec{p} = 0 \qquad \text{or} \qquad \vec{p} = const.$$

In words we say, if no net external force acts on a system of particles, the total linear momentum $\vec{p}$ of the system cannot change. This result is called the law of conservation of linear momentum. It means that the total linear momentum at some initial time is equal to the total one at some later time.

- Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. However, we see that if the component of the net external force on a closed system is zero an axis, then the component of the linear momentum of the system along that axis cannot change.

Collision

- A collision is an isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.
- We must be able to distinguish times that are before, during, and after a collision. During the collision the force on object with mass m_1 due to m_2 is equal in magnitude and opposite in direction to the force on m_2 due to m_1. The momentum of each object changes as a result of the collision, but the total momentum of the system remains constant. We can say for any type of collision, the total momentum of the system just before collision equals the total momentum just after collision.



- We see that the total momentum is always conserved for any type of collision. However, the total kinetic energy is generally not conserved, because some of kinetic energy is converted to thermal energy or internal potential energy when the objects deform.
- An inelastic collision is a collision in which momentum is conserved, but kinetic energy is not.
- A perfectly inelastic collision is an inelastic collision in which the two objects stick together after the collision, so that their final velocities are the same and the momentum of the system is conserved.
- An elastic collision is one in which both momentum and kinetic energy are conserved.

  Elastic and perfectly inelastic collisions are limiting cases. Most actual colisions fall into a category between them.

Example 1

- A 80-kg man stands in the middle of a frozen pond of radius 5 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg coat horizontally toward the nord shore, at a speed of 5 m/s. How long does it take him to reach the south shore?

$$\begin{aligned} 0 &= m_c v_c - m_m v_m \\ v_m &= \frac{m_c v_c}{m_m} = \frac{(1.2 \text{ kg})(5 \text{ m/s})}{80 \text{ kg}} = 0.075 \text{ m/s} \\ t &= \frac{d}{v_m} = \frac{5 \text{ m}}{0.075 \text{ m/s}} = 66.67 \text{ s} \end{aligned}$$

Example 2

- The blocks slide without friction. (a) What is the velocity $\vec{v}$ of the 1.6 kg block after the collision? (b) Is the collision elastic?

$$\begin{aligned} m_1 v_{1,i} + m_2 v_{2,i} &= m_1 v_{1,f} + m_2 v_{2,f} \\ v_{1,f} &= \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_2 v_{2,f}}{m_1} \\ &= \frac{(1.6 \text{ kg})(5.5 \text{ m/s}) + (2.4 \text{ kg})(2.5 \text{ m/s}) - (2.4 \text{ kg})(4.9 \text{ m/s})}{1.6 \text{ kg}} \\ &= +1.9 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \frac{m_1 v_{1,i}^2}{2} + \frac{m_2 v_{2,i}^2}{2} &\stackrel{?}{=} \frac{m_1 v_{1,f}^2}{2} + \frac{m_2 v_{2,f}^2}{2} \\ 24.4 \text{ J} + 7.5 \text{ J} &= 2.888 \text{ J} + 28.812 \text{ J} \end{aligned}$$

The collision is elastic.

ANGULAR POSITION

- We wish to examine the rotation of a rigid body about a fixed axis. A rigid body is a body that can rotate with all its parts locked together and without any change in its shape. We deal with the angular equivalents of the linear quantities: position, displacement, velocity, and acceleration.

The angular position θ of some fixed line in the body, perpendicular to the rotation axis, is the angle of the line relative to the fixed direction. Measured in radians (rad) it is defined as

$$\theta = \frac{s}{r}$$

where s is the length of arc along a circle and between the reference line and the fixed line in the body; r is the radius of that circle. ($1 \text{ rad} = 180^0/\pi \approx 57.3^0$)

Example 1

- The rotor on a helicopter turns at an angular speed of 320 revolutions per minute. Express this in radians per second.

$$320\frac{\text{rev}}{\text{min}} = 320\frac{2\pi \text{ rad}}{60 \text{ s}} = 10.7\pi \text{ rad/s} = 33.6 \text{ rad/s}$$

- A wheel has a radius of 4.1 m. How far (path length) does a point on the circumference travel if the wheel is rotated through angle of 30^0 and 30 rad, respectively?

$$\begin{aligned} s_1 &= r\theta = (4.1 \text{ m})(30^0\frac{\pi \text{ rad}}{180^0}) = 2.14 \text{ m} \\ s_2 &= r\theta = (4.1 \text{ m})(30 \text{ rad}) = 123 \text{ m} \end{aligned}$$

- If the body rotates about the rotational axis changing the fixed line in the body from θ_1 to θ_2, the body undergoes an angular displacement $\Delta\theta$ given by

$$\Delta\theta = \theta_2 - \theta_1$$

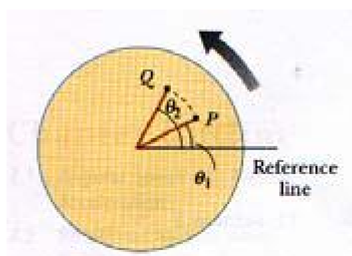
The average angular velocity $\bar{\omega}$ of a rotating rigid body is the ratio of the angular displacement to the time interval

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

By analogy with linear velocity, instantaneous angular velocity, ω, is defined as the limit of the average speed as the time interval approaches zero

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Angular velocity has the units rad/s. We shall take ω to be positive when θ increasing and negative when θ is decreasing. The magnitude of an angular velocity is called the angular speed.



- If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω_1 and ω_2 be its angular velocities at times t_1 and t_2, respectively. The average angular acceleration of the rotating body in the interval from t_1 to t_2 is defined as

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

  The instantaneous angular acceleration is the limit of the average angular acceleration as the time interval Δt approaches zero

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

  Angular acceleration has the units rad/s^2.

  When a rigid object rotates about a fixed axis every portition of the object has the same angular velocity and the same angular acceleration. This is what makes these variables so useful for describing rotational motion.

- When a rigid body rotates around an axis, each particle in the body moves in its own circle around that axis. Since the body is rigid, all the particles make one revolution in the same amount of time; that is, they all have the same angular displacement, angular velocity, and angular acceleration.

  The linear variables for a particular point in a rotating body are relate to the angular variables by the perpenducular distance r

$$\begin{aligned} v &= \omega r && \text{(the magnitude of tangential velocity)} \\ a_t &= \alpha r && \text{(tangential component of acceleration)} \\ a_c &= \frac{v^2}{r} = \omega^2 r && \text{(radial component of acceleration (or centripetal))} \end{aligned}$$

  The radial component a_c of linear acceleration (or centripetal acceleration) is present whenever the angular velocity of the body is not zero. The tangential component a_t is present whenever the angular acceleration is not zero.

Example 1

- A compact disc is designed such as the read head moves out from the center of the disc, the angular speed of the disc changes so that the linear speed at the position of the head will always be at a constant value of about 1.3 m/s. Find the angular speed of the disc when the read head is at a distance of 5 cm from the center.

$$\omega_1 = \frac{v}{r_1} = \frac{1.3\ \mathrm{m/s}}{0.05\ \mathrm{m}} = 26\ \mathrm{rad/s}$$

- A machine part rotates at an angular velocity of 0.6 rad/s; its value is then increased to 2.2 rad/s at an angular acceleration of 0.7 rad/s^2. Find the angle through which the part rotates before reaching this final velocity.

$$\omega = \omega_0 + \alpha t \qquad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{1}{2}\frac{\omega^2 - \omega_0^2}{\alpha} = \frac{1}{2}\frac{(2.2\ \mathrm{rad/s})^2 - (0.6\ \mathrm{rad/s})^2}{0.7\ \mathrm{rad/s^2}} = 3.2\ \mathrm{rad}$$

- Consider the wrench pivoted about the axis O. The applied force acts at an angle of ϕ.

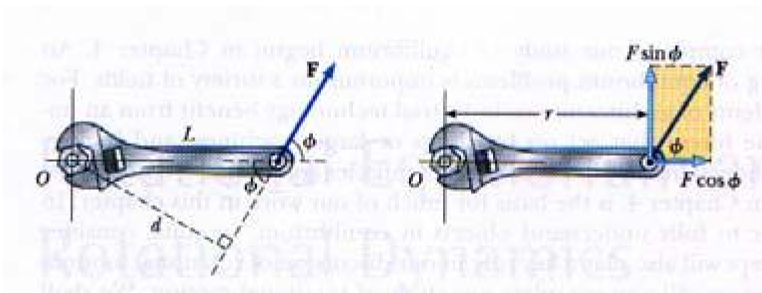
  The tendency of a force to ratate a body about some axis is measured by a quantity called the torque. The torque has the magnitude

$$\tau = Fr\sin\phi$$

  If two or more forces are acting on an object, then each has a tendency to produce a rotation about the pivot O. We shall use the convention that the sign of torque resulting from a force is positive if its turning tendency is counterclockwise and negative if its turning tendency is clockwise.

- As an example, find the torque produced by the 300-N force applied at an angle 60^0 to the door, as in figure.

$$\tau = Fr\sin\phi = (300\ \mathrm{N})(2\ \mathrm{m})\sin 60^0 = 520\ \mathrm{Nm}$$

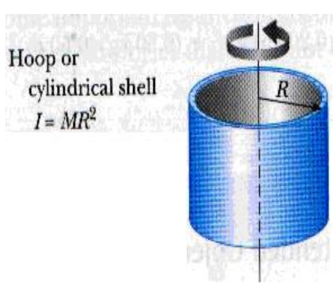


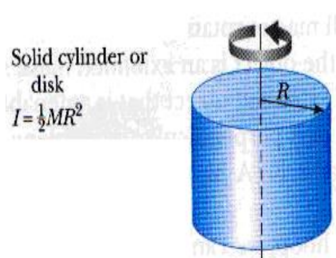
- We define the moment of inertia of a body with respect to the axis of rotation as follows

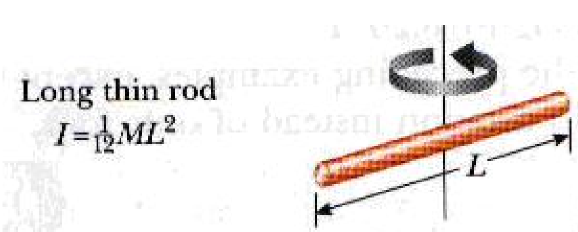
$$I = \sum_i m_i r_i^2$$

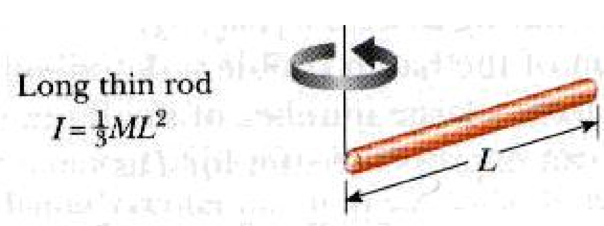
where m_i is the mass of the ith particle and r_i is the perpendicular distance of the ith particle from the given rotation axis. The sum is taken over all the particles in the body.

The moments of inertia about an axis through its center for some common shapes are given without proof (integral calculus are required).









- It is possible to prove that the angular acceleration of an object is proportional to the net torque acting on it. The proportional constant between them is the moment of inertia.

$$\tau_{net} = I\alpha.$$

It is important to note that this equation is rotational counterpart to Newton's second law $F_{net} = ma$. We now see that force and mass in linear motion correspond to torque and moment of inertia in rotational motion.

If we define the product of the angular velocity and the moment of inertia

$$L = I\omega$$

as the angular momentum of the object, then we can write

$$\tau_{net} = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

- When the net external torque acting on the system is zero, we see from the Newton's second law for rotation that the rate of change of the system's angular momentum is zero

$$\Delta L = 0.$$

The angular momentum of a system is conserved when the net external torque acting on the system is zero. That is, when $\tau_{net} = 0$, the initial angular momentum equals the final angular momentum. The magnitude of angular velocity increases when the skater pulls her arms in close to her body, demonstrating that angular momentum is conserved.

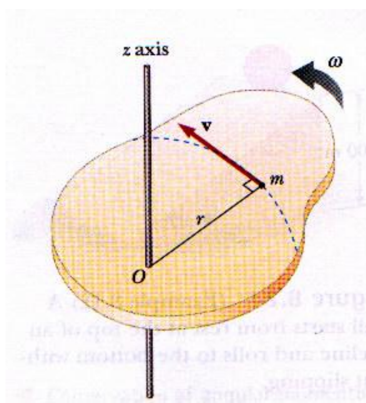
Rotational kinetic energy

- We defined the kinetic energy of a particle moving through space with a velocity v as the quantity $\frac{1}{2}mv^2$. Analogously, a body rotating about some axis with an angular velocity ω has rotational kinetic energy given by

$$E_{k,r} = \frac{1}{2}I\omega^2$$

To prove this, consider a rigid plane body rotating. the body consists of many small particles. All this particles rotate in circular paths about the axis. The total kinetic energy of the body is the sum of all the kinetic energies associated with all the particles making up the body

$$\begin{aligned} E_{k,r} &= \sum(\frac{1}{2}m_i v_i^2) = \sum(\frac{1}{2}m_i r_i^2\omega^2) \\ &= \frac{1}{2}(\sum m_i r_i^2)\omega^2 \\ &= \frac{1}{2}I\omega^2 \end{aligned}$$



- Consider a ball of mass m tied to a string of length r and being whirled in a horizontal circular path. Let us assume that the ball moves with constant speed. Because the velocity changes its direction continuosly during the motion, the ball experiences a centripetal acceleration directed toward the center of motion, with magnitude

$$a_c = \frac{v_t^2}{r}.$$

The strings exerts a force on the ball that makes a circular path. This force is directed along the length of the string toward the center of the circle with magnitude of

$$F_c = m\frac{v_t^2}{r}$$

This force we call centripetal force. Note that a centripetal force is not a new kind of force. The name indicates the direction of the force. It can, in fact, be a frictional force, a gravitational force, or any other force.

NEWTON'S UNIVERSAL LAW OF GRAVITATION

- In 1687 Newton published his work on the universal law of gravitation, which states that
Every particle in the Universe attracts any other particle with a gravitational force. If the particles have masses m_1 and m_2 and their centers are separated by the distance r, the magnitude of the gravitational force between them is

$$F = G\frac{m_1 m_2}{r^2}$$

where G is a universal constant called the constant of universal gravitation

$$G = 6.673 \cdot 10^{-11}\ \mathrm{Nm/kg^2}.$$

Assuming that Earth is a uniform sphere of mass M_E, for the magnitude of gravitational acceleration a_g we find

$$a_g = \frac{GM_E}{R_E^2} = \frac{(6.673 \cdot 10^{-11}\ \mathrm{Nm/kg^2})(6 \cdot 10^{24}\ \mathrm{kg})}{(6.37 \cdot 10^6\ \mathrm{m})^2} = 9.87\ \mathrm{m/s^2}$$