

THE RADIO SKY AT METER WAVELENGTHS: *M*-MODE ANALYSIS IMAGING WITH THE OWENS VALLEY LONG WAVELENGTH ARRAY

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ABSTRACT

A host of new low-frequency radio telescopes are attempting to detect the 21 cm transition at cosmological distances. These measurements hold promise to directly probe star and galaxy formation at redshifts $20 \gtrsim z \gtrsim 7$, but are limited by the dynamic range they can achieve against foreground sources of low-frequency radio emission. Consequently, there is a growing demand for modern, high-fidelity maps of the sky at frequencies below 200 MHz for use in foreground modeling and removal. We use the Owens Valley Long Wavelength Array (OVRO-LWA) to generate 8 new maps of the sky north of $\delta = -30^\circ$ with 15 arcmin angular resolution and at frequencies evenly spaced between 36.528 MHz and 73.152 MHz with 24 kHz bandwidth. Existing full-sky maps at these frequencies have angular resolutions $\geq 2^\circ$ and so these maps are a 10-fold improvement in angular resolution. The thermal noise in each map is ~ 800 mJy/beam. Future iterations will incorporate total power radiometry, and additional bandwidth and integration time. This will improve the thermal noise while simultaneously pushing to higher angular resolutions with the planned expansion of the OVRO-LWA to 2.6 km baselines. These maps are generated from the application of a new widefield imaging technique for drift-scanning interferometers, Tikhonov-regularized *m*-mode analysis imaging. This algorithm constructs images of the entire sky in a single synthesis imaging step with exact treatment of widefield effects. We additionally introduce this technique and describe how CLEAN can be adapted to deconvolve maps generated by *m*-mode analysis imaging. These maps serve as a first step on the path to the use of more sophisticated foreground filters in 21 cm cosmology incorporating the measured angular and frequency structure of all foreground contaminants.

Keywords: cosmology: observations – dark ages, reionization, first stars – radio continuum: galaxies – radio continuum: ISM

1. INTRODUCTION

At redshifts $20 \gtrsim z \gtrsim 7$ the 21 cm hyperfine structure line of neutral hydrogen is expected to produce a 10 to 100 mK perturbation in the Cosmic Microwave Background (CMB) spectrum (Furlanetto et al. 2006; Pritchard & Loeb 2012). The amplitude of this perturbation on a given line-of-sight is a function of the neutral fraction of hydrogen, the baryon overdensity, the spin temperature relative to the CMB temperature at the given redshift, and the line-of-sight peculiar velocity of the gas. The spatial power spectrum of this perturbation is thought to be dominated by inhomogeneous heating of the IGM at $z \sim 20$ (Fialkov et al. 2014), and by growing ionized bubbles during the EoR at $z \sim 7$ where a detection can constrain the ionizing efficiency of early galaxies, the UV photon mean-free-path, and the minimum halo mass that can support star formation (Greig & Mesinger 2015).

Current 21 cm cosmology experiments can be broadly separated into two classes: global experiments that are aiming to detect the spectral signature of the cosmologically redshifted 21 cm transition after averaging over the entire sky (otherwise known as the monopole), and power spectrum experiments that incorporate angular information to attempt to measure the 3D spatial power spectrum of cosmological 21 cm perturbations. Ongoing global experiments include EDGES (Bowman & Rogers 2010; Monsalve et al. 2017), LEDA (Price et al. 2017), BIGHORNS (Sokolowski et al. 2015), SCI-HI (Voytek et al. 2014), and SARAS 2 (Singh et al. 2017). Ongoing power spectrum experiments include PAPER/HERA (Ali et al. 2015; DeBoer et al. 2016), LOFAR (Patil et al. 2017), and the MWA (Beardsley et al. 2016; Ewall-Wice et al. 2016).

Just as for CMB experiments, foreground removal or suppression is an essential component of both classes of 21 cm cosmology experiments. The brightness temperature of the galactic synchrotron emission at high galactic latitudes is measured by Rogers & Bowman (2008) as

$$T \sim 300 \text{ K} \times \left(\frac{\nu}{150 \text{ MHz}} \right)^{-2.5}. \quad (1)$$

Therefore experiments conservatively need to achieve 5 orders of dynamic range against this foreground emission before the cosmological signal can be measured. Current foreground removal methods (for example, Parsons et al. 2012) rely on the assumption that the foreground emission is spectrally smooth. However the low-frequency radio sky is composed of several components: galactic synchrotron emission, supernova remnants, radio galaxies, free-free emission and absorption from H II regions, and a confusing background of radio

sources. Ideally a foreground removal strategy should be informed by the measured spatial structure and frequency spectrum of all foreground components. This possibility is limited by the availability of suitable high-fidelity low-frequency sky maps on angular scales ranging from tens of degrees to arcminutes.

The Global Sky Model (GSM) (de Oliveira-Costa et al. 2008; Zheng et al. 2017a) is currently the most commonly used foreground model. The GSM is a data-driven interpolation of various maps between 10 MHz and 100 GHz. However the majority of information contained in the GSM is derived at frequencies > 1.4 GHz where the majority of the modern, high-fidelity input maps are located. Below 408 MHz, the GSM is a strong function of the Haslam 408 MHz map (Haslam et al. 1981, 1982; Remazeilles et al. 2015). As a consequence, the GSM is not a faithful representation of the sky below 200 MHz. For instance, Dowell et al. (2017) report errors of $\pm 50\%$ between the GSM and their own maps at 74 MHz. This is largely due to the increasing contribution of free-free absorption and modifications to the synchrotron spectral index at low frequencies.

Recently a host of new low-frequency sky surveys have been conducted including MSSS (Heald et al. 2015), GLEAM (Wayth et al. 2015), and TGSS (Intema et al. 2017). However, the primary data product generated by these surveys is a catalog of radio point sources. At 45 MHz, Guzmán et al. (2011) created a map of the sky that captures the diffuse emission with 5° resolution. The LWA1 Low Frequency Sky Survey (Dowell et al. 2017) similarly maps the sky at a range of frequencies between 35 MHz and 80 MHz with resolution between 4.5° and 2° .

This paper presents a series of new low-frequency maps of the sky between 36.528 MHz and 73.152 MHz, capturing the full sky visible from the Owens Valley Radio Observatory (OVRO) with angular resolution of ~ 15 arcmin. We aim for these maps to inform foreground removal strategies in 21 cm cosmology as they represent some of the highest angular resolution full-sky maps ever produced at these frequencies. We anticipate additional ancillary science taking advantage of the combination of high fidelity and high resolution of these maps, including but not limited to studies of the cosmic ray emissivity at low frequencies, searches for giant radio galaxies, and constraining the galactic synchrotron spectrum. The maps will be made freely available online at the Legacy Archive for Microwave Background Data Analysis (LAMBDA)¹.

¹ <https://lambda.gsfc.nasa.gov/>

The structure of this paper is as follows. In §2, we present Tikhonov-regularized m -mode analysis imaging, a new imaging technique that allows us to image the entire visible sky in one coherent synthesis imaging step with exact widefield corrections. In §3 we describe our observations with the Owens Valley Long Wavelength Array (OVRO-LWA). In §4 we present the sky maps, and in §5 we discuss some of the sources of error present in the maps. Finally in §6 the new sky maps are compared against other low-frequency sky maps, and in §7 we present our conclusions.

2. ALL-SKY IMAGING

The goal of all imaging algorithms is to estimate the brightness of the sky $I_\nu(\hat{r})$ in the direction \hat{r} and frequency ν . A radio interferometer measures the visibilities $V_\nu^{ij,pq}$ between pairs of antennas numbered i and j respectively, and between polarizations labeled p and q respectively. We will neglect subtleties associated with polarized imaging, so the Stokes- I visibilities are constructed from the sum of the xx and yy correlations such that $V_\nu^{ij} = (V_\nu^{ij,xx} + V_\nu^{ij,yy})/2$. If the antennas are separated by the baseline \vec{b}_{ij} , and $A_\nu(\hat{r})$ describes an antenna's response to the incident Stokes- I radiation (here assumed to be the same for each antenna), then

$$V_\nu^{ij} = \int_{\text{sky}} A_\nu(\hat{r}) I_\nu(\hat{r}) \exp\left(2\pi i \hat{r} \cdot \vec{b}_{ij}/\lambda\right) d\Omega. \quad (2)$$

Constructing an image from the output of a radio interferometer consists of estimating $I_\nu(\hat{r})$ given the available measurements V_ν^{ij} .

For later convenience we will define the baseline transfer function $B_\nu^{ij}(\hat{r})$ such that

$$V_\nu^{ij} = \int_{\text{sky}} B_\nu^{ij}(\hat{r}) I_\nu(\hat{r}) d\Omega. \quad (3)$$

The baseline transfer function defines the response of a single baseline to the sky, and is a function of the antenna primary beam, and baseline length and orientation.

Naively one might attempt to solve Equation 2 by discretizing, and subsequently solving the resulting matrix equation. If the interferometer is composed of N_{base} baselines, and measures N_{freq} frequency channels over N_{time} integrations, then the entire data set consists of $N_{\text{base}}N_{\text{freq}}N_{\text{time}}$ complex numbers. If the sky is discretized into N_{pix} pixels then the relevant matrix has dimensions of $(N_{\text{base}}N_{\text{freq}}N_{\text{time}}) \times (N_{\text{pix}})$. For making single-channel maps with the OVRO-LWA this becomes a 5 petabyte array (assuming each matrix element is a 64-bit complex floating point number). This matrix equation is therefore prohibitively large, and solv-

ing Equation 2 by means of discretization is usually impossible although [Zheng et al. \(2017b\)](#) demonstrate this technique with the MITEOR telescope.

Instead it is common to make mild assumptions that simplify Equation 2 and ease the computational burden in solving for $I_\nu(\hat{r})$. For example, when all of the baselines \vec{b}_{ij} lie in a plane and the field-of-view is small, Equation 2 can be well-approximated by a two-dimensional Fourier transform ([Thompson et al. 2001](#)). The restriction on baseline co-planarity and field-of-view can be relaxed by using W-projection ([Cornwell et al. 2008](#)). Known primary beam effects can also be accounted for during imaging by using A-projection ([Bhatnagar et al. 2013](#)).

2.1. *m*-Mode Analysis

On the other hand, transit telescopes can take advantage of a symmetry in Equation 2 that greatly reduces the amount of computer time required to image the full-sky with exact incorporation of widefield imaging effects. This technique, called *m*-mode analysis, also obviates the need for gridding and mosaicing. Instead the entire sky is imaged in one coherent synthesis imaging step. We will briefly summarize *m*-mode analysis below, but the interested reader should consult [Shaw et al. \(2014, 2015\)](#) for a complete derivation.

In the context of *m*-mode analysis a transit telescope is any interferometer for which the response pattern of the individual elements does not change with respect to time. This may be an interferometer like the OVRO-LWA where the correlation elements are fixed dipoles, but it may also be an interferometer like LOFAR or the MWA if the steerable beams are held in a fixed position (not necessarily at zenith). The interferometer also does not necessarily have to be homogeneous. Heterogeneous arrays composed of several different types of antennas are allowed as long as care is taken to generalize Equation 2 for a heterogeneous array.

For a transit telescope, the visibilities V_ν^{ij} are a periodic function of sidereal time.² Therefore it is a natural operation to compute the Fourier transform of the visibilities with respect to sidereal time $\phi \in [0, 2\pi]$.

$$V_{m,\nu}^{ij} = \int_0^{2\pi} V_\nu^{ij}(\phi) \exp\left(-im\phi\right) d\phi \quad (4)$$

The output of this Fourier transform is the set of *m*-modes $V_{m,\nu}^{ij}$ where $m = 0, \pm 1, \pm 2, \dots$ is the Fourier

² This is not strictly true. Ionospheric fluctuations and non-sidereal sources (such as the Sun) will violate this assumption. This paper will, however, demonstrate that the impact on the final maps is mild.

conjugate variable to the sidereal time. The m -mode corresponding to $m = 0$ is a simple average of the visibilities over sidereal time. Similarly $m = 1$ corresponds to the component of the visibilities that varies over half-day timescales. Larger values of m correspond to components that vary on quicker timescales.

Shaw et al. (2014, 2015) show that there is a discrete linear relationship between the measured m -modes $V_{m,\nu}^{ij}$ and the spherical harmonic coefficients of the sky brightness $a_{lm,\nu}$.

$$V_{m,\nu}^{ij} = \sum_l B_{lm,\nu}^{ij} a_{lm,\nu}, \quad (5)$$

where the transfer coefficients $B_{lm,\nu}^{ij}$ are computed from the spherical harmonic transform of the baseline transfer function defined by Equation 3. These transfer coefficients define the interferometers response to the corresponding spherical harmonic coefficients.

Equation 5 can be recognized as a matrix equation where the transfer matrix \mathbf{B} is block-diagonal.

$$\underbrace{\begin{pmatrix} \vdots \\ m\text{-modes} \\ \vdots \end{pmatrix}}_v = \underbrace{\begin{pmatrix} \ddots & & \\ & \text{transfer matrix} & \\ & & \ddots \end{pmatrix}}_B \underbrace{\begin{pmatrix} \vdots \\ a_{lm} \\ \vdots \end{pmatrix}}_a \quad (6)$$

$$\mathbf{B} = \begin{pmatrix} m=0 & & & \\ & m=\pm 1 & & \\ & & m=\pm 2 & \\ & & & \ddots \end{pmatrix} \quad (7)$$

The vector \mathbf{v} contains the list of m -modes and the vector \mathbf{a} contains the list of spherical harmonic coefficients representing the sky brightness. In order to take advantage of the block-diagonal structure in \mathbf{B} , \mathbf{v} and \mathbf{a} must be sorted by the absolute value of m . Positive and negative values of m are grouped together because the brightness of the sky is real-valued, and the spherical harmonic transform of a real-valued function has $a_{l(-m)} = (-1)^m a_{lm}^*$.

In practice we now need to pick the set of spherical harmonics we will use to represent the sky. For an interferometer like the OVRO-LWA with many short baselines, a sensible choice is to use all spherical harmonics with $l \leq l_{\max}$ for some l_{\max} . The parameter l_{\max} is determined by the maximum baseline length of the interferometer. For an interferometer without short spacings, a minimum value for l might also be used. This l_{\min} parameter should be determined by the minimum baseline

length. A rough estimate of l for a baseline of length b at frequency ν is $l \sim \pi b \nu / c$. Based on this estimate for the OVRO-LWA and other computational considerations we therefore adapt $l_{\min} = 1$ and $l_{\max} = 1000$ across all frequencies. However this choice of l_{\max} actually limits the angular resolution above 55 MHz, and therefore future work will increase l_{\max} to obtain better angular resolution.

The interferometer's sensitivity to the monopole (a_{00}) deserves special consideration. Venumadhav et al. (2016) prove – under fairly general assumptions – that a baseline with nonzero sensitivity to a_{00} must also have some amount of cross-talk or common-mode noise. In fact the sensitivity to a_{00} is proportional to a sum of these effects. For example, one way a baseline can have nonzero sensitivity to a_{00} is if the baseline is extremely short. In this case the antennas are so close together that voltage fluctuations in one antenna can couple into the other antenna. In order to make an interferometric measurement of a_{00} , this coupling must be measured and calibrated. Consequently we set $a_{00} = 0$ in our analysis. In the future this limitation will be addressed with the inclusion of calibrated total power radiometry.

The size of a typical block in the transfer matrix is $(2N_{\text{base}}N_{\text{freq}}) \times (l_{\max})$. If each element of the matrix is stored as a 64-bit complex floating point number, a single block is 500 MB for the case of single-channel imaging with the OVRO-LWA, which a modern computer can easily store and manipulate in memory. However, with additional bandwidth these blocks quickly become unwieldy and so as a first pass the analysis in this paper is restricted to single-channel imaging. Note also that for the OVRO-LWA $N_{\text{base}} \gg l_{\max}$, so there are more measurements than unknowns in Equation 6.

The power of m -mode analysis is the block-diagonal structure of Equation 6. The computational complexity of many common matrix operations (eg. solving a linear system of equations) is $\mathcal{O}(N^3)$. By splitting the equation into N independent blocks, the number of floating point operations required to operate on the full matrix is reduced by a factor of N^2 . This computational savings is what makes this matrix algebra approach to interferometric imaging feasible.

2.2. m -Mode Analysis Imaging

Imaging in m -mode analysis essentially amounts to inverting Equation 6 to solve for the spherical harmonic coefficients \mathbf{a} . The linear-least squares solution, which minimizes $\|\mathbf{v} - \mathbf{Ba}\|^2$, is given by

$$\hat{\mathbf{a}}_{\text{LLS}} = (\mathbf{B}^* \mathbf{B})^{-1} \mathbf{B}^* \mathbf{v}, \quad (8)$$

where $*$ indicates the conjugate-transpose.

However, usually one will find that \mathbf{B} is not full-rank and hence $\mathbf{B}^*\mathbf{B}$ is not an invertible matrix. For example, an interferometer located in the northern hemisphere will never see a region of the southern sky centered on the southern celestial pole. The m -modes contained in the vector \mathbf{v} must contain no information about the sky around the southern celestial pole, and therefore the act of multiplying by \mathbf{B} must destroy some information about the sky. The consequence of this fact is that \mathbf{B} must have at least one singular value that is equal to zero. It is then trivial to show that $\mathbf{B}^*\mathbf{B}$ must have at least one eigenvalue that is equal to zero, which means it is not an invertible matrix.

Another way of looking at the problem is that because the interferometer is not sensitive to part of the southern hemisphere, there are infinitely many possible solutions to Equation 6 that will fit the measured data equally well. We will regularize the problem and apply an additional constraint that prefers a unique yet physically reasonable solution.

2.2.1. Tikhonov Regularization

The process of Tikhonov regularization minimizes $\|\mathbf{v} - \mathbf{Ba}\|^2 + \varepsilon\|\mathbf{a}\|^2$ for some arbitrary value of $\varepsilon > 0$ chosen by the observer. The solution that minimizes this expression is given by

$$\hat{\mathbf{a}}_{\text{Tikhonov}} = (\mathbf{B}^*\mathbf{B} + \varepsilon\mathbf{I})^{-1}\mathbf{B}^*\mathbf{v}. \quad (9)$$

Tikhonov regularization adds a small value ε to the diagonal of $\mathbf{B}^*\mathbf{B}$, fixing the matrix's singularity. By using the singular value decomposition (SVD) of the matrix $\mathbf{B} = \mathbf{U}\Sigma\mathbf{V}^*$. Equation 9 becomes

$$\hat{\mathbf{a}}_{\text{Tikhonov}} = \mathbf{V}(\Sigma^2 + \varepsilon\mathbf{I})^{-1}\Sigma\mathbf{U}^*\mathbf{v}, \quad (10)$$

where

$$\Sigma = \begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \ddots \end{pmatrix}.$$

The diagonal elements of Σ are the singular values of \mathbf{B} . The contribution of each singular component to the Tikhonov-regularized solution is scaled by $\sigma_i/(\sigma_i^2 + \varepsilon)$, where σ_i is the singular value for the i th singular component. Tikhonov regularization therefore acts to suppress any component for which $\sigma_i \lesssim \sqrt{\varepsilon}$. If $\sigma_i = 0$, the component is set to zero.

In practice the measurement \mathbf{v} is corrupted by noise with covariance \mathbf{N} . For illustrative purposes we will assume that $\mathbf{N} = n\mathbf{I}$ for some scalar n . In this case the covariance of the Tikhonov-regularized spherical harmonic coefficients is

$$\mathbf{C} = n\mathbf{V}(\Sigma^2 + \varepsilon\mathbf{I})^{-2}\Sigma^2\mathbf{V}^*. \quad (11)$$

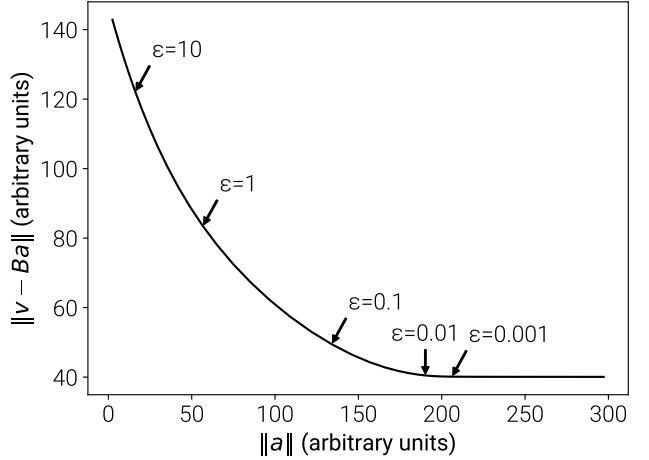


Figure 1. An example L-curve computed from OVRO-LWA data at 36.528 MHz by trialing 200 different values of the regularization parameter ε . The x -axis is the norm of the solution (in this case the spherical harmonic coefficients) given in arbitrary units, and the y -axis is the least-squares norm given in arbitrary units. Where the regularization parameter is small, the norm of the solution grows rapidly. Where the regularization parameter is large, the least-squares norm grows rapidly.

Each singular component is scaled by a factor of $\sigma_i^2/(\sigma_i^2 + \varepsilon)^2$. In the absence of Tikhonov regularization ($\varepsilon = 0$) singular components with the smallest singular values – the ones that the interferometer is the least sensitive to – actually come to dominate the covariance of the measured spherical harmonic coefficients. Tikhonov regularization improves this situation by downweighting these components.

While Tikhonov regularization will force unmeasured modes to zero, if a prior map of the sky already exists, it would be preferable to instead minimize $\|\mathbf{v} - \mathbf{Ba}\|^2 + \varepsilon\|\mathbf{a} - \mathbf{a}_{\text{prior}}\|^2$.

2.2.2. L-Curves

Tikhonov regularization requires the observer to pick the value of ε . If ε is too large then too much importance is placed on minimizing the norm of the solution and the least-squares residuals will suffer. However if ε is too small then the problem will be poorly regularized and the resulting sky map may not represent the true sky. Picking the value of ε therefore requires understanding the trade-off between the two norms.

This trade-off can be analyzed quantitatively by trialing several values of ε , and computing $\|\mathbf{v} - \mathbf{Ba}\|^2$ and $\|\mathbf{a}\|^2$ for each trial. An example is shown in Figure 1. The shape of this curve has a characteristic L-shape, and as a result this type of plot is called an L-curve. The ideal value of ε lies near the turning point of the plot. At this point a small decrease in ε will lead to an

undesired rapid increase in $\|\mathbf{a}\|^2$, and a small increase in ε will lead to an undesired rapid increase in $\|\mathbf{v} - \mathbf{B}\mathbf{a}\|^2$.

In practice, the L-curve should be used as a guide to estimate a reasonable value of ε . However better results can often be obtained by tuning the value of ε . For instance increasing the value of ε can improve the noise properties of the map by down-weighting noisy modes. Decreasing the value of ε can improve the resolution in the map by up-weighting the contribution of longer baselines, which are likely fewer in number. In this respect choosing the value of ε is analogous to picking the weighting scheme in traditional imaging where robust weighting schemes can be tuned to similar effect (Briggs 1995). For the OVRO-LWA we selected $\varepsilon = 0.01$ across all frequency channels. The distribution of singular values of the transfer matrix with respect to $\sqrt{\varepsilon}$ is summarized in Table 1.

2.2.3. The Moore-Penrose Pseudoinverse

The Moore-Penrose pseudoinverse (denoted in this paper with a superscript \dagger), is commonly applied to find the minimum-norm linear-least squares solution to a set of linear equations. This can be used in place of Tikhonov regularization as

$$\hat{\mathbf{a}}_{\text{Moore-Penrose}} = \mathbf{B}^\dagger \mathbf{v}. \quad (12)$$

Much like Tikhonov regularization, the Moore-Penrose pseudoinverse sets components with small singular values (below some user-defined threshold) to zero. Components with large singular values (above the user-defined threshold) are included in the calculation at their full amplitude with no down-weighting of modes near the threshold. The essential difference between using the Moore-Penrose pseudoinverse and Tikhonov regularization is that the pseudoinverse defines a hard transition from on to off. Modes are either set to zero or included in the map at their full amplitude. On the other hand Tikhonov regularization smoothly interpolates between these behaviors. Because of this, Tikhonov regularization tends to produce better results in practical applications.

2.2.4. Other Regularization Schemes

In compressed-sensing applications, the L^1 -norm is used in place of the L^2 -norm in order to encourage sparsity in the reconstructed signal. By expressing the sky in terms of spherical harmonics there is no such expectation of sparsity in the solution, and so the L^1 -norm is inappropriate for this application.

Wiener filtering can also be used in place of Tikhonov regularization in the event that the observer has a prior expectation on the covariance of the spherical harmonic

coefficients in order to interpolate missing information. This technique is demonstrated for simulated measurements by Berger et al. (2016).

In this work we elect to pursue Tikhonov regularization due to its computational simplicity, smooth down-weighting of noisy modes, and independence of prior information.

2.3. CLEAN

In traditional radio astronomy imaging, CLEAN (Högbom 1974) is a physically motivated algorithm that interpolates between measured visibilities on the uv -plane. In the absence of this interpolation, gaps in the interferometer's uv -coverage are assumed to be zero, and – in the image plane – sources are convolved with a point spread function (PSF) that is characteristic of the uv -coverage. Fundamentally, the interferometer's PSF is determined by which modes were assumed to be zero in the initial imaging process.

In m -mode analysis imaging, we assumed modes were zero in two separate ways:

1. We selected a set of spherical harmonic coefficients a_{lm} to describe the sky brightness distribution. All modes with $l > l_{\max}$ are neglected and assumed to be zero.
2. Tikhonov regularization forces linear combinations of spherical harmonic coefficients with $\sigma_i \lesssim \sqrt{\varepsilon}$ towards zero.

As a consequence the final map of the sky is not assembled from a complete set of spherical harmonics. Therefore just as in traditional imaging, m -mode analysis imaging produces dirty maps in which sources are convolved with a PSF. This PSF can be improved by increasing the number and variety of baselines, which increases the number of modes for which $\sigma_i \gg \sqrt{\varepsilon}$. Alternatively by collecting more data, the signal-noise ratio of the measured m -modes increases, which allows the observer to lower the value of ε without increasing the noise in the maps. Finally, the CLEAN algorithm can be applied to interpolate some of the missing information that was assumed to be zero.

The PSF of a dirty m -mode analysis map may be computed with

$$\mathbf{a}_{\text{PSF}}(\theta, \phi) = (\mathbf{B}^* \mathbf{B} + \varepsilon \mathbf{I})^{-1} \mathbf{B}^* \mathbf{B} \mathbf{a}_{\text{PS}}(\theta, \phi), \quad (13)$$

where $\mathbf{a}_{\text{PSF}}(\theta, \phi)$ is the vector of spherical harmonic coefficients representing the PSF at the spherical coordinates (θ, ϕ) , and $\mathbf{a}_{\text{PS}}(\theta, \phi)$ is the vector of spherical harmonic coefficients for a point source at (θ, ϕ) given by

$$\mathbf{a}_{\text{PS}}(\theta, \phi) = \begin{pmatrix} \vdots \\ Y_{lm}^*(\theta, \phi) \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ Y_{lm}^*(\theta, 0) \times e^{im\phi} \\ \vdots \end{pmatrix}. \quad (14)$$

In general the PSF can be a function of the right ascension and declination. However point sources at the same declination take the same track through the sky and therefore (barring any ionospheric effects) will have the same PSF. The PSF is therefore only a function of the declination. For example, sources at low elevations will tend to have an extended PSF along the north-south axis due to baseline foreshortening. For the OVRO-LWA antenna configuration (Figure 2), example PSFs at three separate frequencies are shown in Figure 3. Adapting CLEAN for m -mode analysis requires either pre-computing Equation 13 at a grid of declinations, or a method for rapidly evaluating Equation 13 on the fly.

For an interferometer with more baselines than spherical harmonics used in the maps (eg. the OVRO-LWA), $\mathbf{B}^* \mathbf{B}$ can be a much smaller matrix than the full transfer matrix \mathbf{B} . Therefore pre-computing $\mathbf{B}^* \mathbf{B}$ can allow the entire matrix to fit into memory on a single machine. This greatly reduces the amount of disk I/O necessary for solving Equation 13.

Additionally we can pre-compute the Cholesky decomposition of $\mathbf{B}^* \mathbf{B} + \varepsilon \mathbf{I} = \mathbf{U}^* \mathbf{U}$, where \mathbf{U} is an upper-triangular matrix. Inverting an upper triangular matrix is an $\mathcal{O}(N^2)$ operation (instead of $\mathcal{O}(N^3)$ for a general matrix inverse).³ Equation 13 can then be rapidly evaluated from right to left as:

$$\mathbf{a}_{\text{PSF}} = \mathbf{U}^{-1} (\mathbf{U}^*)^{-1} (\mathbf{B}^* \mathbf{B}) \mathbf{a}_{\text{PS}}. \quad (15)$$

Furthermore, Equation 15 does not need to be separately evaluated for each CLEAN component. Instead we can identify N CLEAN components, accumulate \mathbf{a}_{PS} for each component, and evaluate Equation 15 on the accumulation. This can greatly reduce the number of times this equation needs to be evaluated, but care must be taken to ensure that the N components are not so close together that sidelobes from one may interact with another.

Altogether the adaptation of CLEAN applied to the maps presented in this paper is summarized below.

Precondition: \mathbf{a} is the solution to Equation 9

³ Instead of computing \mathbf{A}^{-1} , we solve the linear equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ each time the matrix inverse is needed so as to avoid numerical instabilities.

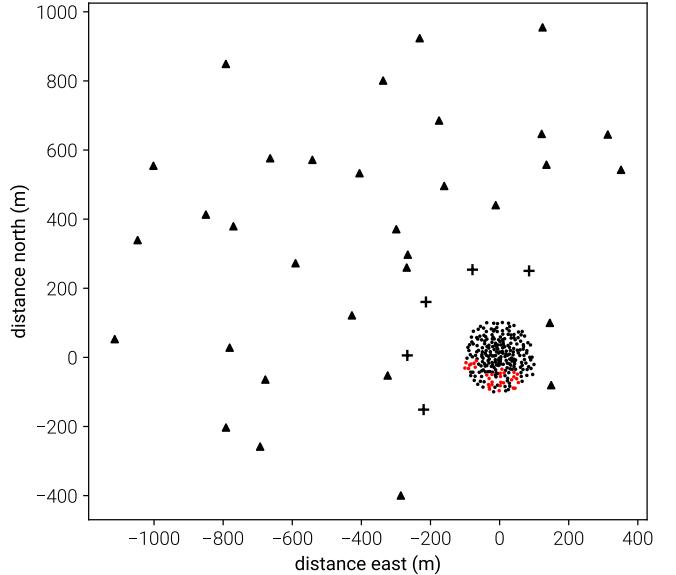


Figure 2. This figure shows the antenna layout for the OVRO-LWA. Black dots correspond to antennas within the 200 m diameter core of the array. The 32 triangles are the expansion antennas built in early 2016 in order to increase the longest baseline to ~ 1.5 km. The red dots are core antennas that are disconnected from the correlator in order to accommodate these antennas. The 5 crosses are antennas equipped with noise-switched front ends.

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1: function CLEAN( $\mathbf{a}$ )
2:    $\mathbf{M} \leftarrow \mathbf{B}^* \mathbf{B}$ 
3:    $\mathbf{U} \leftarrow \text{chol}(\mathbf{M} + \varepsilon \mathbf{I})$   $\triangleright$  Cholesky decomposition
4:   while noise in map > threshold do
5:     find  $N$  pixels with the largest residual flux
6:      $\mathbf{x} \leftarrow \sum_{i=1}^N (\text{pixel flux}) \times \mathbf{a}_{\text{PS}}(\theta_i, \phi_i)$ 
7:      $\mathbf{y} \leftarrow \mathbf{U}^{-1} (\mathbf{U}^*)^{-1} \mathbf{M} \mathbf{x}$ 
8:      $\mathbf{a} \leftarrow \mathbf{a} - (\text{loop gain}) \times \mathbf{y}$ 
9:     record subtracted components
10:     $\mathbf{a} \leftarrow \mathbf{a} + (\text{restored components})$ 
11:   return  $\mathbf{a}$ 
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In summary, Tikhonov-regularized m -mode analysis imaging constructs a widefield synthesis image of the sky from a complete earth rotation, and with exact treatment of widefield-effects. This is accomplished by solving a regularized block-diagonal matrix equation (Equation 9). The solution to this equation generates a map where sources are convolved with a PSF characteristic of the interferometer (a function of the frequency, antenna response, and baseline distribution with a full Earth rotation). The CLEAN algorithm is adopted to deconvolve the PSF and produce the final sky maps.

3. OBSERVATIONS

3.1. The Owens Valley Long Wavelength Array

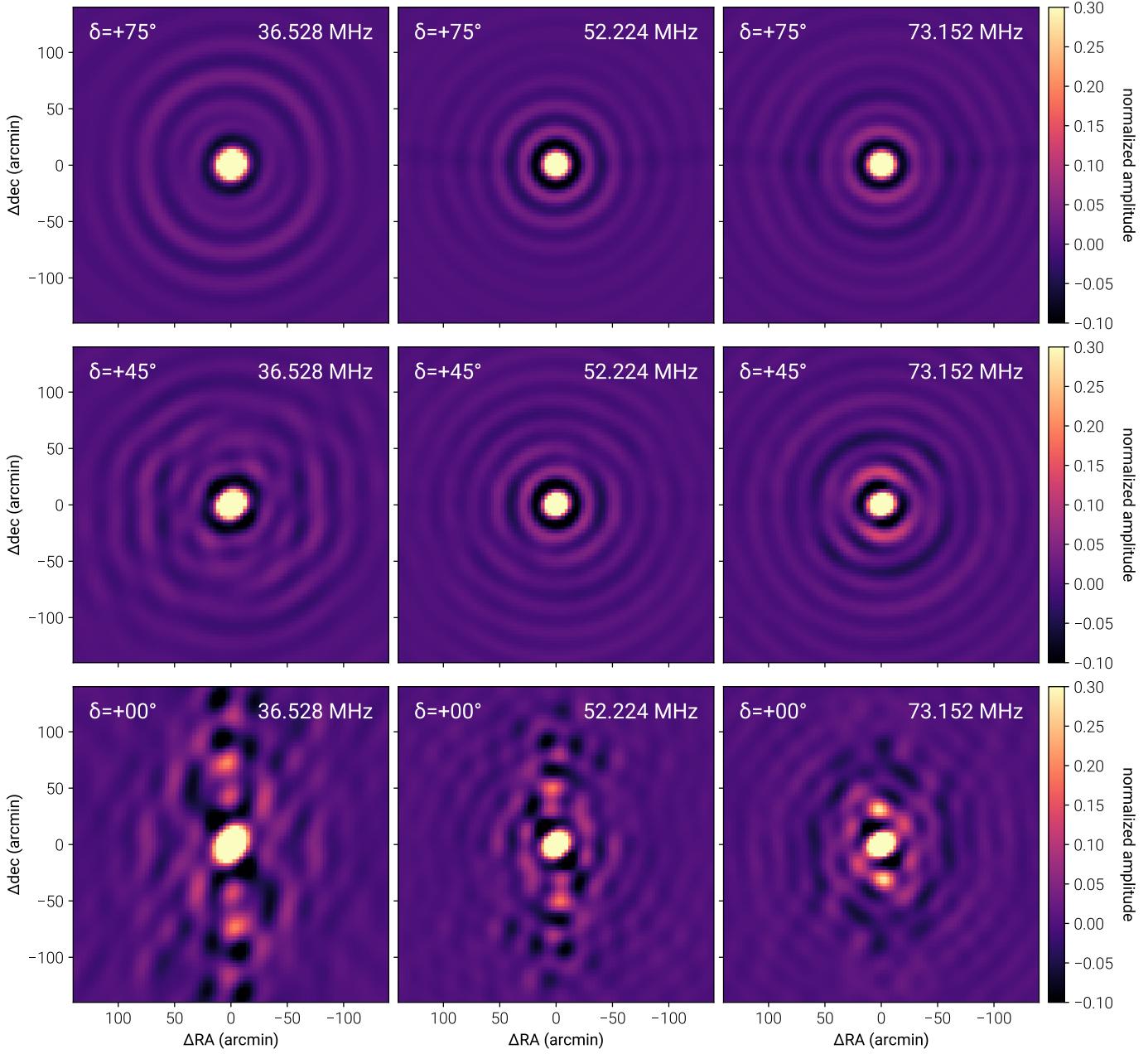


Figure 3. The m -mode analysis imaging PSF at three declinations (top row: $\delta = +75^\circ$, middle row: $\delta = +45^\circ$, bottom row: $\delta = +00^\circ$) and three frequencies (left column: 36.528 MHz, middle column: 52.224 MHz, right column: 73.152 MHz). The PSF is computed by evaluating Equation 13. Above 55 MHz, the angular extent of the PSF does not follow the expected scaling with frequency because the angular resolution is limited by the selection of $l_{\max} = 1000$. The FWHM at $\delta = +45^\circ$ is listed in Table 1.

The Owens Valley Long Wavelength Array (OVRO-LWA) is a 288-element interferometer located at the Owens Valley Radio Observatory near Big Pine, California (Hallinan et al. in prep.). The OVRO-LWA is a low-frequency instrument with instantaneous bandwidth covering 27 MHz to 85 MHz and with 24 kHz channelization. Each antenna stand hosts two perpendicular broadband dipoles so that there are 288×2 signal paths in total. These signal paths feed into the 512-input

LEDA correlator (Kocz et al. 2015), which allows the OVRO-LWA to capture the entire visible hemisphere in a single snapshot image.

The 288 antennas are arranged in a pseudo-random configuration optimized to minimize sidelobes in snapshot imaging (see Figure 2). Of these 288 antennas, 251 are contained within a 200 m diameter core, 32 are placed outside of the core in order to extend the maximum baseline length to ~ 1.5 km, and the final 5

antennas are equipped with noise-switched front ends for calibrated total power measurements of the global sky brightness. These antennas are used as part of the LEDA experiment (Price et al. 2017) to measure the global signal of 21 cm absorption from the cosmic dawn. In the current configuration 32 antennas (64 signal paths) from the core are disconnected from the correlator in order to accommodate the 32 antennas on longer baselines. A final stage of construction will involve 64 additional antennas installed on long baselines out to a maximum length of 2.6 km.

The dataset used in this paper spans 28 consecutive hours beginning at 2017-02-17 12:00:00 UTC time. During this time the OVRO-LWA operated as a zenith-pointing drift scanning interferometer. The correlator dump time was selected to be 13 seconds such that the correlator output evenly divides a sidereal day. Due to the computational considerations presented in §2.1, eight 24 kHz channels are selected for imaging from this dataset: 36.528 MHz, 41.760 MHz, 46.992 MHz, 52.224 MHz, 57.456 MHz, 62.688 MHz, 67.920 MHz, and 73.152 MHz.

3.2. Complex Gain Calibration

The complex gain calibration is responsible for correcting per-antenna amplitude and phase errors. This is accomplished using a sky model and a variant of alternating least squares colloquially known as “Stefcal” (Mitchell et al. 2008; Salvini & Wijnholds 2014)⁴.

Cyg A and Cas A are – by an order of magnitude – the brightest point-like radio sources in the northern hemisphere. Therefore the optimal time to solve for the interferometer’s gain calibration is when these sources are at high elevations. The antenna complex gains are measured from a 22 minute track of data when Cyg A and Cas are at high elevations. The gains measured in this way are then used to calibrate the entire 28 hour dataset. The calibration sky model consists only of Cyg A and Cas A. The model flux of Cyg A is set to the Baars et al. (1977) spectrum while the flux of Cas A is measured from the data itself (using a preliminary calibration solved for with a fiducial Cas A spectrum).

Calibrating in this manner generates \sim arcminute errors in the astrometry of the final sky maps due to ionospheric refractive offsets during the time of calibration. These residual errors in the astrometry are corrected post-imaging by registering the images with re-

⁴ The calibration routine is written in the Julia programming language (Bezanson et al. 2017), and is publicly available online (<https://github.com/mwestwood/TTCal.jl>) under an open source license (GPLv3 or any later version).

spect to all VLSSr (Lane et al. 2014) sources that are bright (> 30 Jy with a consistent flux density measured with the OVRO-LWA), and not too close to other bright sources (at least 1° separation).

Temperature fluctuations of the analog electronics generate 0.1 dB sawtooth oscillations in the analog gain. These oscillations occur with a variable 15 to 17 minute period associated with HVAC cooling cycles within the electronics shelter that houses these electronics. The amplitude of these gain fluctuations is calibrated by smoothing the autocorrelation amplitudes on 45 minute timescales. The ratio of the measured auto-correlation power to the smoothed auto-correlation power defines a per-antenna amplitude correction that is then applied to the cross-correlations.

3.3. Primary Beam Measurements

In order to generate wide-field images of the sky, the response of the antenna to the sky must be known. Drift-scanning interferometers like the OVRO-LWA can empirically measure their primary beam under a mild set of symmetry assumptions (Pober et al. 2012). The symmetry assumptions are necessary to break the degeneracy between source flux and beam amplitude when the flux of the source is unknown. In this work we assume symmetries that are apparent in the antenna design, but real-world defects and coupling with nearby antennas will contribute towards breaking these symmetries at some level. In particular we assume that the x - and y -dipoles have the same response to the sky after rotating one by 90° , and that the beam is invariant under north-south and east-west flips.

We measure the flux of several bright sources (Cyg A, Cas A, Tau A, Vir A, Her A, Hya A, Per B⁵, and 3C 353) as they pass through the sky and then fit a beam model composed of Zernike polynomials to those flux measurements. We select the basis functions to have the desired symmetry (Z_0^0 , Z_2^0 , Z_4^0 , Z_4^4 , Z_6^0 , Z_6^4 , Z_8^0 , Z_8^4 , and Z_8^8) and the beam amplitude at zenith is constrained to be unity. See Figure 4 for an illustration of a fitted beam model at several frequencies. This process is repeated for each frequency channel.

3.4. Ionospheric Conditions

The geomagnetic conditions during this time were mild. The Disturbance storm time (Dst) index, which measures the z -component of the interplanetary magnetic field, was > -30 nT during the entirety of the

⁵ At 74 MHz, Per B is about two times brighter than Per A.

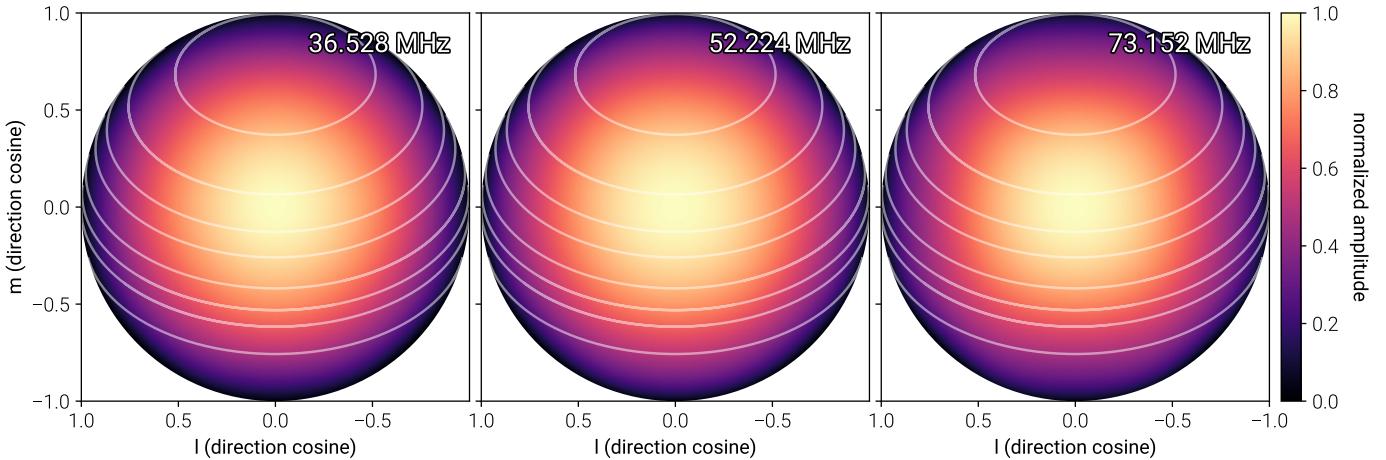


Figure 4. Empirical fits to the OVRO-LWA Stokes-I primary beam (the response of the x - and y -dipoles has been summed) at three frequencies: 36.528 MHz (left panel), 52.224 MHz (middle panel), and 73.152 MHz (right panel). The source tracks used to measure the beam model are overlaid. From north to south, these tracks correspond to Cas A, Cyg A, Per B, Tau A, Vir A, Her A, 3C 353, and Hya A. The fitting process is described in §3.3, and residuals for Cyg A and Cas A are in Figure 5.

observing period.⁶ Following the classification scheme of Kintner et al. (2008), a weak geomagnetic storm has $\text{Dst} < -30 \text{ nT}$. Stronger geomagnetic storms have $\text{Dst} < -50 \text{ nT}$.

Despite the mild conditions, low frequency interferometric observations are still affected by the index of refraction in the ionosphere. Figure 6 shows the median vertical total electron content (TEC) above OVRO measured from GPS (Iijima et al. 1999). The median is computed over all GPS measurements within 200 km of the observatory. Over the observing period the TEC smoothly varies from 20 TECU at midday to 5 TECU during the night. However, this measurement is only sensitive to large scale fluctuations in the ionosphere and does not capture small scale fluctuations.

Small scale fluctuations are best characterized by source scintillation and refractive offsets. Figure 5 shows the apparent flux and position offset of Cyg A and Cas A as a function of time over the entire observing period. Both sources exhibit rapid scintillation on the timescale of a single integration (13 seconds). For example at 36.528 MHz, it is not unusual for Cyg A to have a measured flux variations of 50% between adjacent 13 second integrations. The variance at 36.528 MHz compared with the variance at 73.152 MHz is consistent with an ionospheric ν^{-2} origin. The measured position offset of each source is a measurement of the ionospheric phase gradient across the array. This varies on slower 10 minute timescales refracting each source by as much as 20 arcmin (at 36.528 MHz) from their true astronom-

ical positions as waves in the ionosphere pass through the line of sight. At 74 MHz on the VLA, Kassim et al. (2007) observe ~ 1 arcmin refractive offsets during the night, and ~ 4 arcmin offsets during the day on similar ~ 10 minute timescales. The impact of these effects on the sky maps is simulated in §5.2.

3.5. Source Removal

3.5.1. Cygnus A and Cassiopeia A

Due to the rapid and large ionospheric fluctuations seen in Figure 5, CLEAN cannot be relied on to accurately deconvolve bright sources. However, without removing bright sources from the data, sidelobes from these sources will dominate the variance in the sky maps. At 74 MHz Cyg A is a 15,000 Jy source (Baars et al. 1977). A conservative estimate for the confusion limit at 74 MHz with a 15 arcmin beam is 1000 mJy (Lane et al. 2014). Therefore we require that Cyg A’s sidelobes be at least -45 dB down from the main lobe to prevent Cyg A’s sidelobes from dominating the variance in the image.

In order to account for propagation effects through the ionosphere direction-dependent calibration and peeling (Mitchell et al. 2008; Smirnov & Tasse 2015) must be used. Direction-dependent calibration allows the per-antenna amplitude and phase towards a bright astronomical source to be free parameters. In the dataset used in this paper, scintillation and refractive-offset events on the timescale of a single integration (13 seconds) are observed (Figure 5). Therefore the direction dependent calibration changes on these timescales and we must solve for one set of complex gains per source per integration.

⁶ The Dst index was obtained from the World Data Center for Geomagnetism, Kyoto University (<http://swdcwww.kugi.kyoto-u.ac.jp/>). Accessed 2017-07-25.

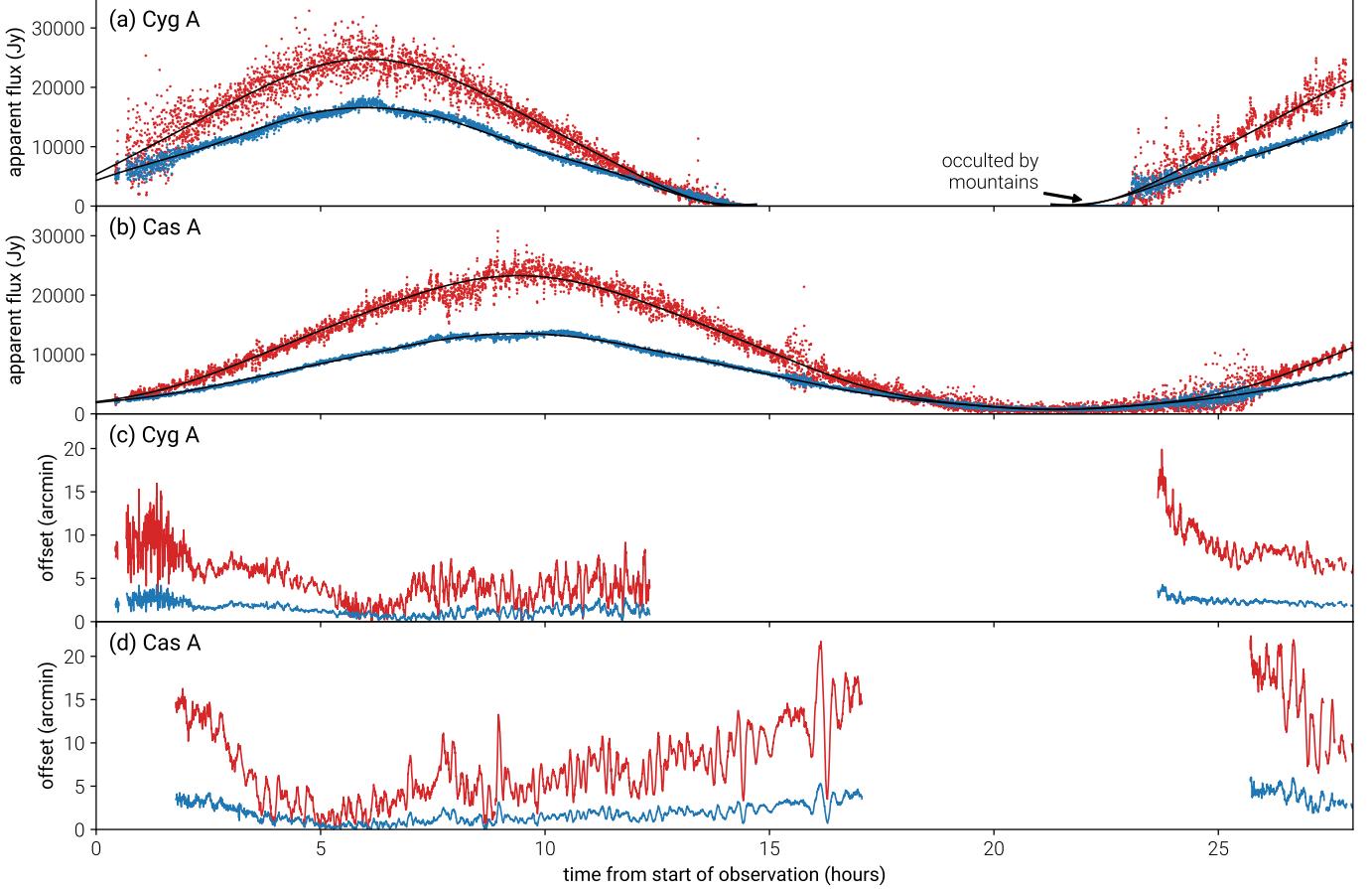


Figure 5. (a, b) These panels show the measured apparent flux of Cyg A and Cas A at 36.528 MHz (red points) and 73.152 MHz (blue points) as a function of time over the observing period. The solid black curves show the expected flux computed using the empirical beam model fits. The thermal noise contribution to each point is about 50 Jy. Cyg A is occulted by the White Mountains when it is low on the horizon to the east. (c, d) These panels show the measured position offset of Cyg A and Cas A relative to their true astronomical positions at 36.528 MHz (red line) and 73.152 MHz (blue line).

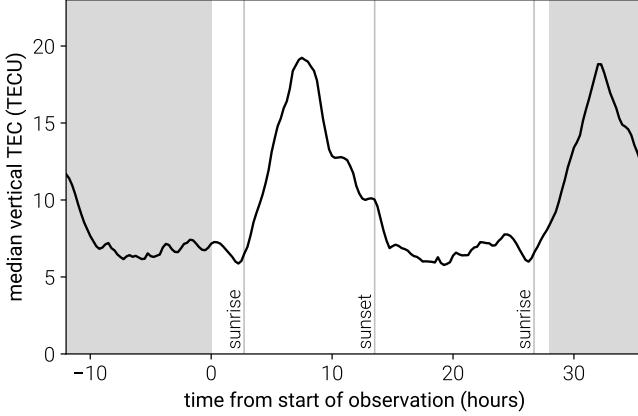


Figure 6. The median vertical TEC within 200 km of OVRO during the time of the observation. The gray shaded regions indicate times outside of the observing period. The gray vertical lines indicate sunrise and sunset (as labeled).

The largest angular scale of Cas A is ~ 8 arcmin, and the largest angular scale of Cyg A is ~ 2 arcmin.

With a ~ 10 arcmin resolution on its longest baselines at 73 MHz, the OVRO-LWA marginally resolves both sources. A resolved source model is needed for both sources. We fit a self-consistent resolved source model to each source. This is performed by minimizing the variance within an aperture located on each source after peeling. By phasing up a large number of integrations before imaging (at least 1 hour) it is possible to smear out the contribution of the rest of the sky. We then use NLOpt's Sbplx routine (Johnson 2008; Rowan 1990) to vary the parameters in a source model until the variance within the aperture is minimized. Cyg A is modeled with 2 Gaussian components, while Cas A is modeled with 3 Gaussian components. Armed with these source models, both sources can be peeled from the dataset to the required dynamic range.

3.5.2. Other Bright Sources

Other bright sources namely Vir A, Tau A, Her A, Hya A, Per B, and 3C 353 are also removed from the

visibilities prior to imaging. Because these sources are much fainter than Cyg A and Cas A, we do not need resolved source models to be able to remove these sources from the visibilities without residual sidelobes contaminating the image.

However, the ionosphere will cause these sources to scintillate and refract. The position and flux of each source is measured separately in each channel and integration. The sources are then subtracted from the visibilities using the updated position and flux of the source. The brightest of these sources (Vir A and Tau A) are peeled using a direction dependent calibration when they are at high elevations.

3.5.3. The Sun

The Sun can be trivially removed from any map of the sky by constructing the map using only data collected at night. A map of the entire sky can be obtained by using observations spaced 6 months apart. However the dataset used in this paper consists of 28 consecutive hours. Therefore we attempt to suppress the contribution of the Sun to the maps with the goal of suppressing its sidelobes. The Sun is well-resolved by the OVRO-LWA and hence a detailed source model is needed. In fact the optical depth $\tau = 1$ surface of the Sun changes with frequency and as a consequence a new model is needed at each frequency. While we could fit a limited number of Gaussian components to Cyg A and Cas A, this is insufficient for the Sun. Additionally while most astronomical sources at these frequencies have negative spectral indices, the Sun has a positive spectral index. Therefore more care will need to be taken in subtracting the Sun at higher frequencies than at lower frequencies.

The strategy used for removing the Sun below 55 MHz involves fitting a shapelet (Refregier 2003) model to the Sun and subtracting without the use of direction dependent gains. The shapelet fitting is performed in the visibility space. Above 55 MHz a model is fit to the Sun by minimizing the residuals after peeling (in the same way that models are obtained for Cyg A and Cas A). The Sun is then peeled from each integration using direction dependent gains.

3.6. Flux Scale

The flux scale of the data was tied to the Baars et al. (1977) spectrum of Cyg A during gain calibration. However gain calibration is also a function of the beam model and the spectrum used for Cas A. Recent work by Scaife & Heald (2012) (hereafter SH12) using archival data from the literature and Perley & Butler (2017) (hereafter PB17) using the VLA has expanded the number of low-frequency radio sources with calibrated flux measurements from one (just Cyg A) to eleven in total. While

the SH12 flux scale is valid between 30 MHz and 300 MHz, the PB17 flux scale is somewhat more limited because the lowest frequency observations come from the VLA 4-band system. As a consequence the PB17 flux scale is not valid below 50 MHz.

Figure 7 shows a comparison between flux measurements made using the all-sky maps from this work, and spectra from the aforementioned flux scales. Generally the OVRO-LWA flux measurements agree to between 5 and 10% with the SH12 spectra. Below 50 MHz there can be substantial departures with respect to the extrapolated PB17 spectra (for eg. 3C 286, 3C 295, and 3C 380), but it is usually the case that we have much better agreement with the SH12 spectra. This indicates that the PB17 spectra cannot be extrapolated below 50 MHz.

4. RESULTS

Eight sky maps were constructed using Tikhonov-regularized m -mode analysis imaging and cleaning with observations from the OVRO-LWA. Each map is individually shown in Figure 8, and Figure 9 is a three-color image constructed from the maps at 36.528 MHz, 52.224 MHz, and 73.152 MHz. The maps cover the sky north of $\delta = -30^\circ$ with ~ 15 arcmin angular resolution. The 8 brightest northern hemisphere point sources are removed from each map (Cyg A, Cas A, Vir A, Tau A, Her A, Hya A, Per B, and 3C 353) as described in §3.5, and there is a small blank region near $l = +45.7^\circ$, $b = -47.9^\circ$ corresponding to the position of the Sun during the observing window. The properties of each map, including frequency, bandwidth, angular resolution, and thermal noise are presented in Table 1.

Due to the considerations presented by Venumadhav et al. (2016), each of these maps is monopole subtracted ($a_{00} = 0$). Consequently a simple pixel-by-pixel power-law fit will be unduly affected by the choice of zero-level correction. Instead, temperature-temperature plots (T-T plot) can be used to measure the spectral index independently of any zero-level corrections (Turtle et al. 1962). This method relies on the assumption that all pixels in a given region are described by the same power law. In Figure 10 the spectral index is locally estimated within a region $\sim 10^\circ$ across by constructing a local T-T plot at every pixel between the maps at 36.528 MHz and 73.152 MHz. The spectral index contours are overlaid on the 3-color image from Figure 9 where free-free regions responsible for flattening the spectral index in the galactic plane are plainly visible as extended blue blobs. Similarly the spectral index can be seen to steepen as galactic latitude increases (this is a transition from blue

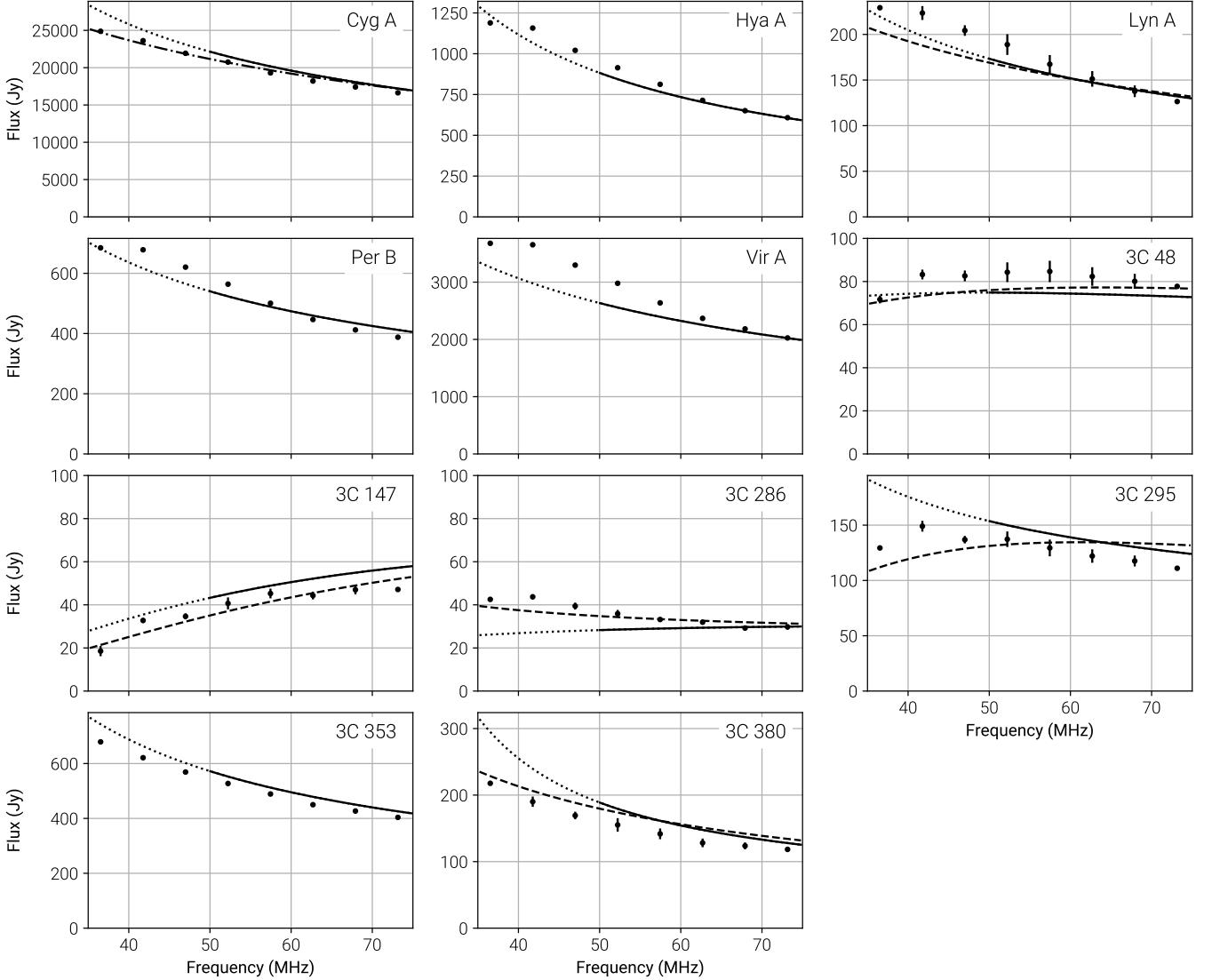


Figure 7. Measured fluxes (black points) of 11 sources plotted against the published spectra from Perley & Butler (2017) (solid line above 50 MHz, dotted line below 50 MHz), Scaife & Heald (2012) (dashed line), and Baars et al. (1977) (dot-dash line).

to white to yellow that is also evident in the spectral index contours).

In order to suppress sources of terrestrial interference, all spherical harmonics with $m = 0$, or $m = 1$ and $l > 100$ are filtered from the map (where the spherical harmonics are defined in the J2017 coordinate system). As will be discussed in §5.1, these spherical harmonics are particularly susceptible to contamination by RFI and common-mode pickup. As a consequence, astronomical emission that circles the J2017 north celestial pole (NCP) is filtered from the maps. This filtering creates negative rings around the NCP at the declination of bright point sources. These rings are naturally removed from the map during CLEANing as long as this filtering step is included in the PSF calculation.

The noise in each map is empirically measured using jackknife resampling. The dataset is split into even- and odd-numbered integrations. These two groups are imaged and cleaned independently, before being compared against the maps constructed from all of the available data using the standard jackknife variance estimator. This estimate of the variance includes all sources of error that operate on ~ 13 second timescales (the integration time) such as thermal noise in rapid ionospheric fluctuations, but does not account for more slowly varying effects (for example, sidereal variation in the system temperature or day-night fluctuations in the ionosphere). These noise calculations are summarized in Table 1. VLSSr source counts (Lane et al. 2014) suggest that the confusion limit at 74 MHz and 15 arcmin angular

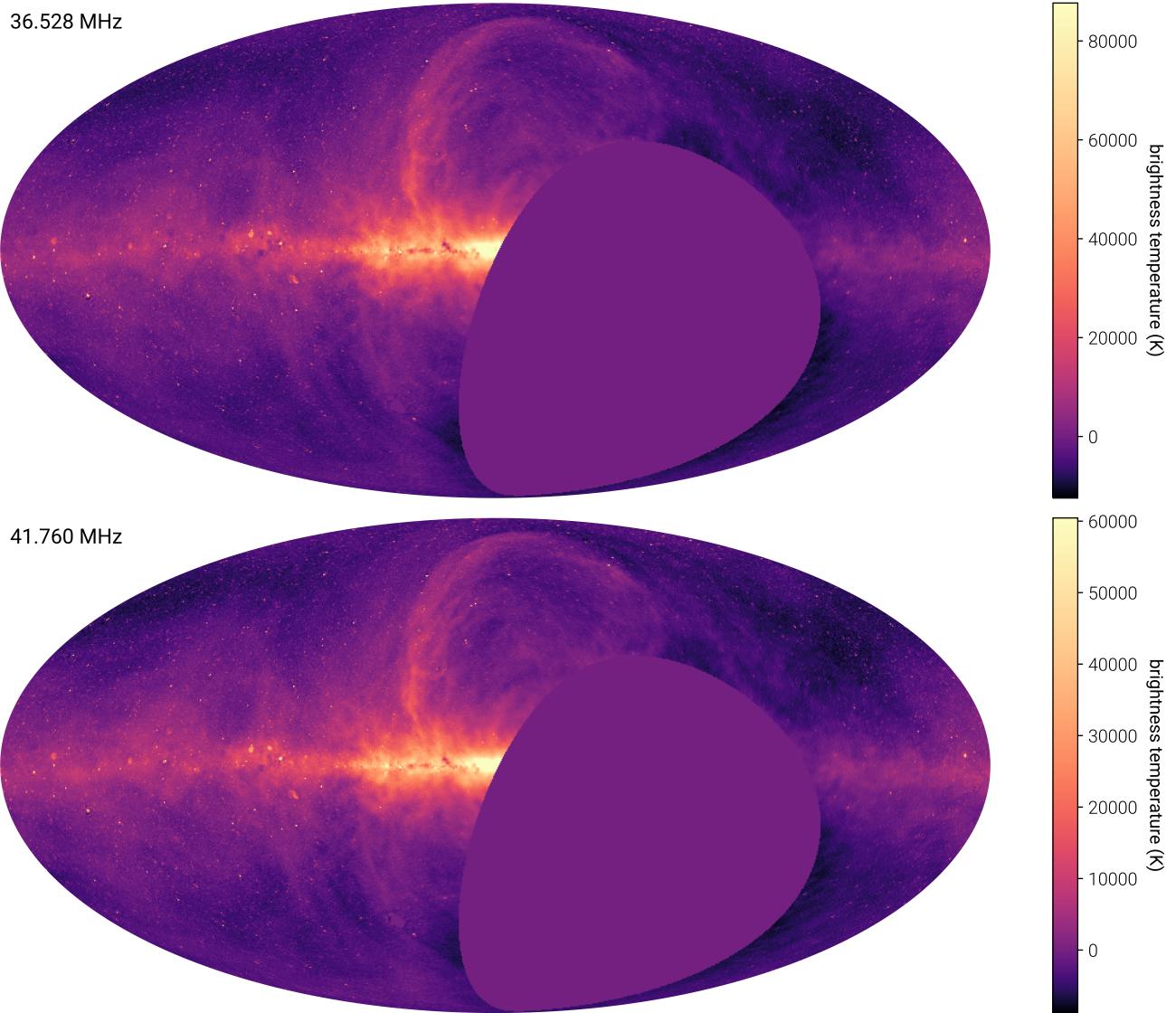


Figure 8. These 8 panels illustrate (with a Mollweide projection and logarithmic color scale) the 8 full-sky maps generated with Tikhonov-regularized m -mode analysis imaging and the OVRO-LWA. Each map covers the sky north of $\delta = -30^\circ$ with angular resolution of ~ 15 arcmin. Eight bright sources have been removed from each map (Cyg A, Cas A, Vir A, Tau A, Her A, Hya A, Per B, and 3C 353). The small blank region near $l = +45.7^\circ$, $b = -47.9^\circ$ corresponds to the location of the Sun during the observation period. A detailed summary of the properties of each map is given in Table 1.

lar resolution is $\sim 1000 \times (\nu/74\text{ MHz})^{-0.7}$ mJy. Each channel map achieves thermal noise < 900 mJy and therefore each map is likely at or near the confusion limit.

Each map from Figure 8 is freely available online in Healpix format (Górski et al. 2005) on LAMBDA (<https://lambda.gsfc.nasa.gov/>).

5. ERROR ANALYSIS

5.1. Terrestrial Interference and Pickup

When writing Equation 2, it is implicitly assumed that the correlated voltage fluctuations measured between pairs of antennas are exclusively generated by as-

trononical sources of radio emission. In practice this assumption can be violated. For instance a low-frequency interferometer located in the vicinity of an arcing power line will see an additional contribution from the radio-frequency interference (RFI) generated by the arcing process. Similarly common-mode pickup along the analog signal path of the interferometer may generate an additional spurious contribution to the measured visibilities. While the amplitude and phase of these contaminating signals may fluctuate with time, they do not sweep across the sky at the sidereal rate characteristic of astronomical sources.

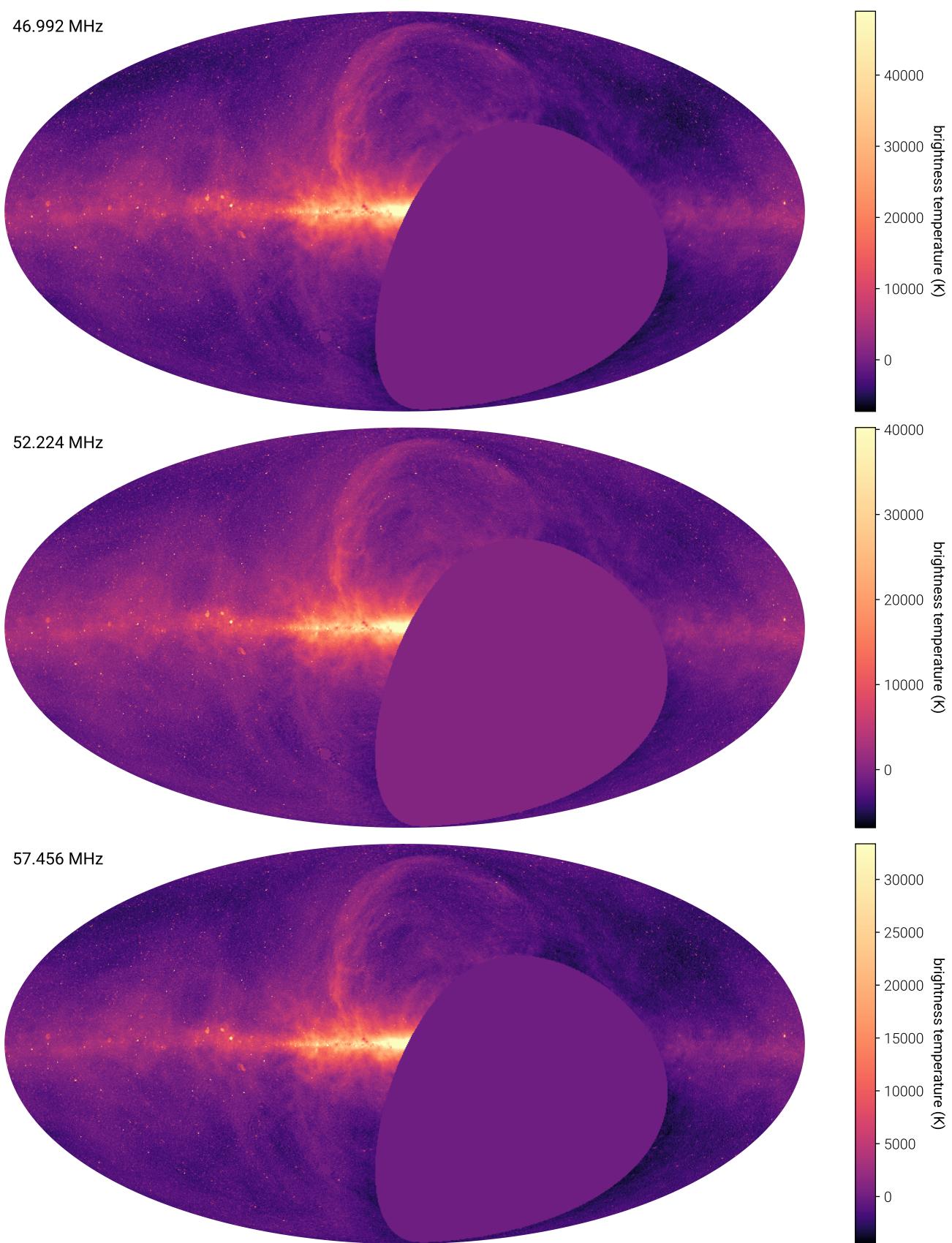


Figure 8. continued

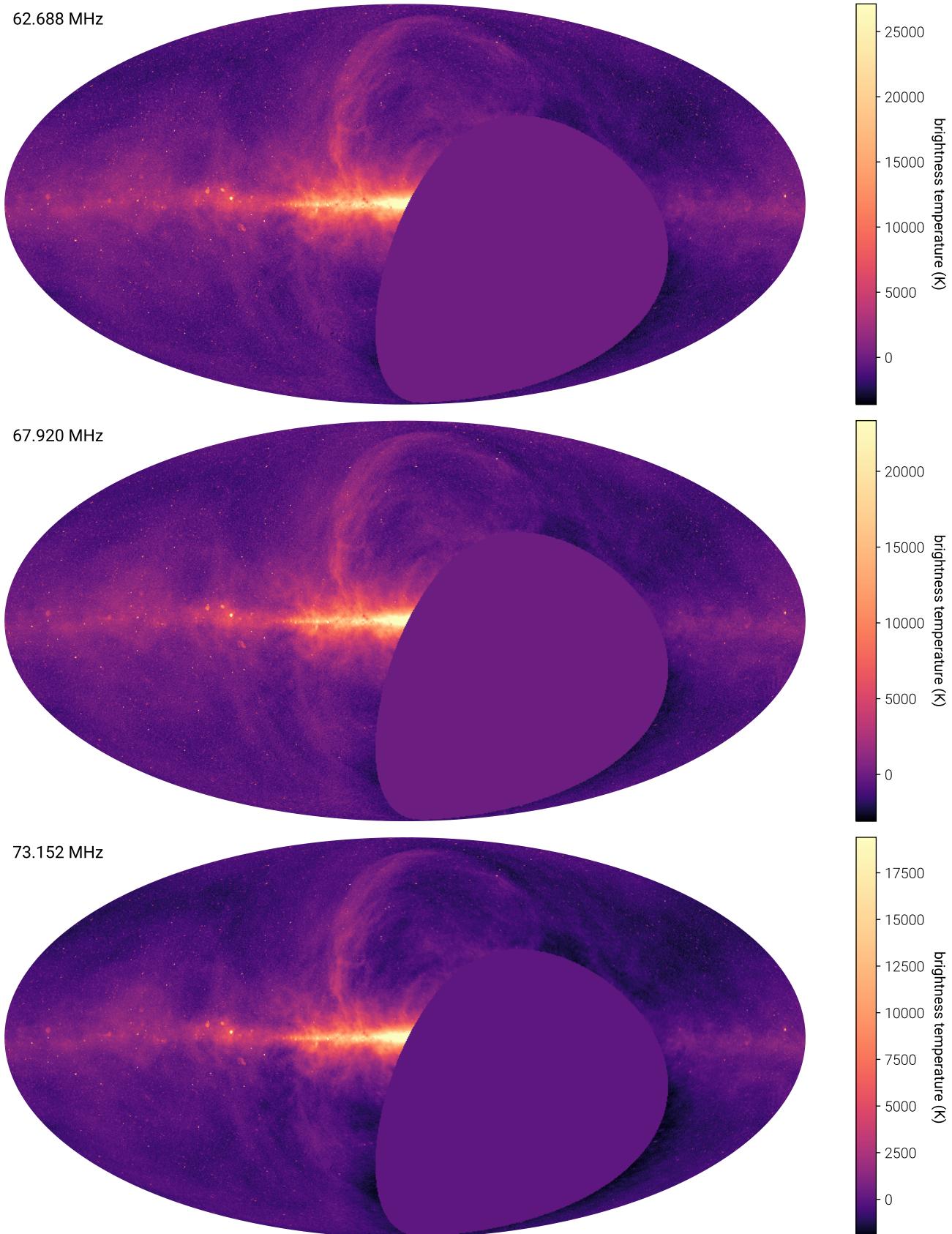


Figure 8. continued

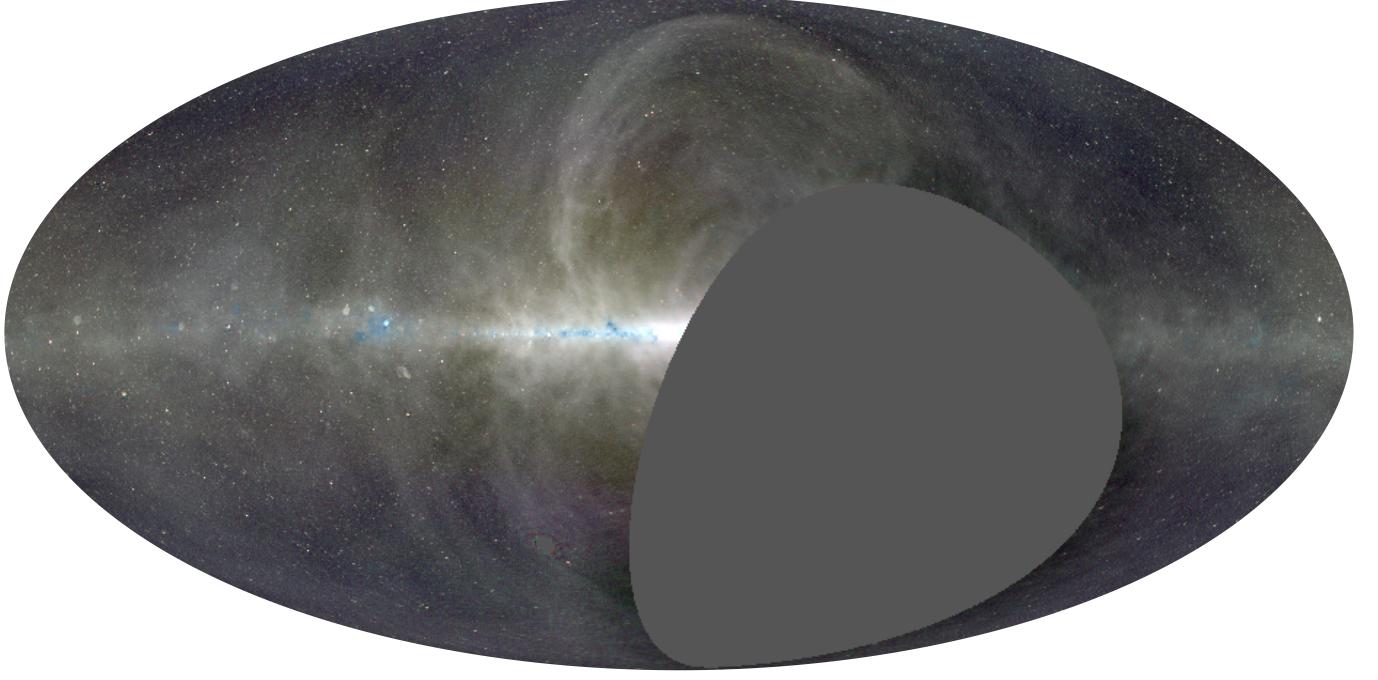


Figure 9. This Mollweide-projected map is constructed from 3 maps of the sky at 36.528 MHz (red), 52.224 MHz (green), and 73.152 MHz (blue). The maps are scaled by $\nu^{2.5}$ before combining and the color scale is logarithmic (as in Figure 8). Therefore regions with a spectral index of -2.5 appear white. Areas with a steeper index appear red and areas with a flatter spectral index appear blue.

#	Frequency MHz	Bandwidth MHz	FWHM ^a arcmin	Noise ^b		Fraction of Modes ^c	
				K	mJy/beam	with $\sigma > \sqrt{\varepsilon}$	with $\sigma > \sqrt{10\varepsilon}$
1	36.528	0.024	18.5	595.	799.	0.391	0.077
2	41.760	0.024	17.2	541.	824.	0.480	0.070
3	46.992	0.024	16.3	417.	717.	0.504	0.056
4	52.224	0.024	15.6	418.	814.	0.535	0.053
5	57.456	0.024	15.4	354.	819.	0.542	0.049
6	62.688	0.024	15.3	309.	843.	0.540	0.046
7	67.920	0.024	15.3	281.	894.	0.529	0.047
8	73.152	0.024	15.7	154.	598.	0.512	0.051

^a The full-width half-maximum (FWHM) of the synthesized beam at $\delta = +45^\circ$.

^b Measured with a jackknife and splitting the dataset into even- and odd-numbered integrations. This estimate therefore includes all noise sources that act on the timescale of a single 13 second integration (eg. thermal, ionospheric, etc.).

^c Singular values of the transfer matrix compared with the value of the regularization parameter ε used while solving Equation 9. As discussed in the text, singular vectors with corresponding singular values $\sigma \ll \sqrt{\varepsilon}$ are set to zero by the Tikhonov regularization procedure.

Table 1. A summary of the generated all-sky maps

The Owens Valley is an important source of water and power for the city of Los Angeles. Unfortunately this means that high voltage power lines run along the valley $\gtrsim 10$ km to the west of the OVRO-LWA. Some of these power line poles have faulty insulators that arc and produce pulsed, broadband RFI. Because these poles exist in the near-field of the array, we have been able to

localize some of them by using the curvature of the incoming wavefront to infer a distance. Efforts are currently underway to work with the utility pole owners to have these insulators replaced.

In the meantime it is possible to suppress their contamination in the dataset. The contribution of these RFI sources to the visibilities can be plainly seen by

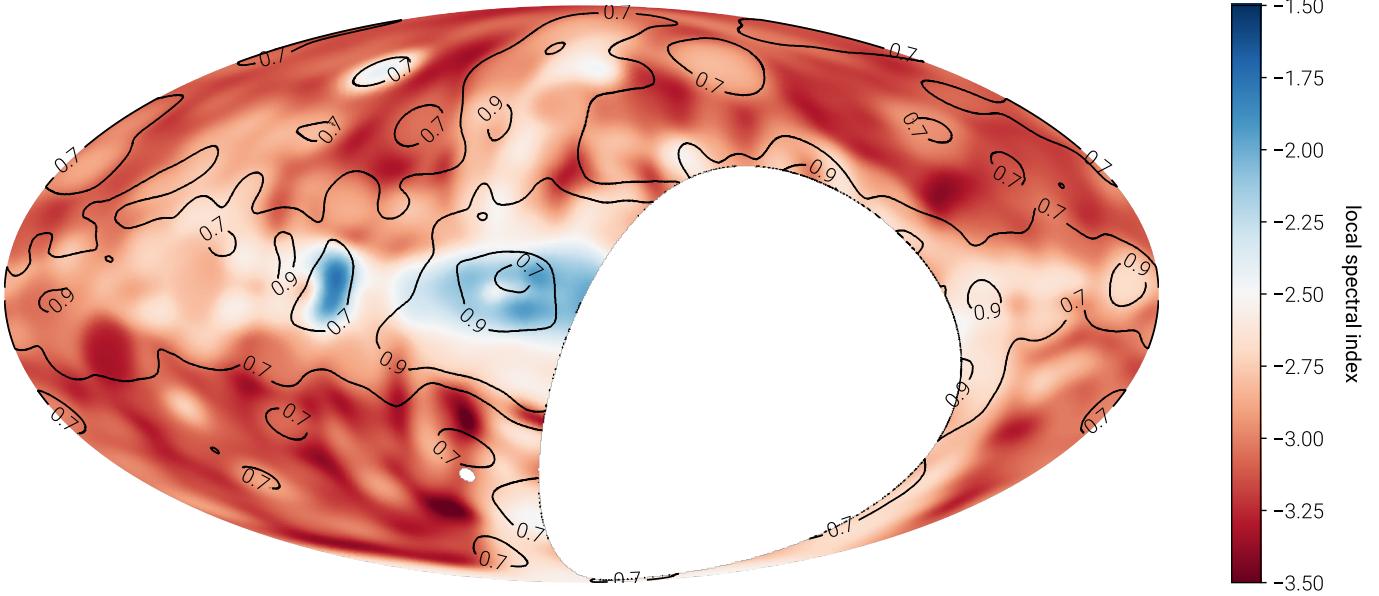


Figure 10. TODO: updated this caption This plot shows the local spectral index (estimated by means of a local T-T plot) as a contour plot. The contours are overlaid on the three-color image from Figure 9. The spectral index fit along the galactic plane is masked because T-T plots assume a single emission mechanism operates in a given region. Along the galactic plane free-free dominated regions lie in close proximity to synchrotron dominated regions. This introduces large systematic errors to the power-law fit unless care is taken to cleanly separate the two types of emission mechanisms.

averaging > 24 hours of data with the phase center set to zenith. In this way, true sky components are smeared along tracks of constant declination while terrestrial sources (ie. the arcing power lines or any contribution due to common-mode pickup) are not smeared. Obtaining a model for the RFI is complicated by the fact that the contaminating sources are at extremely low elevations where the antenna response is essentially unknown (and inhomogeneous due to antenna shadowing effects). Therefore it is not enough to know the physical location of the faulty insulator generating the RFI. In addition we must know the response of each antenna (amplitude and phase) in the appropriate direction. This motivates the use of peeling, which allows the antenna response to be a free parameter. Therefore model visibilities for the RFI can be obtained by peeling the sources after smearing the visibilities over > 24 hours. Figure 11 shows an illustration of some of the removed components at 62.688 MHz.

While attempting to peel RFI sources from the averaged visibilities, it was discovered that frequently peeling would remove components from the visibilities that are not obviously associated with any source on the horizon or elsewhere in the sky (see panels (a) and (b) in Figure 11). These components appear noise-like in the images, but they are actually a constant offset to the measured visibilities and are therefore likely associated with cross-talk or some form of common-mode pickup. If these components are not subtracted from the measured

visibilities, they contribute ring-like structures into the sky maps as seen in Figure 12. This figure is not a simulation, but rather a difference between maps created before and after measuring and subtracting the components in Figure 11 from each integration.

The first step in Equation 9 is to compute $\mathbf{B}^* \mathbf{v}$. In this step we compute the projection of the measurement \mathbf{v} onto the space spanned by the columns of \mathbf{B} . Each column of \mathbf{B} describes the interferometer's response to a corresponding spherical harmonic coefficient of the sky brightness distribution. Therefore the act of computing $\mathbf{B}^* \mathbf{v}$ is to project the measured m -modes onto the space of m -modes which could be generated by astronomical sources. The degree to which a source of terrestrial interferer will contaminate a map generated using m -mode analysis imaging is determined by its amplitude after projection.

For instance, a bright interfering source might contribute $\mathbf{v}_{\text{terrestrial}}$ to the measured m -modes. However, if $\mathbf{v}_{\text{terrestrial}}$ is actually perpendicular to all of the columns of \mathbf{B} , there will be no contamination in the map because $\mathbf{B}^* \mathbf{v}_{\text{terrestrial}} = \mathbf{0}$. In practice this is unlikely. In general the contamination is proportional to the overall amplitude of the interference ($\|\mathbf{v}_{\text{terrestrial}}\|$) and the degree to which the interference mimics an astronomical signal ($\|\mathbf{B}^* \mathbf{v}_{\text{terrestrial}}\| / (\|\mathbf{B}\| \|\mathbf{v}_{\text{terrestrial}}\|)$).

These terrestrial sources do not rotate with the sky and hence their contamination tends to be restricted to modes with small m . In this dataset the contamination

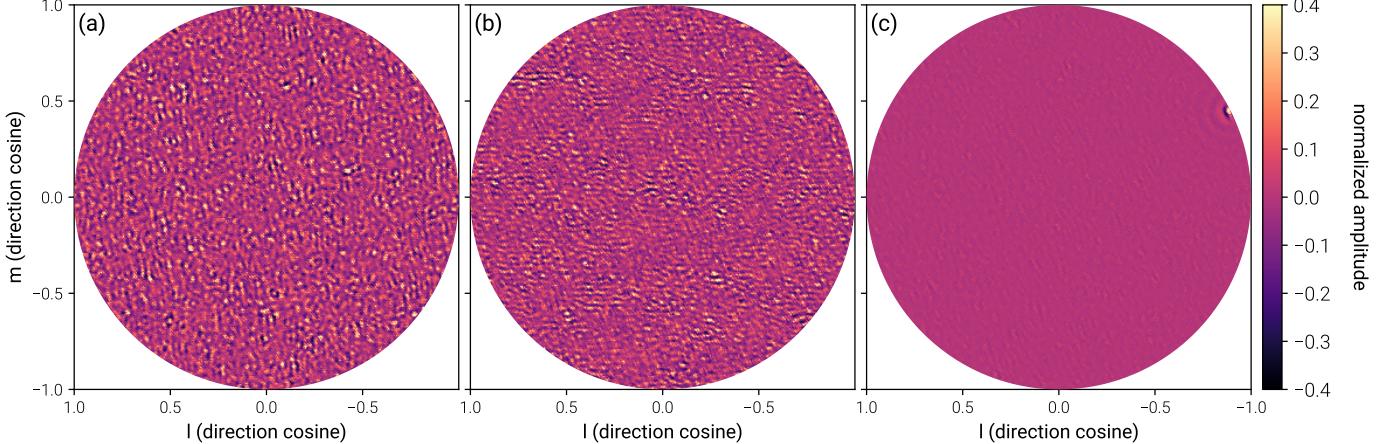


Figure 11. Terrestrial sources of correlated noise that are apparent after averaging the visibilities at 62.688 MHz over the entire 28 hour observing period (keeping the phase center at zenith such that astronomical sources of radio emission are smeared along tracks of constant declination). Each panel represents a different component that is removed from the visibilities. The images are generated using WSClean (Offringa et al. 2014), uniform weighting, and only baselines longer than 15 wavelengths. Panels (a) and (b) illustrate components that appear noise-like in image-space, but are in fact a constant-offset to the measured visibilities likely associated with cross-talk or common-mode pickup. Panel (c) illustrates a component that is clearly associated with an RFI source on the horizon to the west-north-west of the OVRO-LWA. This RFI source is likely an arcing power line. Figure 12 illustrates the characteristic ring-like artifacts introduced into the maps if these 3 components are not removed prior to m -mode analysis imaging. The component shown in panel (a) has about twice the amplitude ($\|\mathbf{B}^* \mathbf{v}_{\text{terrestrial}}\|$) of panels (b) and (c), and for all three components $\|\mathbf{B}^* \mathbf{v}_{\text{terrestrial}}\| / (\|\mathbf{B}\| \|\mathbf{v}_{\text{terrestrial}}\|) \sim 0.035$.

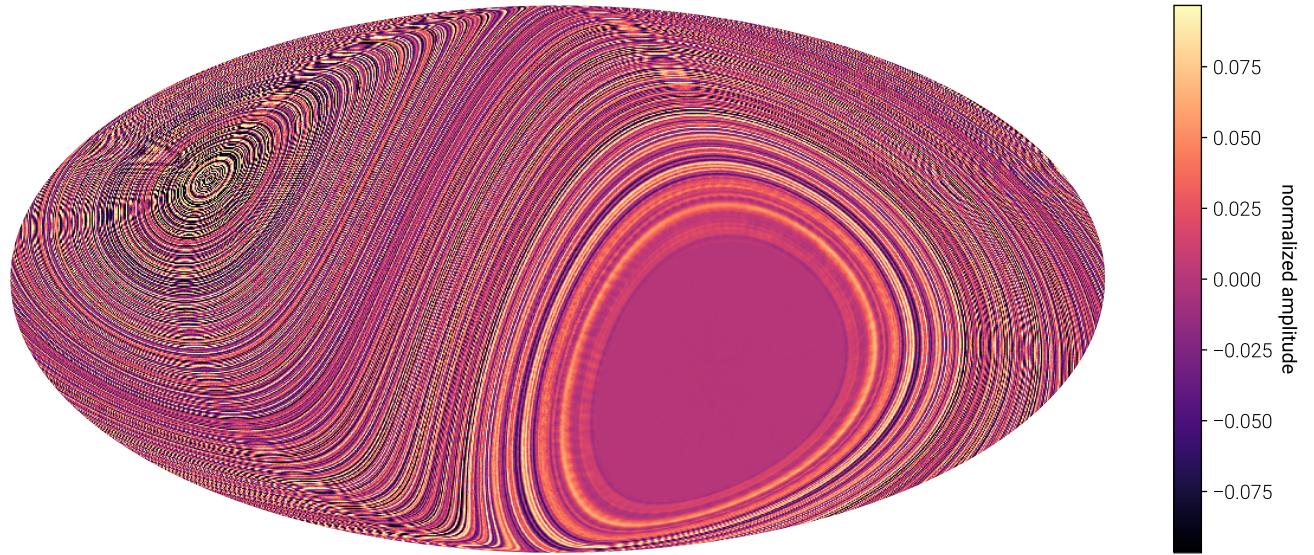


Figure 12. This is a Mollweide-projected image of the artifacts introduced to the m -mode analysis maps by the three terrestrial sources shown in Figure 11. Because these sources are not moving through the sky sidereally, they tend to be smeared along rings of constant declination. The spurs seemingly radiating from the north celestial pole are a Moiré pattern (ie. an artifact of the pixelization).

is largely restricted to $m \lesssim 1$. Although the RFI is capable of fluctuating on short timescales, in this case the artifacts it introduces seem to be restricted to small m (presumably because the phase is not fluctuating). As a result if the contamination is not suppressed, it will manifest itself as rings along stripes of constant declination. This effect is plainly visible in Figure 11. Because

of the distinctive ring-like pattern created by terrestrial sources, we additionally chose to discard spherical harmonics with either $m = 0$ or $m = 1$ and $l > 100$ in order to further suppress the contamination.

5.2. The Ionosphere

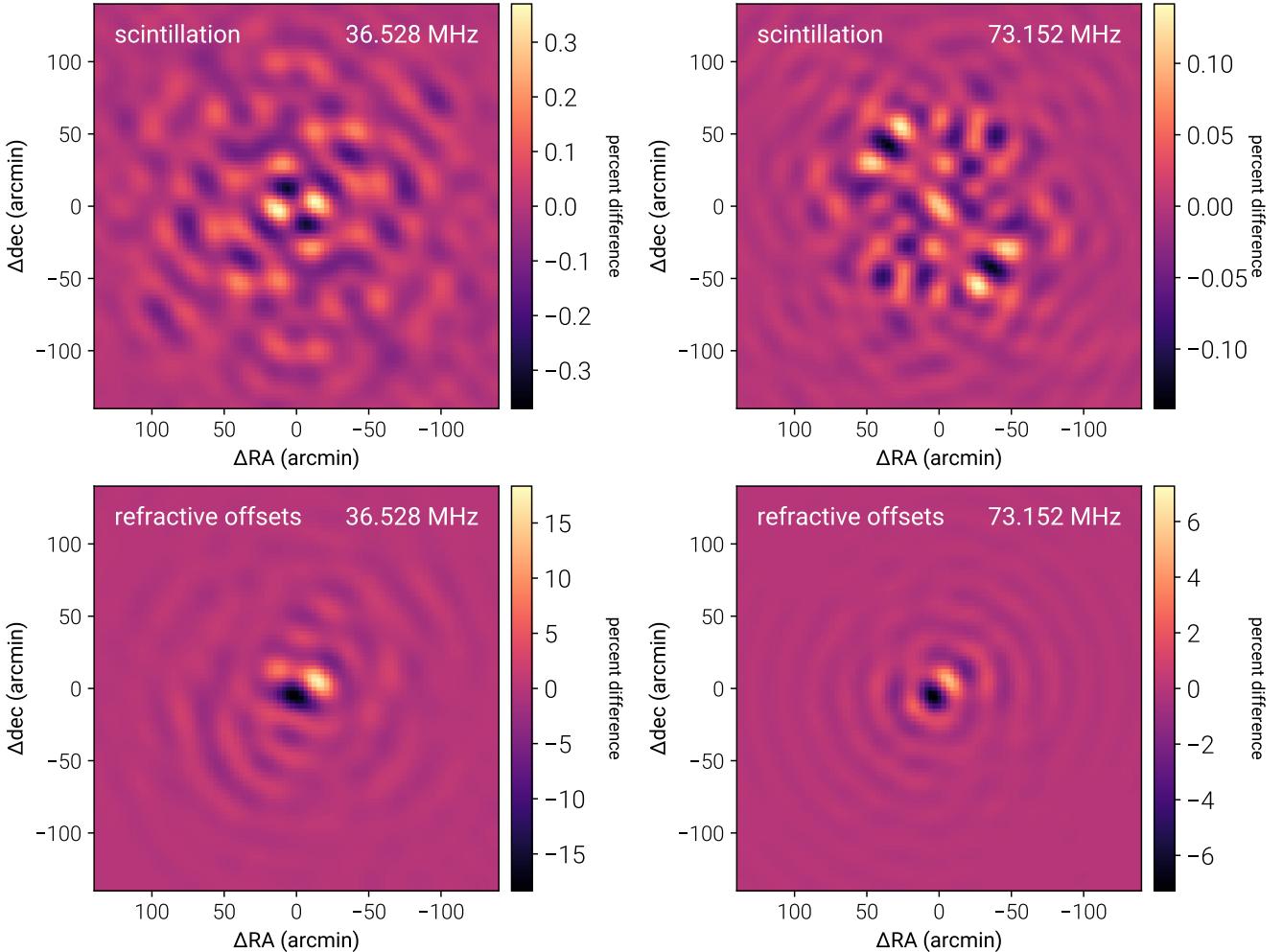


Figure 13. This figure illustrates the corrupting influence of the ionosphere at 36.528 MHz (left column) compared with 73.152 MHz (right column). Each panel shows the *simulated* PSF for a source at the location of Cas A. Each panel illustrates the percent difference (relative to the peak flux of the uncorrected PSF) due to including an ionospheric effect. In the top row, the simulated source scintillates using the measured light-curve for Cas A in Figure 5. In the bottom row, the simulated source is refracted from its true position using the measured refractive offsets for Cas A in Figure 5.

One of the key assumptions made by m -mode analysis is that the sky is static. We assume that the only time-dependent behavior is the rotation of the Earth, which slowly rotates the sky through the fringe patterns of the interferometer. At low frequencies the ionosphere violates this assumption. In particular, ionospheric scintillation and refractive offsets will cause even static sources to exhibit significant variability (Figure 5).

The correlation observed on a given baseline for a single point source is

$$V_\nu(t_{\text{sidereal}}) = I_\nu(t)B_\nu(t_{\text{sidereal}}), \quad (16)$$

where I_ν is the flux of the source at the frequency ν , and B_ν is the baseline transfer function defined by Equation 3. The transfer function is a function of the direction to the source, which is in turn a function of the sidereal time t_{sidereal} . If the source is varying, from in-

trinsic variability or due to scintillation, than the source flux is also a function of the time coordinate t such that

$$V_\nu(t_{\text{sidereal}}) = I_\nu(t)B_\nu(t_{\text{sidereal}}), \quad (17)$$

where $t_{\text{sidereal}} = (t \bmod 23^h 56^m)$.

In order to compute the m -modes we must take the Fourier transform with respect to the sidereal time. As a consequence of the Fourier convolution theorem we find:

$$V_{\nu,m} \sim \sum_{m'} V_{\nu,m'}^{\text{static}} I_{\nu,m-m'}. \quad (18)$$

where $V_{\nu,m}^{\text{static}}$ is the set of observed m -modes if the source was actually static, and $I_{\nu,m-m'}$ is the Fourier transform of the light curve $I_\nu(t)$. Equation 18 indicates that power is scattered between different values of m . As a consequence, the true transfer matrix, which is exactly

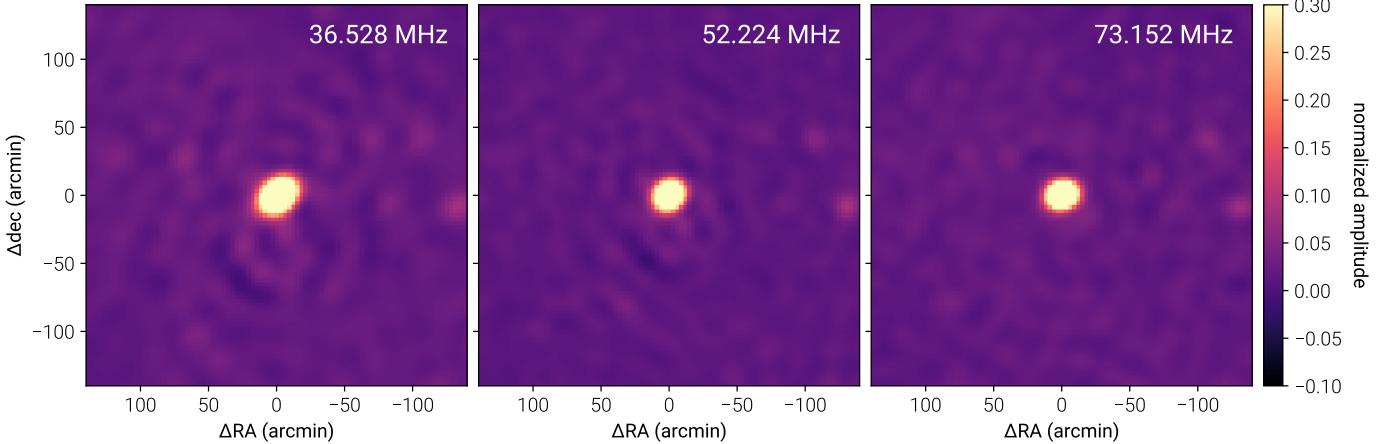


Figure 14. A zoom-in of 3C 134 at 36.528 MHz (left panel), 52.224 MHz (middle panel), and 73.152 MHz (right panel). At 36.528 MHz there are $\sim 10\%$ artifacts around 3C 134 that persist after CLEANing due to ionospheric effects. As expected for an ionospheric origin, these artifacts decrease in amplitude as frequency increases.

block diagonal in the ideal case, is no longer truly block diagonal (Shaw 2016).

The maps presented in Figure 8 do not account for any off-diagonal terms arising from ionospheric fluctuations. The effect of this can be seen in Figure 13. In this simulation at 36.528 MHz, a point source is placed at the location of Cas A. In the first panel the source flux traces the beam model exactly and the source is always located exactly at the location of Cas A. In the middle panel, the source is allowed to scintillate in the same way Cas A does in Figure 5. The image of the point source is effectively unchanged from the ideal PSF. In the right panel, the source position is allowed to vary in the same way Cas A does in Figure 5. The refractive offsets lead to a clear degradation of the sidelobes. Because the sidelobes are altered from that of the ideal PSF, refractive offsets will restrict the dynamic range it is possible to obtain with the CLEAN algorithm described in §2.3. This effect can be clearly seen in Figure ??, where 10% errors within 1° of 3C 134 are seen at 36.528 MHz. As expected for an ionospheric effect these errors decrease to a few percent at 52.224 MHz, and less at 73.152 MHz. We therefore conclude that ionospheric effects directly limit the dynamic range in the vicinity of bright point sources.

5.3. Beam Errors

A model of the antenna beam is essential for widefield imaging. Because m -mode analysis imaging operates on a full sidereal day of data, images are constructed after watching each point in the sky move through a large slice of the beam (excepting the celestial poles). The beam model therefore serves two purposes:

1. setting the flux scale as a function of declination

2. reconciling observations from two separate sidereal times

In the first case, all sources at a given declination take the same path through the antenna primary beam. If the antenna response is overestimated along this track then all sources at this declination will have their flux underestimated. Similarly if the antenna response is underestimated then all the sources will have their flux overestimated. The errors in Figure 7 do not show a clear pattern with declination. Two sources have a clear systematic offset at all frequencies: 3C 353 and 3C 380. 3C 353 is the second southernmost source, but Hya A – the first southernmost source – does not exhibit this systematic error. Similarly 3C 380 is at a comparable declination to Lyn A, which appears, if anything, to have its flux overestimated. The absence of a coherent pattern does not eliminate the possibility of beam errors effecting the flux scale, but it does mean that these errors are at least comparable to the errors inherent to the flux scale itself.

The second case is more subtle. Sources are observed at a wide range of locations in the primary beam of the antenna. The imaging process must reconcile all of these observations together, and the beam model provides the instructions for how to do this. In the event of an error in the beam model, it can be expected that the beam will introduce errors into the sky maps that will limit the dynamic range in the vicinity of bright point sources. Shaw et al. (2015) simulate the effect of beam errors on a cosmological analysis concluding that the beam must be known to one part in 10^4 . Our requirements are significantly less stringent because we are estimating the sky brightness instead of estimating the amplitude of a faint cosmological signal in the presence of foreground emission that dominates the signal by 5 orders of mag-

nitude. In fact in §?? we found that ionospheric effects likely dominate over other sources of error that affect the PSF shape. Therefore we conclude that the beam models generated in §3.3 are sufficient to limit the effect of beam errors on the PSF to at least less than those introduced by the ionosphere.

5.4. *Polarization Leakage*

[Shaw et al. \(2015\)](#) describe how to generalize m -mode analysis to account for a polarized sky observed with a polarized antenna beam. Heretofore this generalization has been neglected in the discussion of m -mode analysis imaging. At low frequencies, increasingly rapid Faraday rotation leads to depolarization. Therefore polarization fractions are generally expected to decrease at low frequencies (varying with ionospheric conditions). [Lenc et al. \(2016\)](#) detected the presence of diffuse polarized emission on degree angular scales with the MWA, also finding typical depolarization ratios of ~ 0.3 for pulsars at 154 MHz relative to 1.4 GHz, although there was a large variance between pulsars. Even more depolarization is expected at frequencies ≤ 73.152 MHz, but crossed-dipole antennas with extremely large primary beams will naturally introduce large polarization leakage terms at low elevations. It is instructive to compute what impact this will have on the unpolarized imaging process.

In order to understand the effect of polarization leakage, a point source with 10% polarization in Stokes- Q was simulated at the location of Cas A. The simulated visibilities were computed using the measured beams for x - and y -dipoles. Because the amplitude of the two beams are not equal in every direction on the sky, this introduces a direction-dependent leakage of Stokes- Q into Stokes- I . At 73.152 MHz, this leakage is $\lesssim 5\%$ above 15° elevation, but rapidly rises to $\gtrsim 50\%$ at lower elevations. Cas A is a circumpolar source and spends about 7 hours every day skirting the horizon where the polarization leakage is large, so by placing the simulated source at the location of Cas A, we are engineering a situation where the polarization leakage from Stokes- Q into Stokes- I will be large. However, the impact on the unpolarized m -mode analysis maps is mild, amounting to a 0.5% error in the flux of the source with no measurable effect on the PSF.

6. COMPARISONS WITH OTHER SKY MAPS

6.1. *Guzmán 45 MHz Map*

A direct comparison between the OVRO-LWA maps interpolated to 45 MHz and the Guzmán 45 MHz map ([Guzmán et al. 2011](#)) can be seen in Figure 15. In order to make this comparison the OVRO-LWA map was de-

graded to a 5° resolution by convolving with a Gaussian kernel, and the Guzmán map has had spherical harmonics with $m = 0$ discarded in order to make it consistent with the maps presented in this paper. This figure shows a $\sim 20\%$ excess of emission in the galactic plane that is consistent with the discrepancy observed by the LWA1 sky survey ([Dowell et al. 2017](#)). However, while the LWA1 sky survey has an excess of emission near the north galactic pole, no such excess is observed in this work. Instead there is a 10% excess of emission near the south galactic pole. Elsewhere off the plane of the galaxy the discrepancy can be as much as -20%.

6.2. *Haslam 408 MHz Map*

The Haslam 408 MHz map ([Haslam et al. 1981, 1982](#)) was reprocessed by [Remazeilles et al. \(2015\)](#) to remove artifacts associated with $1/f$ noise and bright sources. In order to compare the OVRO-LWA maps with the lower resolution Haslam map, the OVRO-LWA maps were degraded to 56 arcmin resolution and the spectral index between the 73.152 MHz map and the 408 MHz Haslam map was computed by means of a local T-T plot at each pixel in the maps (in order to obviate the need for a zero-level correction to the 73.152 MHz map). The results of this computation are shown in Figure 16. Because this comparison uses a local T-T plot to estimate the spectral index, the angular resolution of this computation is somewhat degraded relative to other comparisons in the literature but the spectral indices in Figure 16 are robust to zero-level errors in either map.

The primary difference between the spectral index derived between 73.152 MHz and 408 MHz (Figure 16), and the spectral index derived between 36.528 MHz and 73.152 MHz (Figure 10) is the appearance of a multitude of H II regions that dot the galactic plane in the latter. The free-free absorption from these H II regions contributes to an overall flattening of the spectral index in the galactic plane that is also seen at higher frequencies.

7. CONCLUSION

In this work we presented a new imaging technique – Tikhonov regularized m -mode analysis imaging and cleaning – for drift-scanning telescopes like the OVRO-LWA. This technique exactly corrects for widefield effects in interferometric imaging with a single synthesis imaging step. We applied Tikhonov regularized m -mode analysis imaging to a 28 hour dataset and generated 8 sky maps between 36.528 MHz and 73.152 MHz. These sky maps are a substantial improvement over existing maps at these frequencies due to their 15 arcmin angular resolution and < 600 K thermal noise. The point source flux scale is consistent with that defined by [Scaife](#)

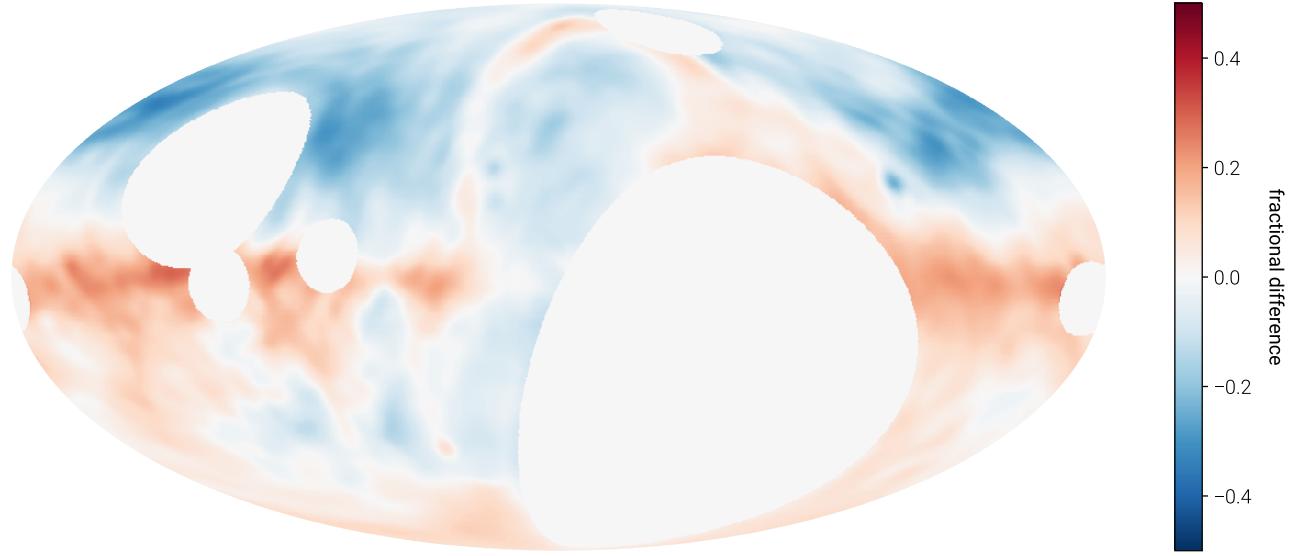


Figure 15. This Mollweide projection map compares the fractional difference between the Guzmán 45 MHz map, and the OVRO-LWA map at 46.992 MHz (degraded to 5° resolution). A positive value indicates regions where the OVRO-LWA map has more emission than the Guzmán map, and a negative value indicates regions where the Guzmán map has more emission than the OVRO-LWA map. Cas A, Cyg A, Vir A, and Tau A are masked due to the fact that they are subtracted from the OVRO-LWA maps but not the Guzmán map.

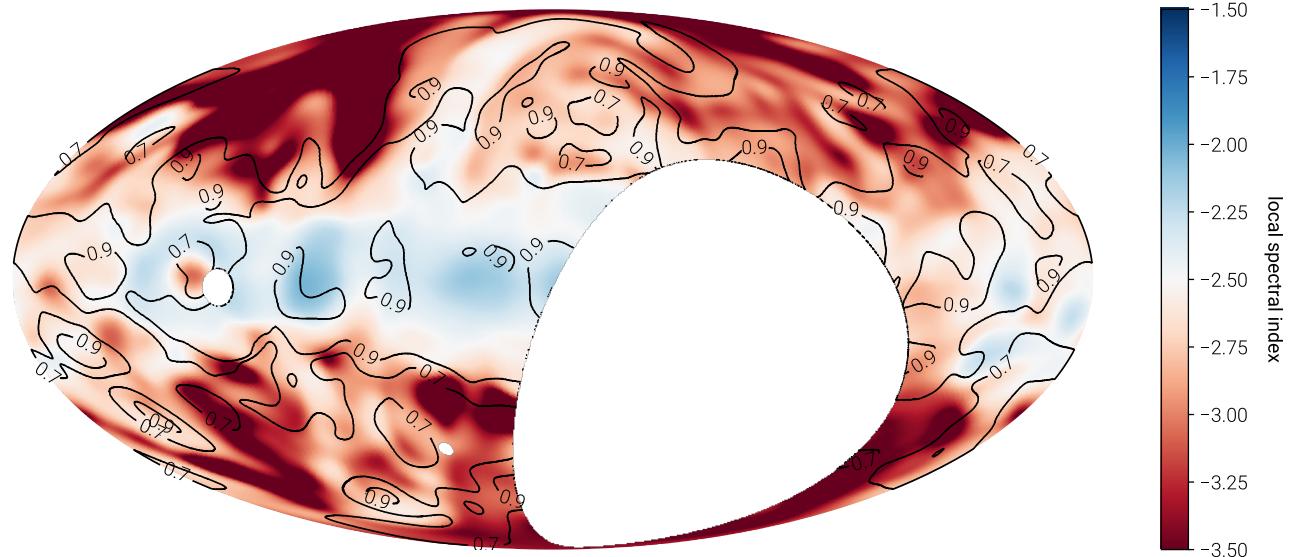


Figure 16. TODO: update caption The contours show the spectral index between the reprocessed Haslam 408 MHz map and the 73.152 MHz map (degraded to 56 arcmin resolution). These contours are overlaid on the same three-color map from Figure 9. The regions with shallow spectral index between 73.152 MHz and 408 MHz coincide with regions with large free-free absorption that is evident between 36.528 MHz and 73.152 MHz.

& Heald (2012) to about 5% and large angular scales are consistent with the work of Guzmán et al. (2011) to within 20%. These maps are primarily intended to be used as part of a foreground modeling and subtraction routine for 21 cm cosmology experiments.

At frequencies above ~ 55 MHz, the angular resolution of these maps is limited by the selection of $l_{\max} = 1000$. Future work will increase l_{\max} to re-

move this restriction, as well as include more time and bandwidth to improve the thermal noise. The usage of nighttime only data can help mitigate dynamic range limitations from the ionosphere and also eliminate solar sidelobe residuals. Observations could also be extended to slightly higher and lower frequencies (~ 27 to 85 MHz) to take advantage of the full frequency range of the OVRO-LWA. The higher frequencies are particu-

larly interesting in order to maximize the overlap with the MWA in the southern hemisphere, which could be used to fill-in the hole around the southern celestial pole.

This work is dedicated to the memory of Professor Marjorie Corcoran, who was an influential mentor to M. W. Eastwood.

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