

Berechne mit Hilfe der partiellen Integration:

$$\int x e^x dx = x e^x - \int e^x dx = (x - 1)e^x + c$$

$$\int x \sin(x) dx = -x \cos(x) - \int -\cos(x) dx = -x \cos(x) + \sin(x) + c$$

$$\begin{aligned} \int_0^1 3x e^x dx &= 3x e^x \Big|_0^1 - \int_0^1 3e^x dx = 3e - 0 - (3e^x) \Big|_0^1 = 3e - 3e + 3e^0 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \int_{\pi}^{2\pi} 2x \cos(x) dx &= (2x \sin(x)) \Big|_{\pi}^{2\pi} \\ &- \int_{\pi}^{2\pi} 2 \sin(x) dx = 0 - (-2 \cos(x)) \Big|_{\pi}^{2\pi} = -(-2 - 2) = 4 \end{aligned}$$

$$\begin{aligned} \int x^2 e^x dx &= x^2 e^x - \int 2x e^x dx = x^2 e^x - \left(2x e^x - \int 2e^x dx \right) \\ &= (x^2 - 2x + 2)e^x \end{aligned}$$

Die partielle Integration wird 2x durchgeführt!

$$\begin{aligned} \int_0^{\pi} a x^2 \sin(x) dx &= a \left[-x^2 \cos(x) \Big|_0^{\pi} - \int_0^{\pi} -2x \cos(x) dx \right] \\ &= a \left[(\pi)^2 + 0 + 2 \left(x \sin(x) \Big|_0^{\pi} - \int_0^{\pi} \sin(x) dx \right) \right] \\ &= a [(\pi)^2 + 0 + 2 \cos(x) \Big|_0^{\pi}] \\ &= a [(\pi)^2 + (-2 - 2)] = a(\pi^2 - 4) \end{aligned}$$