Berechne die Flächen zwischen den Funktionen

1) $f(x) = x^2 + 2$ $\qquad g(x) = 6$

$x^2 + 2 = 6$
$\Rightarrow x^2 = 4$
$x_1 = 2 \quad \vee \quad x_2 = -2$

$$F = \left| \int_{-2}^{2} (x^2 + 2 - 6)\, dx \right| = \left| \frac{1}{3}x^3 - 4x \, \Big|_{-2}^{2} \right| = \left| \left(\frac{8}{3} - 8 \right) - \left(-\frac{8}{3} + 8 \right) \right|$$

$$= \left| -\frac{16}{3} - \frac{16}{3} \right| = \frac{32}{3}$$

2) $f(x) = x^3 - 3x \qquad g(x) = 2x$

$x^3 - 3x = 2x$
$\Rightarrow x^3 - 5x = 0$
$\Rightarrow x(x^2 - 5) = 0$
$\Rightarrow x_1 = 0 \quad \vee \quad x_2 = -\sqrt{5} \quad \vee \quad x_3 = \sqrt{5}$

$$F = \left| \int_{-\sqrt{5}}^{0} (x^3 - 3x - 2x)\, dx \right| + \left| \int_{0}^{\sqrt{5}} (x^3 - 3x - 2x)\, dx \right|$$

$$= \left| \frac{1}{4}x^4 - \frac{5}{2}x^2 \, \Big|_{-\sqrt{5}}^{0} \right| + \left| \frac{1}{4}x^4 - \frac{5}{2}x^2 \, \Big|_{0}^{\sqrt{5}} \right|$$

$$= \left| 0 - \left(\frac{25}{4} - \frac{25}{2} \right) \right| + \left| \left(\frac{25}{4} - \frac{25}{2} \right) - 0 \right| = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}$$

3) $f(x) = x^3 - 3x^2$ $\qquad\qquad$ $g(x) = x - 3$

$\quad x^3 - 3x^2 = x - 3$

$\Rightarrow\ x^3 - 3x^2 - x + 3 = 0$

$\Rightarrow\ (x - 1)(x^2 - 2x - 3) = 0$

$\Rightarrow\ (x - 1)(x + 1)(x - 3) = 0$

$\Rightarrow\ x_1 = -1\ \ \lor\ \ x_2 = 1\ \ \lor\ \ x_3 = 3$

$$F = \left|\int_{-1}^{1} x^3 - 3x^2 - x + 3\, dx\right| + \left|\int_{1}^{3} x^3 - 3x^2 - x + 3\, dx\right|$$

$$= \left|\frac{1}{4}x^4 - x^3 - \frac{1}{2}x^2 + 3x \Big|_{-1}^{1}\right|$$

$$+ \left|\frac{1}{4}x^4 - x^3 - \frac{1}{2}x^2 + 3x \Big|_{1}^{3}\right|$$

$$= \left|\left(\frac{1}{4} - 1 - \frac{1}{2} + 3\right) - \left(\frac{1}{4} + 1 - \frac{1}{2} - 3\right)\right|$$

$$+ \left|\left(\frac{81}{4} - 27 - \frac{9}{2} + 9\right) - \left(\frac{1}{4} - 1 - \frac{1}{2} + 3\right)\right|$$

$$= \left|\frac{7}{4} + \frac{9}{4}\right| + \left|-\frac{9}{4} - \frac{7}{4}\right| = 4 + 4 = 8$$

4) $f(x) = 2x^2 + 5x - 1$ $\qquad\qquad$ $g(x) = 5x + 3$

$\quad 2x^2 + 5x - 1 = 5x + 3$

$\Rightarrow\ 2x^2 = 4$

$\Rightarrow\ x^2 = 2$

$\Rightarrow\ x_1 = \sqrt{2}\ \ \lor\ \ x_2 = -\sqrt{2}$

$$F = \left|\int_{-\sqrt{2}}^{\sqrt{2}} (2x^2 + 5x - 1) - (5x + 3)\, dx\right| = \left|\int_{-\sqrt{2}}^{\sqrt{2}} (2x^2 - 4)\, dx\right|$$

$$= \left|\frac{2}{3}x^3 - 4x \Big|_{-\sqrt{2}}^{\sqrt{2}}\right| = \left|\left(\frac{4}{3}\sqrt{2} - 4\sqrt{2}\right) - \left(-\frac{4}{3}\sqrt{2} + 4\sqrt{2}\right)\right|$$

$$= \left|-\frac{8}{3}\sqrt{2} - \frac{8}{3}\sqrt{2}\right| = \frac{16}{3}\sqrt{2}$$