1. Eaton and Kortum 2002

This is easily one of the more important papers written in the last 20 years. There are *multiple* facets of this paper that make a contribution, but let me first focus on a trade issue: Reviving the Ricardian model and focus on technology as a determinant of trade flows. Recall, leading into 2000s essential trade was motivated by variety (either Armington or monopolistic competition models)

The big issue with the Ricardian Model, i.e. technology driven comparative advantage shaping trade flows, was hard to connect with the "real world" in any substantive way. For example, *Leamer and Levinsohn (1995)* said "It's just too simple." At this point there was a very important contribution to the Ricardian model which was the DFS model with two countries and a continuum of goods. How'd they solve it? DFS used a natural ordering between the two countries, $A(z)$ schedule, which says:

$$A(z') > A(z) \quad \text{if} \quad z' < z \tag{1}$$

meaning the home country has a greater comparative advantage in the good z' than z. How to do this? We could bilaterally order them, but that would be silent regarding the third option of the country left out. Willson (cite) tried to work this out and it was a mess. The bottom line was that Ricardian ideas behind trade was not taken seriously empirically for a long time.

Eaton and Kortum was a paradigm shit with multiple innovations. I'll talk through a couple that are relevant for these notes.

First, was how to handle the "ordering problem." Their idea was to treat the productivity draws for each individual good as random variables. Then track aggregate trade flows as the probability that a country is a low-cost supplier or not. Note that this is a conceptually important point that should be thought of as separate from the next contribution.

A second contribution: make very particular assumptions on the distributions which result in/preserve a lot of intuition from DFS and the old Ricardian model but then provide a tight mapping of aggregate trade flows to primitives through a Gravity equation. Basically, aggregate trade flows boil down to four things:

1. **TFP** — There is be a technology term (Ts) which is like TFP in a sense. Countries with better technology trade less, but other countries want to buy more from them.

2. **Trade Frictions** — They play the same role as in DFS by generating sets of non-traded goods and reduce overall flows on the intensive and extensive margin.

3. **Factor Costs** — This is the key equilibrium object. A country may have high TFP but also high wages. This is a force that allows other countries a chance to trade.

4. **A notion of comparative advantage** — How much technology, trade frictions, factor costs impede or facilitate trade depends upon the motives for trade. In the EK this boils down to a parameter θ which controls how different, at an idiosyncratic level, countries are. This turns out to play the exact same role as in the Armington model, i.e. how much I care about a "fruit salad."

One more thing which I won't go into detail in these notes is that while the distribution assumption they make seems peculiar it has some core micro-foundations that relate to earlier work by Jonathan and Sam and specifically Sam's job market paper. So if someone just looked at the Eaton and Kortum 2002 randomly, it would seem odd. To them, it was built upon a thought process over a decade that this was the right way to think about technology.

2. EK Set Up

Commodity space: $\omega \in [0,1]$ goods.

Technologies: Competitive producers in country i, to produce a good ω, receive a random draw $z(\omega)$ which determines their productivity. All producers of good ω in country i have the same draw $z(\omega)$. For simplicity, let's just assume they have the following production function:

$$q(\omega) = z(\omega)N, \tag{2}$$

so labor (N) is the only factor of production.

Trade frictions: For now, let's keep things simple by just focusing on ice-berg trade costs, here I'm using the EK notation where the first sub-script is the importer, the second sub-script is the exporter. So to supply good ω in market n, from i, they face an iceberg cost to trade:

$$d_{ni} \geq 1, \quad d_{ii} = 1 \tag{3}$$

and a triangle inequality is assumed to hold. In this model, this means the only relevant set of possible sources are direct routes i to n, we don't need to think about indirect routes.

Consumers: In each country, there is a representative consumer with preferences:

$$U_n = \left[\int_0^1 q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where σ is the elasticity of substitution across goods. Important! Notice how the goods with names ω enter into this utility function symmetrically, thus we **do not need** to keep track of the name of the good ω. Hence, I will drop the indexing of z by ω and work directly with z we will index everything by productivity from here on out.

A. The Firms Problem

The problem of a representative producer in i, to supply to n of good z, has the following problem:

$$\max_{N} \quad p_{ni}(z)\frac{z}{d_{ni}}N - w_i N \tag{5}$$

The solution implies:

$$p_{ni}(z) = \frac{w_i d_{ni}}{z} \tag{6}$$

This is the price consumers face.

B. The Consumers Problem

In each country, there is a representative consumer earning wage income w_n and face the problem:

$$\max_{\{q(z)\}} \quad \left[\int_0^1 q(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

subject to:

$$\int p_n(z)q(z)dz \leq w_n N_n \tag{8}$$

where the price p_n is the best prevailing price for that variety, that is

$$p_n(z) = \min\{p_{n1}(z), \ldots, p_{nN}(z)\}. \tag{9}$$

C. Thinking about trade

The goal is do be able to connect how much someone in country n demands from country i within the (i) model and (ii) and then connect it to data. In other words, how does a country's demand relate to technological properties in the other country. To make this connection, with the CES utility function, we have expenditure share for each good:

$$\frac{p_n(z)q_n(z)}{w_n N_n} = \frac{p_n(z)^{1-\sigma}}{\int p_n(z)^{1-\sigma}dz} \tag{10}$$

where the bottom term is related to the aggregate price index. Now define the expenditure share (which in the data is something like imports divided by output) on goods from country i as:

$$\frac{p_n^i(z)q_n^i(z)}{w_n N_n} = \frac{\int_{i \text{ is low cost in n}} \left(p_n(z)\right)^{1-\sigma} dz}{\int p_n(z)^{1-\sigma}dz} \tag{11}$$

Now there are several challenges to make this work. First, inspecting the tiny words one can see that his integral is not that simple since it is conditioned on i being the lowest cost supplier in county n. The question (and it relates to the ordering problem above), how does one make *any* progress on this. Second, and stepping back before the selection issues, is what is the distribution of prices consumed in a country. This is where the Frechet distribution comes in.

D. The Frechet Distribution

Now let's put more structure on good-level productivity. EK's assumption is that good-level technology is characterized by a probability distribution, so that a producer of any good in country i receives a draw from a country-specific **Type II extreme value** or **Frechet** distribution with CDF

$$F_i(z) = \exp\left\{-T_i z^{-\theta}\right\} = \text{Prob}\left\{z_i \leq z\right\} \tag{12}$$

where my notation is that z_i is the random variable and is less than value z and the support of this is defined on the positive real line. And each country's draws are independent of another countries draws.

This distribution has two parameters which are important to keep in mind:

- The Ts are the centering parameters and these control the mean of the distribution with high Ts meaning that larger zs are more likely. This is the sense that it determines a countries TFP or how advanced its technology is. But note, since this is a distribution with full support on the positive real line, this means that even a country that is on average very productive, it will have some bad draws (and very good ones).
- Then there is the θ which controls the dispersion of the draws. So the way this is setup is if θ is **large**, there is little dispersion (not many bad not good draws). If θ is **small**, then there is a lot of dispersion (many bad and

many good draws). In this setting, there is a restriction on the size of θ, it can't be below one and it also must be bounded below by σ so that the utility is well defined. This is a detail to see in the paper.

As a preview, the thing that determines things like gains from trade, how much trade responds to changes in trade costs, etc. are fundamentally tied to θ. There is tons of intuition about this in the sense that if countries are very different (low θ), then there is more scope for us to be trading as there are more instances where one country is good, another is bad at producing a particular good. Thus, gains from trade will be large. Intuitively, think to your self how this would work with high θ (we are very similar).

Other details: Micro foundation, max stability, correlation. Fill this in or ask me later.

E. Next steps...

Given the parameterized distribution, we need figure out how to compute the integrals in the import demand equation. I'm going to do this is several steps:

1. Characterize the distribution of prices in a country.
2. Characterize the distribution of prices **conditional** on being a low cost supplier.
3. Characterize the import share.

3. Characterizing the distribution of prices

So far, we know:

$$\text{Prob}\left\{z_i \leq z\right\} = \exp\left\{-T_i z^{-\theta}\right\} \tag{13}$$

Then from marginal cost pricing, we know:

$$p_{ni} = \frac{w_i}{z_i} d_{ni} \quad \iff \quad z_i = \frac{w_i d_{ni}}{p_i} \tag{14}$$

So we will plug this observation in:

$$\text{Prob}\left\{\frac{w_i d_{ni}}{p_{ni}} \leq \frac{w_i d_{ni}}{p}\right\} = \exp\left\{-T_i (w_i d_{ni})^{-\theta} p^{\theta}\right\} \tag{15}$$

Then, to make this look decent:

$$\text{Prob}\left\{p_i > p\right\} = \exp\left\{-T_i (w_i d_{ni})^{-\theta} p^{\theta}\right\} \tag{16}$$

Then this tells us the cumulative distribution function (CDF):

$$\text{Prob}\left\{p_{ni} \leq p\right\} = 1 - \exp\left\{-T_i (w_i d_{ni})^{-\theta} p^{\theta}\right\} \tag{17}$$

Now we want to compute the distribution of prices purchased. The stuff about is just about what the prices might be if they in country n from i. But not what is actually purchased. So we want to compute:

$$\text{Prob}\left\{p_n = \min\left\{p_{n1}, \ldots, p_{nN}\right\} \leq p\right\} \tag{18}$$

Claim #1 (and note I'm flipping the inequality sign to make the algebra work)

$$\text{Prob}\left\{p_{n1} > p, p_{n2} > p, \ldots, p_{nN} > p\right\} = \text{Prob}\left\{p_n = \min\left\{p_{n1}, \ldots, p_{nN}\right\} > p\right\} \tag{19}$$

This just follows from the fact that if p_n is the minimum and is greater than p, then **all** values of p_{ni} must be greater than p.

Claim #2 we can express this probability as the product of the individual price distributions

$$\text{Prob}\left\{p_{n1} > p, p_{n2} > p, \ldots, p_{nN} > p\right\} = \prod_{i=1}^{N}\left(1 - \text{Prob}\left\{p_{ni} \leq p\right\}\right) \tag{20}$$

The product comes from assumed independence. The inner part is just the claim that this is the probability $p_{ni} > p$. So from above and the CDF of prices offered, we know:

$$\text{Prob}\left\{p_{n1} > p, \ldots, p_{nN} > p\right\} = \prod_{i=1}^{N}\left(1 - 1 + \exp\left(-T_i(w_i d_{ni})^{-\theta} p^{\theta}\right)\right) \tag{21}$$

$$= \exp\left\{-\sum_{i=1}^{N} T_i(w_i d_{ni})^{-\theta} p^{\theta}\right\} = \text{Prob}\left\{p_n > p\right\} \tag{22}$$

Now what we really want is:

$$\text{Prob}\left\{p_n \leq p\right\} \tag{23}$$

Thus, this is:

$$1 - \exp\left\{-\sum_{i=1}^{N} T_i(w_i d_{ni})^{-\theta} p^{\theta}\right\} \tag{24}$$

Define:

$$\Phi_n = \sum_{i} T_i(w_i d_{ni})^{-\theta} \tag{25}$$

Now the Φ object is a **key** object. In this model, all that matters for trade is how it shapes the prices that a consumer faces. Φ is a summary statistic for this with a lower Φ meaning there are the consumer is getting lower prices. With that then notice how this works:

- Summarizes how technology and factor costs *diffuse* through trade.
- Barriers to trade affect prices and "weight" this diffusion.
- Note: if $d_{ni} = 1$ for all pairs, this implies all countries face the same price distribution, satisfying the **law of one price**.

- An opposite limit is if the country is in autarky. Then only that countries T matters, there is no benefit of being able to import goods from countries produced using their technology.

One more note. This is actually looking a lot like the inside term of the CES, ideal price index. It turns out that they are related. Can you figure that out?

4. Characterizing import demand in the EK model

Now we have the distribution of prices, so we can compute the bottom part of (11). I'm going to hold off on that task and focus on computing the top part of (11). To compute that top part, we need to characterize this probability:

$$\text{Prob}\left\{p_n < p \mid p_{ni} \leq \min p_{n\ell}, \forall \ell\right\} \tag{26}$$

which tells us essentially the distribution of prices, conditional on i being the low cost supplier to country n. If we know this probability, then we can simply compute the top part of (11) and be done with it. Computing this object is the **key heavy lift** at it will also reveal some interesting properties of the EK model and the Frechet distribution. Now to go through the derivation, I'm going to simplify this to two countries to show it:

$$\text{Prob}\left\{p_n < p \mid p_{12} \leq p_{11}\right\} \tag{1}$$

Now the probability above can be expressed as

$$\text{Prob}\left\{p_n < p \mid p_{12} \leq p_{11}\right\} = \frac{\text{Prob}\left\{p_n < p, p_{12} \leq p_{11}\right\}}{\text{Prob}\left\{p_{12} \leq p_{11}\right\}} \tag{27}$$

so what we need to do is (i) compute the denominator which is the probability that country 2 is the low cost supplier and (ii) the numerator which is interesting because its the union of several random outcomes. First lets do the denominator.

A. Computing the probability that a country is a low cost supplier

So I buy from country i if

$$p_{ni} = \min\left\{p_{n1}, p_{n2}, \dots\right\} \tag{28}$$

$$= \min\left\{\frac{w_1 d_{n1}}{z_1}, \frac{w_2 d_{n2}}{z_2}, \dots, \frac{w_N d_{nN}}{z_N}\right\} \tag{29}$$

Note that this is equivalent to

$$\max\left\{\frac{z_1}{w_1 d_{n1}}, \frac{z_2}{w_2 d_{n2}}\right\} \tag{30}$$

Insert digression on order statistics, connection with auctions, Roy model.

For simplicity, we consider two countries. We want to compute the probability that country 2 is the low-cost supplier:

$$\text{Prob}\left\{z_1 \leq z_2 \frac{w_1}{w_2} d_{12}\right\} = \int_0^\infty \exp\left\{-T_1\left(z_2 \tilde{w}\right)^{-\theta}\right\} f_2(z_2)\, dz_2 \tag{31}$$

This last equation is important to understand an interpret about what is going on. The first term inside the integral is the CDF of country 1's productivity distribution evaluated at the random variable $z_2\tilde{w}$. So this is saying, given z_2, what is the total probability that z_1 is lower. But, z_2 is also a random variable, we add these probabilities over all possible outcomes of z_2 weighted by probability z_2 occurs, i.e. the pdf $f_2(z_2)$. Then if we insert the pdf for the z_2 distribution you get

$$\text{Prob}\left\{z_1 \leq z_2 \frac{w_1}{w_2} d_{12}\right\} = \int_0^\infty \exp\left\{-T_1(z_2\tilde{w})^{-\theta}\right\} \exp\left\{-T_2 z_2^{-\theta}\right\} T_2 \theta z_2^{-\theta}\, dz_2 \tag{32}$$

At this point, one can see where the common θ is critical. We can start to pull things together where we have

$$= \int_0^\infty \exp\left\{-(T_1\tilde{w}_{12}^{-\theta} + T_2) z_2^{-\theta}\right\} T_2 \theta z_2^{-\theta}\, dz_2 \tag{33}$$

This looks ugly, but it turns out to have a clean solution. The key observation is that there is a nice change of variables where one would set $u = (T_1\tilde{w}_{12}^{-\theta} + T_2) z_2^{-\theta}$ and (I won't type out the details), but the observation is that after making this change of variables there is the a gamma function equal to one and then the following constant which is the choice probability:

$$= \frac{T_2 w_2^{-\theta} d_{12}^{-\theta}}{T_1 w_1 + T_2 w_2^{-\theta} d_{12}^{-\theta}} \tag{34}$$

which comes about after multiplying through by $(w_2 d_{12})^{-\theta}$. So we have

$$\pi_{12} = \text{Prob}\left\{\text{country 1 buys from 2}\right\} = \frac{T_2(w_2 d_{12})^{-\theta}}{T_1 w_1 + T_2(w_2 d_{12})^{-\theta}} \tag{35}$$

This generalizes to:

$$\pi_{ni} = \frac{T_i(w_i d_{ni})^{-\theta}}{\sum\limits_{\ell=1}^{N} T_\ell (w_\ell d_{n\ell})^{-\theta}} \tag{36}$$

One more step, notice what the denominator is...it's Φ_n derived above, so the probability that country is a low cost supplier is then

$$\pi_{ni} = \frac{T_i(w_i d_{ni})^{-\theta}}{\Phi_n} \tag{37}$$

BOOM!

B. Computing the joint probability of price and low cost supplier

Now the second probability to compute is the joint probability of the price taking some value and it being low cost. To recap

$$\text{Prob}\left\{p_n < p, p_{12} < p_{12}\right\} = \int_0^p \left(\int_{P_{12}}^\infty g_{12}(p_{12}) g_{11}(P_{11})\, dP_{11}\right) dP_{12} \tag{38}$$

where g is the pdf of the distribution of offered prices (not realized) in each country. Then, notice that we can pull $g_{12}(P_{12})$ out from the second integral sign. This leaves:

$$\int_0^P g_{12}(p_{12}) \left(\int_{P_{12}}^{\infty} g_{11}(P_{11})\, dP_{11} \right) dP_{12} \tag{39}$$

Notice how the inside bracket is like a cumulative distribution function (CDF), so basically what we are doing here is: Fix a P_{12} and compute the probability that P_{11} is higher. Then add these probabilities up by integrating over all the possible p_{12} combinations. Now, the term in brackets equals $1 - G_{11}(p_{12})$. Why? $G_{11}(p_{12})$ is everything below p_{12}, and hence $1 - G_{11}(p_{12})$ is everything above.

So what we have is:

$$\text{Prob}\left\{p_n < p, p_{12} < p_{11}\right\} = \int_0^p g_{12}(p_{12}) \left(1 - G_{11}(p_{12})\right) dp_{11} \tag{40}$$

Then note that:

$$g_{12}(p_{12}) = T_2(w_2 d_{12})^{-\theta} p_{12}^{\theta-1} \exp\left\{-T_2(w_2 d_{12})^{-\theta} p_{11}^{\theta}\right\} \tag{41}$$

$$1 - G_{11}(p) = \exp\left\{-T_1 w_1^{-\theta} p^{\theta}\right\} \tag{42}$$

This integral is then:

$$\int_0^p \exp\left\{-T_2(w_2 d_{12})^{-\theta} p_{12}^{\theta}\right\} \exp\left\{-T_1 w_1^{-\theta} p_{12}^{\theta}\right\} T_2(w_2 d_{12})^{-\theta} p_{12}^{\theta-1} dp_{12} \tag{43}$$

The rewriting we have

$$\int_0^p \exp\left\{-\sum_{i=1}^{2} T_i(w_i d_{1i})^{-\theta} p_{12}^{\theta}\right\} T_2(w_2 d_{12})^{-\theta} p_{12}^{\theta-1} dp_{12} \tag{44}$$

Now here is where it's important to notice that the inside part of the $\exp$ function is Φ_n, the centering parameter on the price distribution. So we can write this as

$$\frac{T_2(w_2 d_{12})^{-\theta}}{\Phi_1} \int_0^p \exp\left\{-\Phi_1 p_{12}^{\theta}\right\} \Phi_1 p_{12}^{\theta-1} dp_{12} \tag{45}$$

where I multiplied and divided the outside part by Φ and then swaped it out. So we have:

$$\text{Prob}\left\{p_n < p, p_{12} < p_{11}\right\} = \frac{T_2(w_2 d_{12})^{-\theta}}{\Phi_1} \int_0^p \exp\left\{-\Phi_1 p_{12}^{\theta}\right\} \Phi_1 p_{12}^{\theta-1} dp_{12} \tag{46}$$

Now notice that the front part is the probability that country 2 is the low cost supplier

$$\pi_{12} = \frac{T_2(w_2 d_{12})^{-\theta}}{\Phi_1} = \text{Prob}\left\{p_{12} < p_{11}\right\} \tag{47}$$

And then notice that the integral is the cumulative distribution function (CDF) of the price distribution derived:

$$G_1 = 1 - \exp\left\{-\Phi_1 p^{\theta}\right\} \tag{48}$$

Thus:

$$\text{Prob}\left\{p_n < p, p_{12} < p_{11}\right\} = \pi_{12} G_1 \tag{49}$$

Pause. Recall that we wanted to characterise

$$\text{Prob}\left\{p_n < p \mid p_{12} < p_{11}\right\} = \frac{\text{Prob}\left\{p_n < p, p_{12} \leq p_{11}\right\}}{\text{Prob}\left\{p_{12} \leq p_{11}\right\}} = \frac{\pi_{11} G_1}{\pi_{11}} = G_1 \tag{50}$$

So this is a crazy property that we just derived. This says that the distribution of prices, sold in country n, are the same **independent of the source.** This is very peculiar and unique to this model. What it means is that if we took the histogram of prices that are purchased from Canada (in the US) and compared it to the histogram of prices from Botswana (into the US) they would be exactly the same. Without the Frechet distribution, this would not be going on.

C. Pulling it all together...

Recall that we wanted to compute this:

$$\frac{p_n^i(z) q_n^i(z)}{w_n N_n} = \frac{\int_{i \text{ is low cost in n}} \left(p_n(z)\right)^{1-\sigma} dz}{\int p_n(z)^{1-\sigma} dz}$$

where lets define the bottom integral as $Ep_n^{1-\sigma}$. But notice that the top integral can be expressed as

$$\frac{p_n^i(z) q_n^i(z)}{w_n N_n} = \frac{\int_{i \text{ is low cost in n}} \left(p_n(z)\right)^{1-\sigma} dz}{Ep_n^{1-\sigma}} = \frac{\pi_{ni} Ep_n^{1-\sigma}}{Ep_n^{1-\sigma}} = \pi_{ni} \tag{51}$$

so the CES expenditure share is equivalent to the probability that the i is the low cost supplier to n! The fact that this worked out is crazy and notice a critical step to this was the observation that the price distribution that i sells into n is the same as the entire distribution that n faces.

5. Interpretation of EK 2002 and determinants of trade and gains from trade

Connection to Armington and gravity models. Stare at this equation...

$$\pi_{ni} = \frac{T_i (w_i d_{ni})^{-\theta}}{\Phi_n} \tag{52}$$

Gains from trade