The two-layer equations we want to solve are

$$\partial_t q_1 + \mathbf{u}_1 \cdot \nabla q_1 = F_1 + \nu \nabla^2 q_1 \tag{1}$$

$$\partial_t q_2 + \mathbf{u}_2 \cdot \nabla q_2 = F_2 - r\nabla^2 \psi_2 + \nu \nabla^2 q_2 \tag{2}$$

$$\partial_t T + \left(\frac{\mathbf{u}_1 + \mathbf{u}_2}{2}\right) \cdot \nabla T = -\frac{u_1 + u_2}{2} + \nu \nabla^2 T \tag{3}$$

The PV inversion equations are

$$q_1 = \nabla^2 \psi_1 + \frac{k_d^2}{2}(\psi_2 - \psi_1) \tag{4}$$

$$q_2 = \nabla^2 \psi_2 + \frac{k_d^2}{2}(\psi_1 - \psi_2) \tag{5}$$

$$\tag{6}$$

The depth-independent equations that I solved in Phys. Rev. Fluids 2017 are

$$\partial_t q + \mathbf{u} \cdot \nabla q = F_q - r\nabla^2 \psi + \nu \nabla^2 q \tag{7}$$

$$\partial_t T + \mathbf{u} \cdot \nabla T = -u + \nu \nabla^2 T \tag{8}$$

$$q = \nabla^2 \psi \tag{9}$$

We want the depth-independent part of (1)–(6) to match the PRF paper. The depth independent part of (1)–(6) comes from averaging across the layers, and the new system is depth-independent if $q_1 = q_2$ and $\psi_1 = \psi_2$. (Friction in the bottom layer only prevents the new system from actually being depth-independent; we're just trying to make a fair comparison.) To match (1)–(6) with the old system we should set $F_1 = F_2 = F_q$ and use the same ν. It remains to figure out how to set r in the new system. If you average (1) and (2) and assume $F_1 = F_2 = F_q$ and $q_1 = q_2$ etc then you get

$$\partial_t q + \mathbf{u} \cdot \nabla q = F_q - \frac{r}{2}\nabla^2 \psi + \nu \nabla^2 q.$$

This doesn't quite match the old system, so we should make r in the new system equal to twice the r from the old system. r has units of 1/time, and the original system was non-dimensionalized so that $r \to 1$. So the new system should be

$$\partial_t q_1 + \mathbf{u}_1 \cdot \nabla q_1 = F_q + \nu \nabla^2 q_1 \tag{10}$$

$$\partial_t q_2 + \mathbf{u}_2 \cdot \nabla q_2 = F_q - 2\nabla^2 \psi_2 + \nu \nabla^2 q_2 \tag{11}$$

$$\partial_t T + \left(\frac{\mathbf{u}_1 + \mathbf{u}_2}{2}\right) \cdot \nabla T = -\frac{u_1 + u_2}{2} + \nu \nabla^2 T \tag{12}$$

The PV inversion equations are

$$q_1 = \nabla^2 \psi_1 + \frac{k_d^2}{2}(\psi_2 - \psi_1) \tag{13}$$

$$q_2 = \nabla^2 \psi_2 + \frac{k_d^2}{2}(\psi_1 - \psi_2) \tag{14}$$

$$\tag{15}$$

The domain should be a periodic square of width 2 and all the coefficients (ν and forcing amplitude) should be the same as the PRF paper. In equation (12) on the right hand side, u_1 and u_2 are not vectors; they're the first element of $\mathbf{u}_1$ and $\mathbf{u}_2$.