Assignment_6

November 16, 2024

#Assignment 6 Due: Thursday Nov 7th EOD

Note: The lab on Monday the 11th will have an oral discussion based on the topic and your answers

1 Q1

You are calculating the temperature profile in an nuclear plate fuel that is 1mm thick; much thinner than the other dimensions. In a 1D model through the thickness, the temperature obeys the Fourier heat balance law with a heat source:

$$\nabla \cdot \lambda \nabla T = -Q$$

The boundary conditions are $T = 300$ on all outer surfaces.

1.1 1a

Write the finite difference scheme for Fourier heat balance equation.

{answer}

1.2 1b

Assuming $Q = 1kW/m^3$, and $\lambda = 2\frac{mW}{m \cdot K}$, use the finite difference method to find a solution with step size $10\mu m$

{implementation, answer}‘

1.3 1c

Write the Finite Difference formula if $\lambda = 2 + \frac{T}{300} \quad \frac{mW}{m \cdot K}$

{answer}

1.4 1d

Solve for T with the temperature dependent thermal conductivity

{Implementation, answer}

1.5 Bonus

Could you have used the method of solution of 1d to solve 1b (without any prior knowledge?)

{answer}

#Q2

The Fourier series shows that periodic functions can be written as an infinite sum of sine and cosine waves:

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \ \cos(nx) + B_n \ \sin(nx)$$

Due to function orthogonality, the values of A_n and B_n can be computed using the following formulae:

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\ dx$$
$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\ dx$$

Periodic functions can be approximated by truncating the Fourier series at some $n = N$.

1.6 2a

Write a function that takes a $2-\pi$ periodic function and a degree n, then outputs the $n-th$ Fourier coefficients $[A_n, B_n]$. Use the equations above with a suitable integration method. (Don't use packaged Fourier analysis tools)

Test it for a suitable set of functions for which you have an analytic answer.

{implementation, answer}

1.7 2b

Find the coefficients of the following functions and expansion orders: {All answers}

[ ]:
```
f = lambda x: np.mod(x, np.pi/2)
N = 5
```

[ ]:
```
f = lambda x: np.mod(x, np.pi/2)
N = 20
```

[ ]:
```
f = lambda x: (x > -np.pi/2) & (x < np.pi/2)
N = 2
```

[ ]:
```
f = lambda x: (x > -np.pi/2) & (x < np.pi/2)
N = 20
```

2 Q3:

Given a cubic $f(x)$ with

$$\int_{-1}^{1} f(x)dx = 3$$

and

$$f(-3^{-\frac{1}{2}}) = 1$$

What is $f(3^{-\frac{1}{2}})$? Why?

{answer}