Position-Normalized CTR Formulation

Variable Definitions

i : query-ad pair

j : ad position

v_{ij} : number of impressions

c_{ij} : number of clicks

p : vector of ad relevance CTR p_is

q : vector of positional priors q_js

Formulation

Number of clicks can be assumed to follow binomial distribution:

$c_{ij} \sim \text{Binomial}\ (v_{ij}, p_i q_j), \forall i, j$

However, if p is small and as n is sufficiently large, you can model it instead as a Poisson distribution:

$c_{ij} \sim \text{Poisson}\ (v_{ij} p_i q_j), \forall i, j$

In order to regularize lower positions that get very few impressions and clicks, we add in a gamma prior on the positional factor, since it is a good fit for that distribution:

$q_j \sim \text{Gamma}\ (\alpha, \beta), \forall j$

The regularization only applies to the E-step, and when $alpha = 1$ and $beta \to \infty$, the gamma distribution approaches uniform, i.e. no prior.

EM Steps

You can look at the derivation yourself in the paper, but the final formulas for the expectation and maximization steps are:

E-Step: $q_j^{'} \leftarrow \frac{\sum_i c_{ij} + (\alpha - 1)}{\sum_i v_{ij} p_i + 1/\beta}$

M-Step: $p_i^{'} \leftarrow \frac{\sum_j c_{ij}}{\sum_j v_{ij} q_j}$

I will set the initial p values to the average CTRs across all i and the q values to the average CTRs across all j