Kanza Mazhar

Truc Vo

Mudiha Wazirali

COMPANY A SHAMPOO SALES REPORT

INTRODUCTION

Company A carries many different types of beauty products such as cosmetics, skin care, fragrances, nails, bath and body, tools, haircare. Company A has increased their sales over the last three years for their new shampoo product.

EXECUTIVE SUMMARY

Company A wants to evaluate each of the product type to forecast the future sales and determine if it is still a good product that it need to carry in the stores. They have hired Group 4 Data Analyst team to conduct a time series analysis on their shampoo sales for the last three years and provide business recommendations. We are focusing on the shampoo sales for this project and evaluating the three-year sales to forecast the future demand of Shampoo for their target market.

Group 4 has conducted a time series analysis on Company A's shampoo sales with the following business questions:

1. Is there a trend in Company A's shampoo sales over the last three years?
2. What time series model(s) best fits this dataset?
3. What are the forecasted shampoo sales for Company A for the months of September, October, November and December 2019?

In this paper, we will review the initial analysis of our dataset, the model selection process, final model selection, forecasts and our business recommendations based on this forecast.

INITIAL ANALYSIS OF DATASET

Month is the equally spaced index predictor variable for the time series. Data was pulled on the 17th of each month from January 17, 2017 to October 17, 2019. The number of sales of shampoo over a period of a month at each index point in time is in terms of millions.

Looking at Figure 1 below, there is an increasing trend in the mean of the Y(t) over time. Furthermore, the variance of Y(t) is not constant. There is also an obvious level shift starting September 2018 that we need to explore. A differencing of the data will help to stabilize the mean. The dataset is not stationary, so we need stabilize the mean before fitting a model.

Looking at Figure 2, the ACF above is damped sinusoidal and the PACF has a high lag at lag 1 and 2 but not after lag 2.

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Figure 1

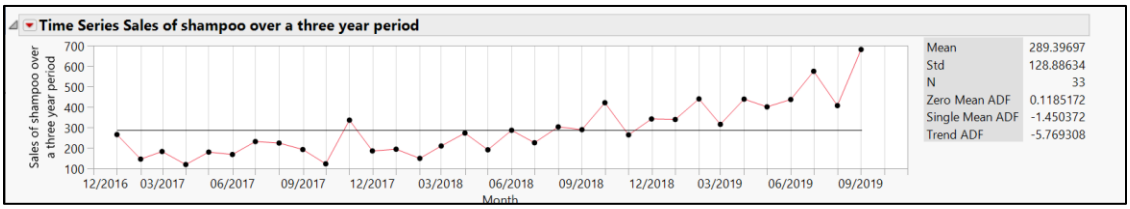
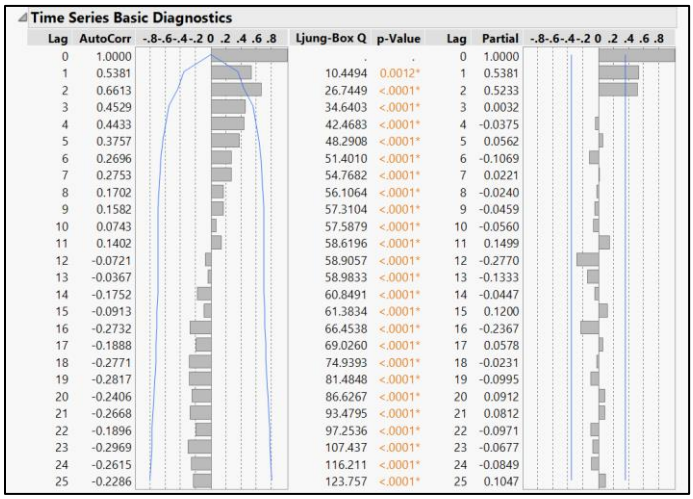


Figure 2



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.5381	10.4494	0.0012*	1	0.5381
2	0.6613	26.7449	<.0001*	2	0.5233
3	0.4529	34.6403	<.0001*	3	0.0032
4	0.4433	42.4683	<.0001*	4	-0.0375
5	0.3757	48.2908	<.0001*	5	0.0562
6	0.2696	51.4010	<.0001*	6	-0.1069
7	0.2753	54.7682	<.0001*	7	0.0221
8	0.1702	56.1064	<.0001*	8	-0.0240
9	0.1582	57.3104	<.0001*	9	-0.0459
10	0.0743	57.5879	<.0001*	10	-0.0560
11	0.1402	58.6196	<.0001*	11	0.1499
12	-0.0721	58.9057	<.0001*	12	-0.2770
13	-0.0367	58.9833	<.0001*	13	-0.1333
14	-0.1752	60.8491	<.0001*	14	-0.0447
15	-0.0913	61.3834	<.0001*	15	0.1200
16	-0.2732	66.4538	<.0001*	16	-0.2367
17	-0.1888	69.0260	<.0001*	17	0.0578
18	-0.2771	74.9393	<.0001*	18	-0.0231
19	-0.2817	81.4848	<.0001*	19	-0.0995
20	-0.2406	86.6267	<.0001*	20	0.0912
21	-0.2668	93.4795	<.0001*	21	0.0812
22	-0.1896	97.2536	<.0001*	22	-0.0971
23	-0.2969	107.437	<.0001*	23	-0.0677
24	-0.2615	116.211	<.0001*	24	-0.0849
25	-0.2286	123.757	<.0001*	25	0.1047

After taking the difference of the mean, we see in Figure 3 that the mean and variance have stabilized. The new ACF below in Figure 4 is damped sinusoidal and PACF has a high lag at lag 1 but not after lag 1.

Figure 3

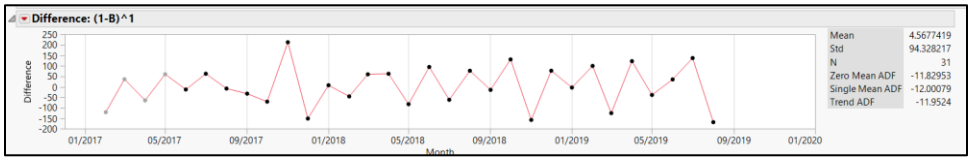
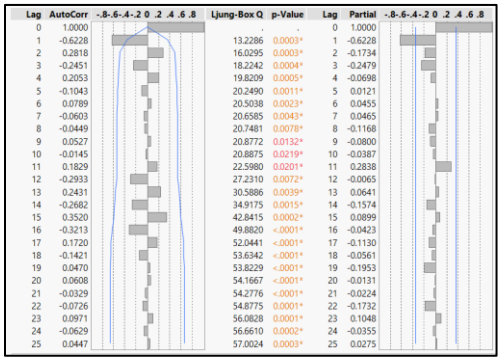


Figure 4



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.6228	13.2286	0.0003*	1	-0.6228
2	0.2818	16.0295	0.0003*	2	-0.1734
3	-0.2451	18.2242	0.0004*	3	-0.2479
4	0.2053	19.8209	0.0005*	4	-0.0698
5	-0.1043	20.2490	0.0011*	5	0.0121
6	0.0789	20.5038	0.0023*	6	0.0455
7	-0.0603	20.6585	0.0043*	7	0.0465
8	-0.0449	20.7481	0.0078*	8	-0.1168
9	0.0527	20.8772	0.0132*	9	-0.0800
10	-0.0145	20.8875	0.0219*	10	-0.0387
11	0.1829	22.5980	0.0201*	11	0.2838
12	-0.2933	27.2310	0.0072*	12	-0.0065
13	0.2431	30.5886	0.0039*	13	0.0641
14	-0.2682	34.9175	0.0015*	14	-0.1574
15	0.3520	42.8415	0.0002*	15	0.0899
16	-0.3213	49.8820	<.0001*	16	-0.0423
17	0.1720	52.0441	<.0001*	17	-0.1130
18	-0.1421	53.6342	<.0001*	18	-0.0561
19	0.0470	53.8229	<.0001*	19	-0.1953
20	0.0608	54.1667	<.0001*	20	-0.0131
21	-0.0329	54.2776	<.0001*	21	-0.0224
22	-0.0726	54.8775	0.0001*	22	-0.1732
23	0.0971	56.0828	0.0001*	23	0.1048
24	-0.0629	56.6610	0.0002*	24	-0.0355
25	0.0447	57.0024	0.0003*	25	0.0275

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Mudiha Wazirali

Top Model Selections

To identify the best model for the data set, we executed the SCAN and ESACF function in SAS, and the result show a few suggested ARIMA values.

SCAN Chi-Square[1] Probability Values

Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	<.0001	<.0001	0.0166	0.0199	0.0630	0.1807
AR 1	<.0001	0.1156	0.4035	0.5310	0.9470	0.8764
AR 2	0.3584	0.7522	0.9839	0.4089	0.8890	0.9130
AR 3	0.5352	0.8824	0.4351	0.3938	0.9874	0.8721
AR 4	0.6906	0.2465	0.9004	0.6658	0.8778	0.7496
AR 5	0.0496	0.3543	0.6875	0.7461	0.8485	0.9739

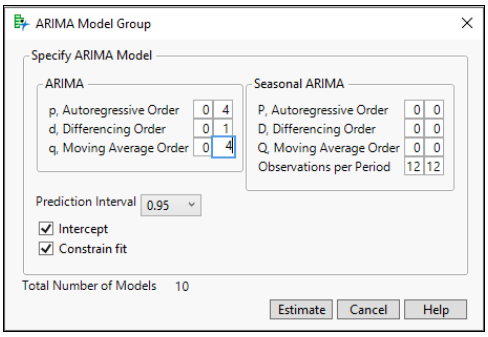
ESACF Probability Values

Lags	MA 0	MA 1	MA 2	MA 3	MA 4	MA 5
AR 0	0.0020	0.0025	0.0968	0.1323	0.2317	0.4103
AR 1	0.0003	0.1244	0.4371	0.6090	0.9900	0.8795
AR 2	0.0999	0.2663	0.6870	0.6287	0.7137	0.7784
AR 3	0.0122	0.7416	0.6846	0.3801	0.5633	0.8316
AR 4	0.0157	0.1979	0.6383	0.0814	0.5396	0.5816
AR 5	0.8885	0.3145	0.2402	0.1673	0.9054	0.5665

ARMA(p+d,q) Tentative Order Selection Tests

SCAN		ESACF	
p+d	q	p+d	q
1	1	2	0
0	4	1	1
		0	2
		4	1
		5	0

SCAN and ESACF result show the highest value in q is 4 and highest value in p+d is 5, so we extend our ARIMA model group to 4.



ARIMA Model group result sorted by SBC rank

Model Comparison

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	SBC Rank	MAPE	MAE
☑	☐	IMA(1, 2)	28	4287.3277	351.49497	355.79693	0.660	345.49497	0.079426	1	24.131879	52.416445
☑	☐	ARIMA(1, 1, 1)	28	4373.1751	351.91145	356.21341	0.656	345.91145	0.064495	2	23.818617	52.103031
☑	☐	IMA(1, 1)	29	4791.8803	353.71551	356.58348	0.621	349.71551	0.026168	3	24.822230	55.771933
☐	☐	ARIMA(3, 1, 2)	25	3112.8424	349.21901	357.82294	0.727	337.21901	0.247844	4	22.074923	47.635312
☐	☐	ARI(1, 1)	29	5047.7733	354.97534	357.84331	0.601	350.97534	0.013938	5	23.856965	56.827335
☐	☐	ARIMA(1, 1, 2)	27	3805.7885	353.18248	358.91843	0.660	345.18248	0.034160	6	24.646190	52.751545
☐	☐	ARI(3, 1)	27	4451.5193	353.41973	359.15568	0.660	345.41973	0.030339	7	23.573942	52.682660
☐	☐	IMA(1, 3)	27	4444.3791	353.48318	359.21913	0.660	345.48318	0.029392	8	24.135686	52.499138
☐	☐	ARI(2, 1)	28	4902.9242	355.09607	359.39803	0.623	349.09607	0.013122	9	23.791680	54.123055
☐	☐	ARIMA(2, 1, 1)	27	4534.9455	353.90540	359.64135	0.656	345.9054	0.023798	10	23.806263	52.161751
☐	☐	ARI(4, 1)	26	4298.7342	353.50153	360.67147	0.678	343.50153	0.029123	11	23.073374	50.530646
☐	☐	ARIMA(4, 1, 2)	24	3154.4239	350.63361	360.67152	0.730	336.63361	0.122181	12	22.796327	48.421359
☐	☐	ARIMA(3, 1, 3)	24	3190.4768	350.79233	360.83025	0.729	336.79233	0.112859	13	22.536464	47.999348
☐	☐	ARIMA(3, 1, 1)	26	4401.3394	354.11350	361.28343	0.672	344.1135	0.021446	14	23.398062	51.473569
☐	☐	IMA(1, 4)	26	4434.9799	354.46985	361.63978	0.671	344.46985	0.017946	15	22.852518	50.334635
☐	☐	ARIMA(2, 1, 2)	26	3894.1668	354.73860	361.90854	0.664	344.7386	0.015690	16	24.692710	52.881774
☐	☐	ARIMA(1, 1, 3)	26	3946.6922	355.11504	362.28497	0.660	345.11504	0.012998	17	24.582588	52.694535
☐	☐	ARIMA(2, 1, 3)	25	3975.1658	355.45630	364.06022	0.677	343.4563	0.010959	18	24.324782	51.488087

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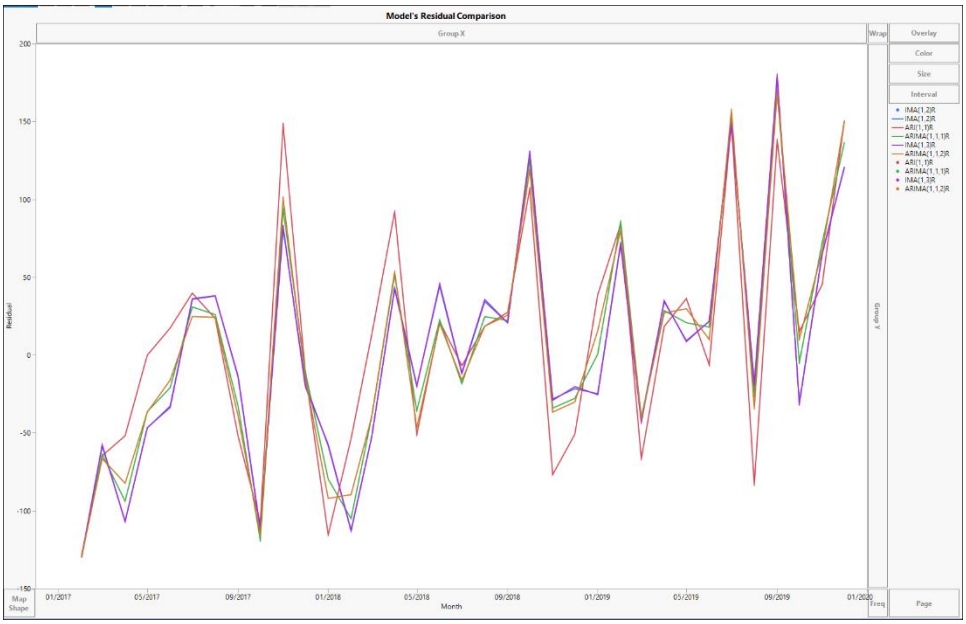
The first 6 models with the lowest SBC rank Ljung-Box Q values are not statistically significant, so they are all valid models with our data set, so we decide to explore these 6 models.

Our comparison on the forecast value and residual from each model show that these 6 models seem to provide similar results, so we decide to go with the top two models that has the lowest SBC which is IMA(1,2) and ARIMA(1,1,1)

Forecast Comparison

Residual Comparison



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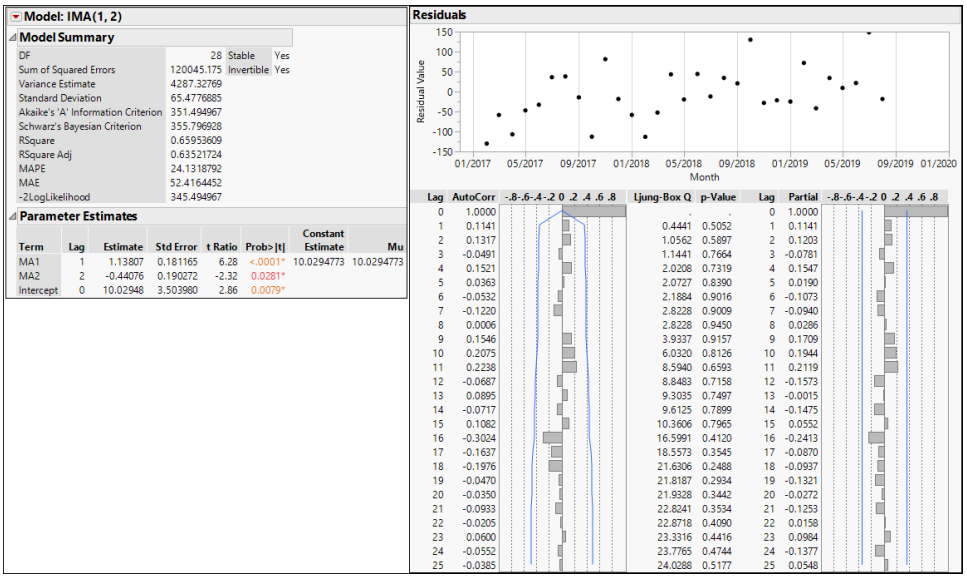
IMA (1,2)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. IMA(1,1) is a valid model.
2. This model is not parsimonious due to the parameter MA2 is not statistically significant.



Model: IMA(1, 2)

Model Summary

DF	28	Stable	Yes
Sum of Squared Errors	120045.175	Invertible	Yes
Variance Estimate	4287.32769		
Standard Deviation	65.4776885		
Akaike's 'A' Information Criterion	351.494967		
Schwarz's Bayesian Criterion	355.796928		
RSquare	0.65953609		
RSquare Adj	0.63521724		
MAPE	24.1318792		
MAE	52.4164452		
-2LogLikelihood	345.494967		

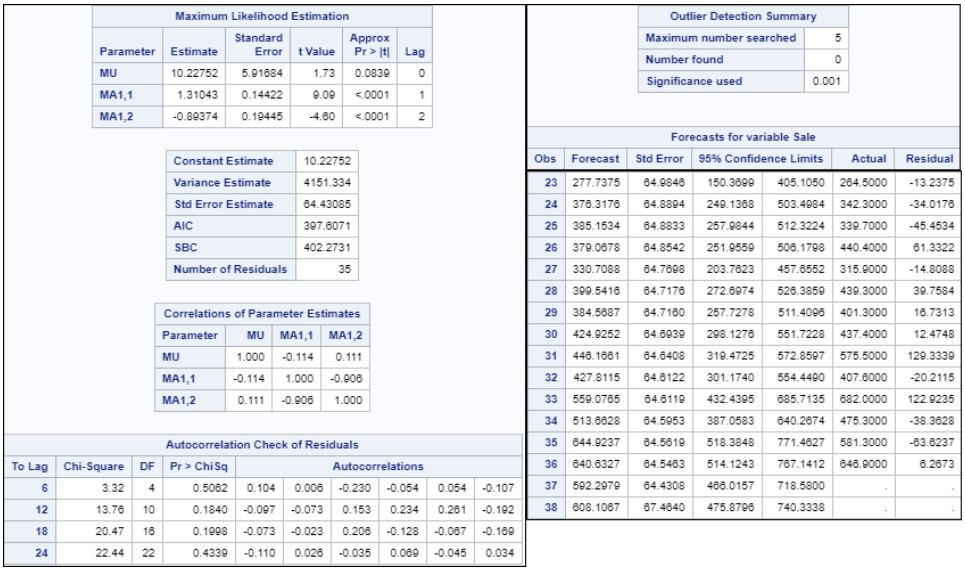
Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
MA1	1	1.13807	0.181165	6.28	<.0001*	10.0294773	10.0294773
MA2	2	-0.44076	0.190272	-2.32	0.0281*		
Intercept	0	10.02948	3.503980	2.86	0.0079*		

Residuals

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.1141	0.4441	0.5052	1	0.1141
2	0.1317	1.0562	0.5897	2	0.1203
3	-0.0491	1.1441	0.7664	3	-0.0781
4	0.1521	2.0208	0.7319	4	0.1547
5	0.0363	2.0727	0.8390	5	0.0190
6	-0.0532	2.1884	0.9016	6	-0.1073
7	-0.1220	2.8228	0.9009	7	-0.0940
8	0.0006	2.8228	0.9450	8	0.0286
9	0.1546	3.9337	0.9157	9	0.1709
10	0.2075	6.0320	0.8126	10	0.1944
11	0.2238	8.5940	0.6593	11	0.2119
12	-0.0687	8.8483	0.7158	12	-0.1573
13	0.0895	9.3035	0.7497	13	-0.0015
14	-0.0717	9.6125	0.7899	14	-0.1475
15	0.1082	10.3606	0.7965	15	0.0552
16	-0.3024	16.5991	0.4120	16	-0.2413
17	-0.1637	18.5573	0.3545	17	-0.0870
18	-0.1976	21.6306	0.2488	18	-0.0937
19	-0.0470	21.8187	0.2934	19	-0.1321
20	-0.0350	21.9328	0.3442	20	-0.0272
21	-0.0933	22.8241	0.3534	21	-0.1253
22	-0.0205	22.8718	0.4090	22	0.0158
23	0.0600	23.3316	0.4416	23	0.0984
24	-0.0552	23.7765	0.4744	24	-0.1377
25	-0.0385	24.0288	0.5177	25	0.0548

3. Model in SAS with outlier and Forecast



Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	10.22752	5.91684	1.73	0.0839	0
MA1,1	1.31043	0.14422	9.09	<.0001	1
MA1,2	-0.89374	0.19445	-4.60	<.0001	2

Constant Estimate	10.22752
Variance Estimate	4151.334
Std Error Estimate	64.43085
AIC	397.6071
SBC	402.2731
Number of Residuals	35

Correlations of Parameter Estimates

Parameter	MU	MA1,1	MA1,2
MU	1.000	-0.114	0.111
MA1,1	-0.114	1.000	-0.906
MA1,2	0.111	-0.906	1.000

Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	3.32	4	0.5062	0.104	0.006	-0.230	-0.054	0.054	-0.107
12	13.76	10	0.1840	-0.097	-0.073	0.153	0.234	0.261	-0.192
18	20.47	16	0.1998	-0.073	-0.023	0.206	-0.128	-0.067	-0.169
24	22.44	22	0.4339	-0.110	0.026	-0.035	0.069	-0.045	0.034

Outlier Detection Summary

Maximum number searched	5
Number found	0
Significance used	0.001

Forecasts for variable Sale

Obs	Forecast	Std Error	95% Confidence Limits		Actual	Residual
23	277.7375	64.9846	150.3699	405.1050	264.5000	-13.2375
24	376.3176	64.8894	249.1368	503.4984	342.3000	-34.0176
25	385.1534	64.8833	257.9844	512.3224	339.7000	-45.4534
26	379.0678	64.8542	251.9559	506.1798	440.4000	61.3322
27	330.7088	64.7698	203.7623	457.6552	315.9000	-14.8088
28	399.5416	64.7176	272.6974	526.3859	439.3000	39.7584
29	384.5687	64.7160	257.7278	511.4096	401.3000	16.7313
30	424.9252	64.6939	298.1276	551.7228	437.4000	12.4748
31	446.1661	64.6408	319.4725	572.8597	575.5000	129.3339
32	427.8115	64.6122	301.1740	554.4490	407.6000	-20.2115
33	559.0765	64.6119	432.4395	685.7135	682.0000	122.9235
34	513.6628	64.5953	387.0583	640.2674	475.3000	-38.3628
35	644.9237	64.5619	518.3848	771.4627	581.3000	-63.6237
36	640.6327	64.5463	514.1243	767.1412	646.9000	6.2673
37	592.2979	64.4308	466.0157	718.5800	.	.
38	608.1067	67.4640	475.8796	740.3338	.	.

ARIMA(1,1,1)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. ARIMA(1,1,1) is a valid model.
2. This model is not parsimonious due to the parameter AR1 is not statistically significant.

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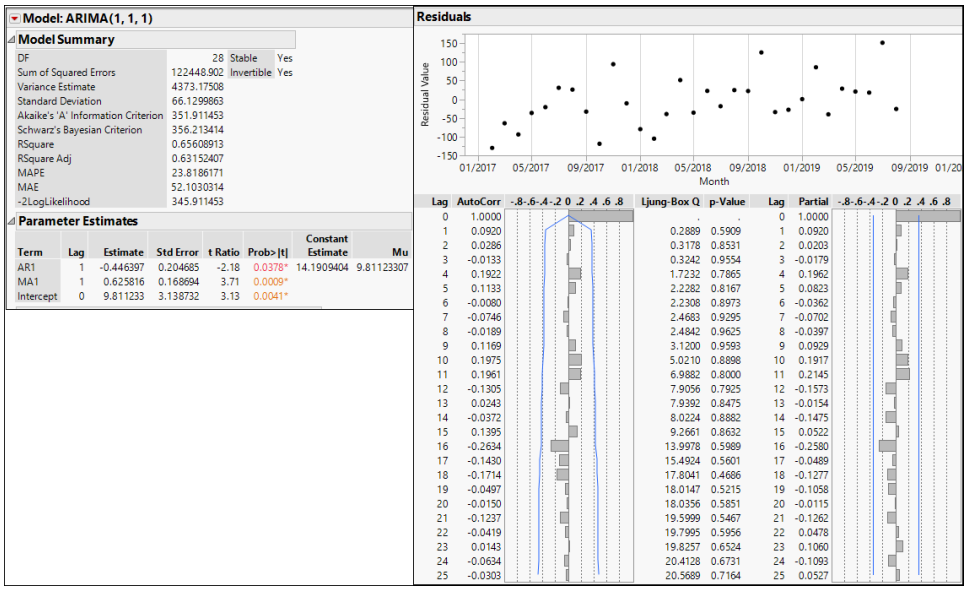


Model: ARIMA(1, 1, 1)

Model Summary

DF	28	Stable	Yes
Sum of Squared Errors	122448.902	Invertible	Yes
Variance Estimate	4373.17508		
Standard Deviation	66.1299863		
Akaike's 'A' Information Criterion	351.911453		
Schwarz's Bayesian Criterion	356.213414		
RSquare	0.65608913		
RSquare Adj	0.63152407		
MAPE	23.8186171		
MAE	52.1030314		
-2LogLikelihood	345.911453		

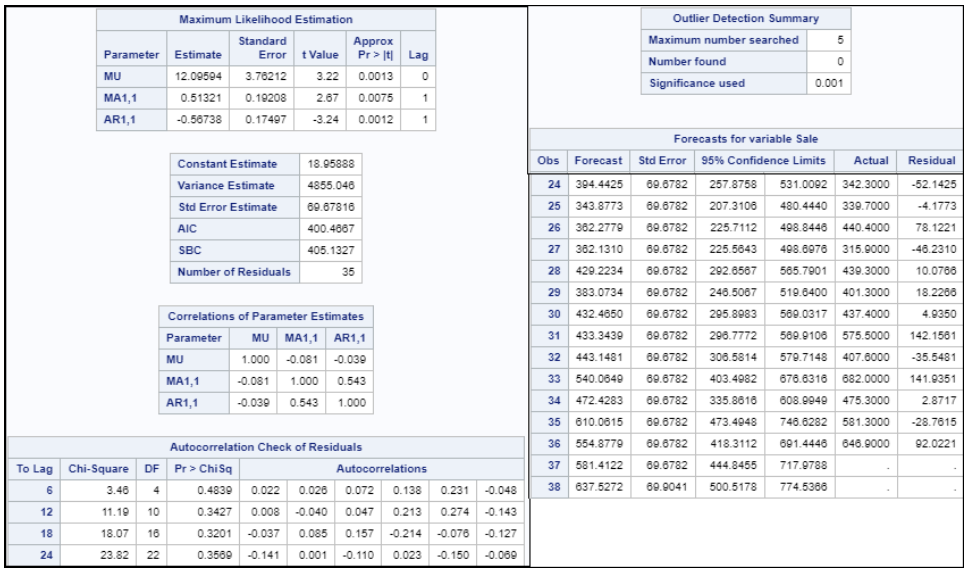
Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
AR1	1	-0.446397	0.204685	-2.18	0.0378*	14.1909404	9.81123307
MA1	1	0.625816	0.168694	3.71	0.0009*		
Intercept	0	9.811233	3.138732	3.13	0.0041*		

Residuals

Lag	AutoCorr	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	0.0920		0.2889	0.5909	1	0.0920	
2	0.0286		0.3178	0.8531	2	0.0203	
3	-0.0133		0.3242	0.9554	3	-0.0179	
4	0.1922		1.7232	0.7865	4	0.1962	
5	0.1133		2.2282	0.8167	5	0.0823	
6	-0.0080		2.2308	0.8973	6	-0.0362	
7	-0.0746		2.4683	0.9295	7	-0.0702	
8	-0.0189		2.4842	0.9625	8	-0.0397	
9	0.1169		3.1200	0.9593	9	0.0929	
10	0.1975		5.0210	0.8898	10	0.1917	
11	0.1961		6.9882	0.8000	11	0.2145	
12	-0.1305		7.9056	0.7925	12	-0.1573	
13	0.0243		7.9392	0.8475	13	-0.0154	
14	-0.0372		8.0224	0.8882	14	-0.1475	
15	0.1395		9.2661	0.8632	15	0.0522	
16	-0.2634		13.9978	0.5989	16	-0.2580	
17	-0.1430		15.4924	0.5601	17	-0.0489	
18	-0.1714		17.8041	0.4686	18	-0.1277	
19	-0.0497		18.0147	0.5215	19	-0.1058	
20	-0.0150		18.0356	0.5851	20	-0.0115	
21	-0.1237		19.5999	0.5467	21	-0.1262	
22	-0.0419		19.7995	0.5956	22	0.0478	
23	0.0143		19.8257	0.6524	23	0.1060	
24	-0.0634		20.4128	0.6731	24	-0.1093	
25	-0.0303		20.5689	0.7164	25	0.0527	

3. Model in SAS with outlier and Forecast



Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	12.09594	3.76212	3.22	0.0013	0
MA1,1	0.51321	0.19208	2.67	0.0075	1
AR1,1	-0.56738	0.17497	-3.24	0.0012	1

Constant Estimate	18.95888
Variance Estimate	4855.046
Std Error Estimate	69.67816
AIC	400.4867
SBC	405.1327
Number of Residuals	35

Correlations of Parameter Estimates

Parameter	MU	MA1,1	AR1,1
MU	1.000	-0.081	-0.039
MA1,1	-0.081	1.000	0.543
AR1,1	-0.039	0.543	1.000

Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	3.46	4	0.4839	0.022	0.026	0.072	0.138	0.231	-0.048
12	11.19	10	0.3427	0.008	-0.040	0.047	0.213	0.274	-0.143
18	18.07	16	0.3201	-0.037	0.085	0.157	-0.214	-0.076	-0.127
24	23.82	22	0.3569	-0.141	0.001	-0.110	0.023	-0.150	-0.069

Outlier Detection Summary

Maximum number searched	5
Number found	0
Significance used	0.001

Forecasts for variable Sale

Obs	Forecast	Std Error	95% Confidence Limits		Actual	Residual
24	394.4425	69.6782	257.8758	531.0092	342.3000	-52.1425
25	343.8773	69.6782	207.3106	480.4440	339.7000	-4.1773
26	362.2779	69.6782	225.7112	498.8446	440.4000	78.1221
27	362.1310	69.6782	225.5643	498.6976	315.9000	-46.2310
28	429.2234	69.6782	292.6567	565.7901	439.3000	10.0766
29	383.0734	69.6782	246.5067	519.6400	401.3000	18.2266
30	432.4650	69.6782	295.8983	569.0317	437.4000	4.9350
31	433.3439	69.6782	296.7772	569.9106	575.5000	142.1561
32	443.1481	69.6782	306.5814	579.7148	407.6000	-35.5481
33	540.0649	69.6782	403.4982	676.6316	682.0000	141.9351
34	472.4283	69.6782	335.8616	608.9949	475.3000	2.8717
35	610.0615	69.6782	473.4948	746.6282	581.3000	-28.7615
36	554.8779	69.6782	418.3112	691.4446	646.9000	92.0221
37	581.4122	69.6782	444.8455	717.9788	.	.
38	637.5272	69.9041	500.5178	774.5366	.	.

FINAL MODEL

Model Recommendation

The ARIMA (1,1,1) model turned out be balanced in its predictions. Both, its AIC and SBC values, are very close in values and in ranks fifth for AIC and second for SBC indicating this conclusion. Based on it forecasting nature, its predicted results were the most accurate in comparison to the actual values of the last four months of shampoo sales.

Kanza Mazhar

Truc Vo

Mudiha Wazirali

Weaknesses

Due to the small dataset, ARIMA(1,1,1) model may not capture possible seasonality, and outliers because in the process of exploring the ARIMA (1,1,1), we found no outliers or level shifts. We were initially curious as to whether there may be some seasonality from the original time series plot, but as in other models, the ARIMA (1,1,1) did not indicate any level shifts or outliers. However, we do not want to dismiss the concern there might be seasonality present. Our recommendation is to increase our data set to further validate the conclusion that seasonality is not present.

Pros

ARIMA(1,1,1) was both stable and invertible. Residual analysis presented the model to be valid due to no correlation being present. Its parameters were significant except for AR1. The iteration history also shows that while ARIMA (1,1,1,) went through 23 steps which was greater than the other two comparable models. The cost of processing it was not much greater because there was not much of difference in iteration steps from IMA(1,2): 18 steps. The forecasting model showed that there was consistency in the residual difference as time proceeded. It did not increase into a cone shape. However, when comparing the predicted values with the actual values, there was an apparent need in increasing the data set so that prediction may be more accurate and parsimonious of the actual trend of the data. It was the best model available for the small data set present, but it also made clear the need for including more data into the analysis process.

FORECAST & BUSINESS RECOMMENDATIONS

The ARIMA(1,1,1) model shows a forecast of 512.87M sales for September, 480.07M sales for October, 508.90M for November, and 510.22M sales for December.

Month	Actual Sale	Forecast-IMA(1,2)	Forecast - ARIMA(1,1,1)
Jul-19	575.50	426.98	424.25
Aug-19	407.60	426.00	433.39
Sep-19	682.00	504.03	512.87
Oct-19	475.30	505.95	480.07
Nov-19	581.30	515.97	508.90
Dec-19	646.90	526.00	510.22

Overall, we see a steady project increase in shampoo sales for Company A by the end of the year with sales nearly doubling in the last 3 years. We recommend Company A to further invest in the shampoo sales market as our forecasts show an increase in sales and profits by the end of the year.

Kanza Mazhar

Truc Vo

Mudiha Wazirali

For future work, we recommend Company A to review their sales by end of 2020, when they have more data for a model to be able to detect any underlying pattern or seasonality that our current recommended model is not able to detect.

Kanza Mazhar

Truc Vo

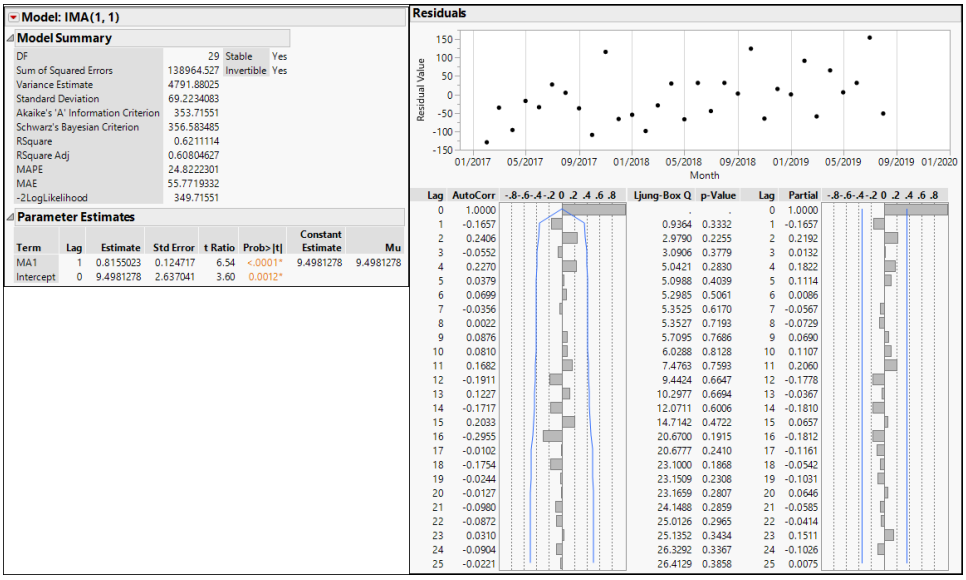
Mudiha Wazirali

APPENDIX

Additional Models that we explored

IMA(1,1)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. IMA(1,1) is a valid model.
2. This model is parsimonious due to the parameter is statistically significant.



Model: IMA(1, 1)

Model Summary

DF	29	Stable	Yes
Sum of Squared Errors	138964.527	Invertible	Yes
Variance Estimate	4791.88025		
Standard Deviation	69.2234083		
Akaike's 'A' Information Criterion	353.71551		
Schwarz's Bayesian Criterion	356.583485		
RSquare	0.6211114		
RSquare Adj	0.60804627		
MAPE	24.8222301		
MAE	55.7719332		
-2LogLikelihood	349.71551		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
MA1	1	0.8155023	0.124717	6.54	<.0001*	9.4981278	9.4981278
Intercept	0	9.4981278	2.637041	3.60	0.0012*		

Residuals

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.1657	0.9364	0.3332	1	-0.1657
2	0.2406	2.9790	0.2255	2	0.2192
3	-0.0552	3.0906	0.3779	3	0.0132
4	0.2270	5.0421	0.2830	4	0.1822
5	0.0379	5.0988	0.4039	5	0.1114
6	0.0699	5.2985	0.5061	6	0.0086
7	-0.0356	5.3525	0.6170	7	-0.0567
8	0.0022	5.3527	0.7193	8	-0.0729
9	0.0876	5.7095	0.7686	9	0.0690
10	0.0810	6.0288	0.8128	10	0.1107
11	0.1682	7.4763	0.7593	11	0.2060
12	-0.1911	9.4424	0.6647	12	-0.1778
13	0.1227	10.2977	0.6694	13	-0.0367
14	-0.1717	12.0711	0.6006	14	-0.1810
15	0.2033	14.7142	0.4722	15	0.0657
16	-0.2955	20.6700	0.1915	16	-0.1812
17	-0.0102	20.6777	0.2410	17	-0.1161
18	-0.1754	23.1000	0.1868	18	-0.0542
19	-0.0244	23.1509	0.2308	19	-0.1031
20	-0.0127	23.1659	0.2807	20	0.0646
21	-0.0980	24.1488	0.2859	21	-0.0585
22	-0.0872	25.0126	0.2965	22	-0.0414
23	0.0310	25.1352	0.3434	23	0.1511
24	-0.0904	26.3292	0.3367	24	-0.1026
25	-0.0221	26.4129	0.3858	25	0.0075

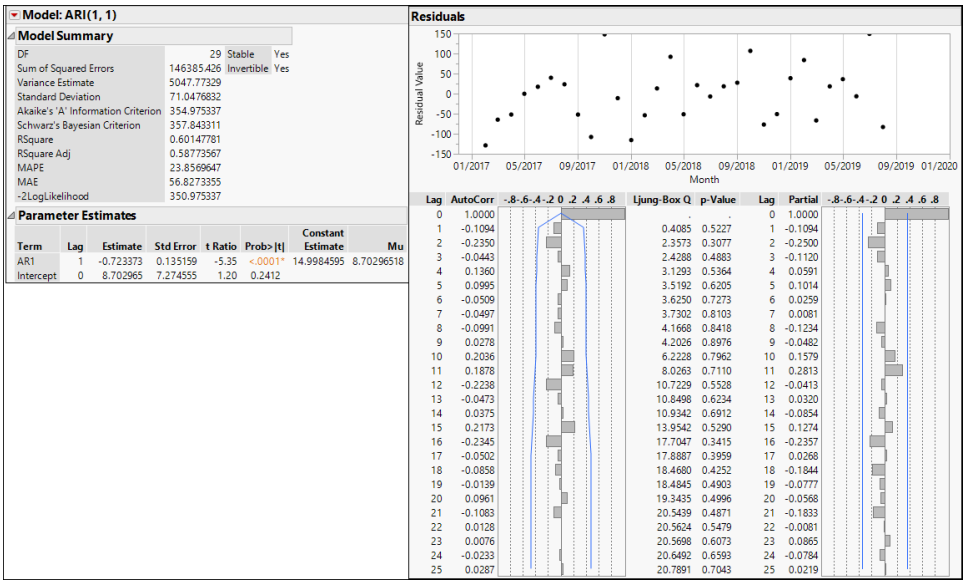
ARI(1,1)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. ARI(1,1) is a valid model.
2. This model is parsimonious due to the parameter is statistically significant.

Kanza Mazhar

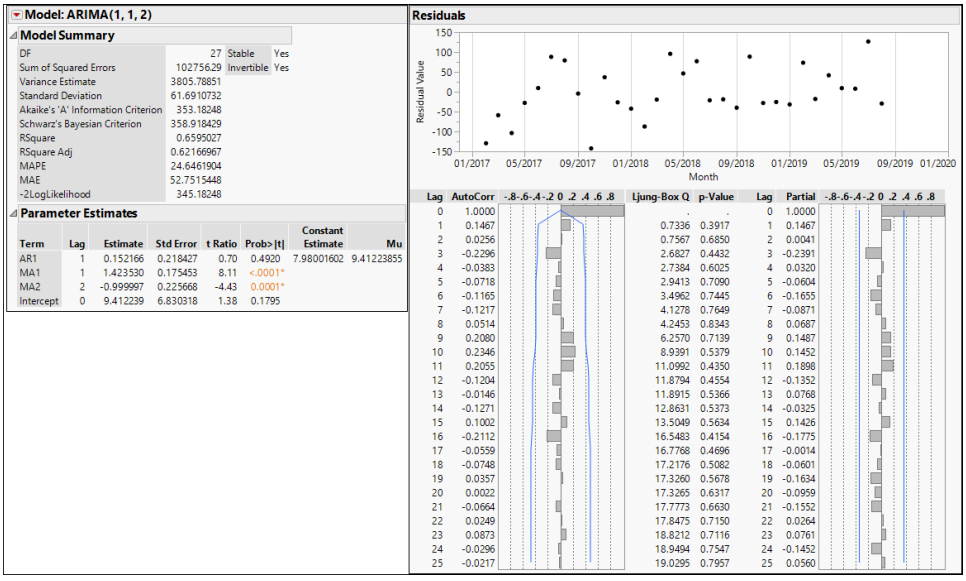
Truc Vo

Mudiha Wazirali



ARIMA(1,1,2)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. ARIMA(1,1,2) is a valid model.
2. This model is not parsimonious due to the parameter AR1 is not statistically significant.



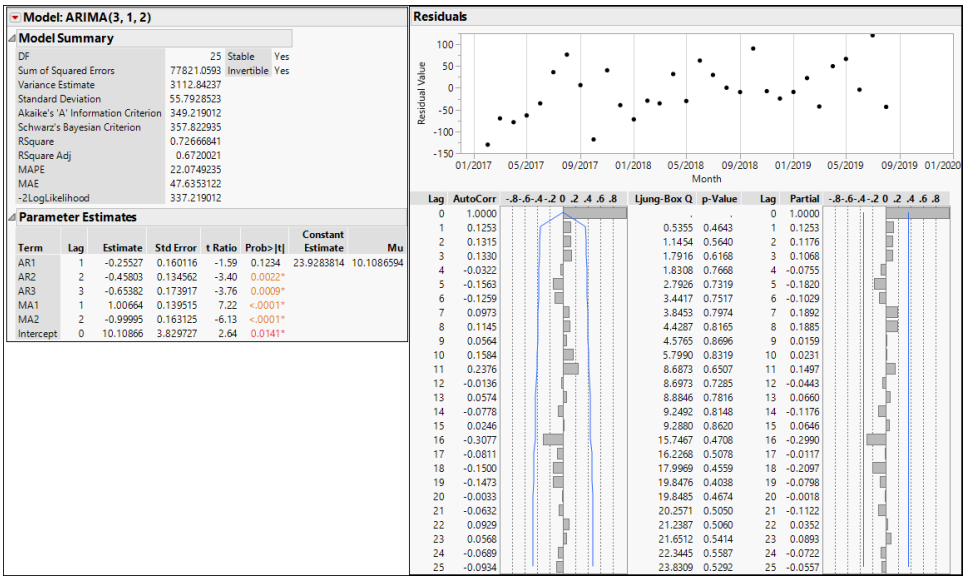
ARIMA(3,1,2)

1. Residual consistence with white noise due to Ljung-Box Q value is not statistically significant. ARIMA(3,1,2) is a valid model.
2. This model is not parsimonious due to the parameter AR1 is not statistically significant.

Kanza Mazhar

Truc Vo

Mudiha Wazirali



Model: ARIMA(3, 1, 2)

Model Summary

DF	25	Stable	Yes
Sum of Squared Errors	77821.0593	Invertible	Yes
Variance Estimate	3112.84237		
Standard Deviation	55.7928523		
Akaike's 'A' Information Criterion	349.219012		
Schwarz's Bayesian Criterion	357.822935		
RSquare	0.72666841		
RSquare Adj	0.6720021		
MAPE	22.0749235		
MAE	47.6353122		
-2LogLikelihood	337.219012		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate	Mu
AR1	1	-0.25527	0.160116	-1.59	0.1234	23.9283814	10.1086594
AR2	2	-0.45803	0.134562	-3.40	0.0022*		
AR3	3	-0.65382	0.173917	-3.76	0.0009*		
MA1	1	1.00664	0.139515	7.22	<.0001*		
MA2	2	-0.99995	0.163125	-6.13	<.0001*		
Intercept	0	10.10866	3.829727	2.64	0.0141*		

Residuals

Lag	AutoCorr	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	0.1253		0.5355	0.4643	1	0.1253	
2	0.1315		1.1454	0.5640	2	0.1176	
3	0.1330		1.7916	0.6168	3	0.1068	
4	-0.0322		1.8308	0.7668	4	-0.0755	
5	-0.1563		2.7926	0.7319	5	-0.1820	
6	-0.1259		3.4417	0.7517	6	-0.1029	
7	0.0973		3.8453	0.7974	7	0.1892	
8	0.1145		4.4287	0.8165	8	0.1885	
9	0.0564		4.5765	0.8696	9	0.0159	
10	0.1584		5.7990	0.8319	10	0.0231	
11	0.2376		8.6873	0.6507	11	0.1497	
12	-0.0136		8.6973	0.7285	12	-0.0443	
13	0.0574		8.8846	0.7816	13	0.0660	
14	-0.0778		9.2492	0.8148	14	-0.1176	
15	0.0246		9.2880	0.8620	15	0.0646	
16	-0.3077		15.7467	0.4708	16	-0.2990	
17	-0.0811		16.2268	0.5078	17	-0.0117	
18	-0.1500		17.9969	0.4559	18	-0.2097	
19	-0.1473		19.8476	0.4038	19	-0.0798	
20	-0.0033		19.8485	0.4674	20	-0.0018	
21	-0.0632		20.2571	0.5050	21	-0.1122	
22	0.0929		21.2387	0.5060	22	0.0352	
23	0.0568		21.6512	0.5414	23	0.0893	
24	-0.0689		22.3445	0.5587	24	-0.0722	
25	-0.0934		23.8309	0.5292	25	-0.0557	