Assignment_3_answers

November 16, 2024

Assignment 3

Due EOD Tuesday Oct 1st

#Question 1

A nuclear fuel pellet is a cylinder, 1.5 cm in lenth and 1 cm in diameter. Assume the surface temperature is 300 C everywhere. Given temperature probe data below, determine the radial temperature profile in the middle of a nuclear fuel pellet (i.e.: T(r, z = 0.75)) using radial basis functions.

[1]:
```python
import numpy as np

# 20 data points presented in columns: | x | y | z | T |

data = np.array([
    [5.1690e-02, 2.3766e-01, 6.7059e-01, 5.2645e+02],
    [1.1353e-01, 9.4708e-02, 5.3856e-01, 5.5201e+02],
    [1.6676e-01, 1.4358e-01, 4.6936e-01, 5.0802e+02],
    [1.3610e-01, 3.7207e-02, 2.1694e-01, 4.3663e+02],
    [8.9225e-02, 3.7293e-01, 1.1270e+00, 3.9234e+02],
    [1.9001e-01, 3.7240e-01, 8.4774e-01, 3.8872e+02],
    [5.4849e-02, 3.5425e-01, 5.7478e-01, 4.3784e+02],
    [1.7001e-01, 2.0241e-01, 1.2960e+00, 4.0159e+02],
    [2.0606e-01, 3.1594e-01, 6.4077e-01, 4.2652e+02],
    [2.5382e-01, 2.5859e-01, 4.8610e-01, 4.2481e+02],
    [5.6038e-02, 8.2231e-02, 4.2029e-01, 5.3244e+02],
    [3.1242e-01, 8.0489e-02, 1.1530e+00, 4.2453e+02],
    [6.0186e-02, 4.4891e-01, 3.9941e-01, 3.4207e+02],
    [1.5070e-01, 3.4794e-01, 1.5595e-01, 3.4750e+02],
    [1.8215e-01, 3.4388e-01, 1.0478e+00, 3.9963e+02],
    [1.1633e-01, 4.1011e-01, 5.5001e-01, 3.7611e+02],
    [1.2377e-01, 3.3703e-01, 3.7672e-02, 3.1423e+02],
    [4.6378e-02, 3.3653e-01, 1.4434e+00, 3.2345e+02],
    [2.9063e-02, 3.2584e-02, 2.3977e-01, 4.5993e+02],
    [2.1162e-02, 3.8590e-01, 2.5905e-01, 3.6901e+02]
])
```

Consider what you know about this system. What extra information do you have in terms of

0.0.1 a) type(s) of symmetry?

{answer}

Answer: 2 - azimuthal symmetry and mirror symmetry about the midplane

0.0.2 b) Boundary conditions?

{answer}

Answer: We konw the boundary conditions are 300 C around the periphery. This data is not in the data points above and needs to be added in manually.

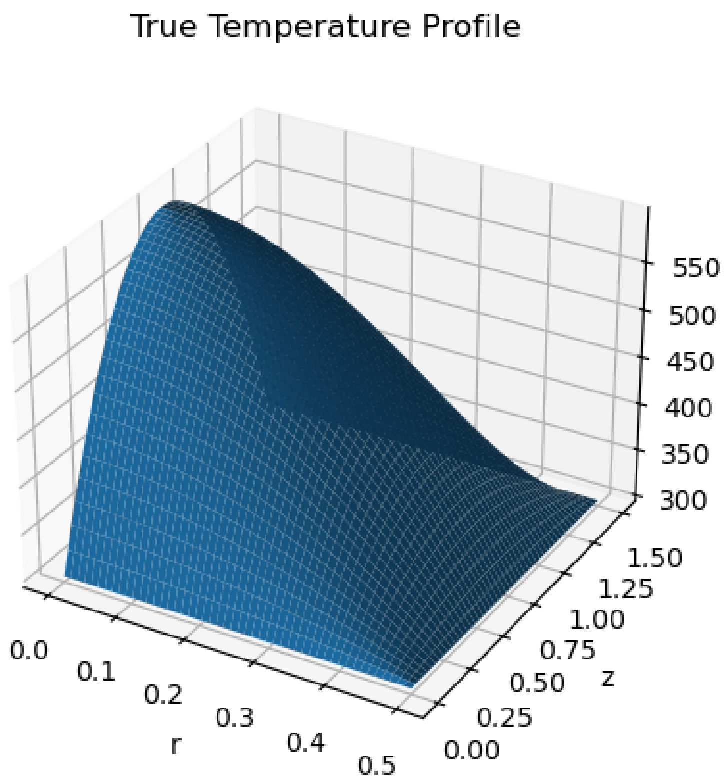
0.1 c) Plot the best guess of the radial temperature gradient

WARNING: RBFs will fail with a linear solver error if two data points exactly overlap.

{Method, implementation, answer (2 points)}

##Answer

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

def T_true(r, z):
  return 300 + 300*(1- (r / 0.5) ** 2) * (1 - ((z - 0.75) / 0.75) ** 2)

# Create a grid of r and z values
r = np.linspace(0, 0.5, 50)
z = np.linspace(0, 1.5, 50)
R, Z = np.meshgrid(r, z)

# Calculate the temperature at each point in the grid
T = T_true(R, Z)

# Create a 3D plot of the temperature profile
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(R, Z, T)
ax.set_xlabel('r')
ax.set_ylabel('z')
ax.set_zlabel('T')
ax.set_title('True Temperature Profile')

plt.show()
```



[3]:
```
# make radial symmetry
dataR = np.sqrt(data[:,0]**2 + data[:,1]**2)
# make z symmetric about midplane
dataZ = abs(data[:,2] - 0.75)

# Data in R Z T columns
dataRZ = np.column_stack((dataR, dataZ, data[:,3]))



n = 10
data_BC_top = np.column_stack([np.linspace(0, 0.49, n), np.ones(n)*.75, np.
 ↪ones(n)*300])
data_BC_side = np.column_stack([np.ones(n)*.5, np.linspace(0, 0.75, n), np.
 ↪ones(n)*300])

dataRZ2 = np.vstack([dataRZ, data_BC_top, data_BC_side])

print(dataRZ2)



from scipy.interpolate import RBFInterpolator
```

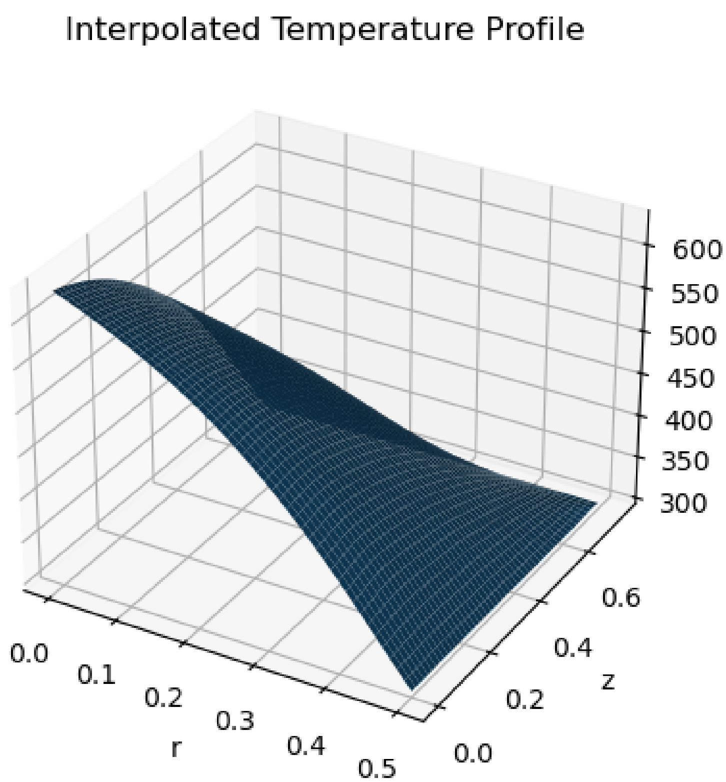
```
rbf = RBFInterpolator(dataRZ2[:,:2], dataRZ2[:,2])
```

```
[[2.43216224e-01 7.94100000e-02 5.26450000e+02]
 [1.47846766e-01 2.11440000e-01 5.52010000e+02]
 [2.20054798e-01 2.80640000e-01 5.08020000e+02]
 [1.41094191e-01 5.33060000e-01 4.36630000e+02]
 [3.83455194e-01 3.77000000e-01 3.92340000e+02]
 [4.18073630e-01 9.77400000e-02 3.88720000e+02]
 [3.58471024e-01 1.75220000e-01 4.37840000e+02]
 [2.64335409e-01 5.46000000e-01 4.01590000e+02]
 [3.77198631e-01 1.09230000e-01 4.26520000e+02]
 [3.62344284e-01 2.63900000e-01 4.24810000e+02]
 [9.95097724e-02 3.29710000e-01 5.32440000e+02]
 [3.22621660e-01 4.03000000e-01 4.24530000e+02]
 [4.52926642e-01 3.50590000e-01 3.42070000e+02]
 [3.79173751e-01 5.94050000e-01 3.47500000e+02]
 [3.89142746e-01 2.97800000e-01 3.99630000e+02]
 [4.26289668e-01 1.99990000e-01 3.76110000e+02]
 [3.59037928e-01 7.12328000e-01 3.14230000e+02]
 [3.39710700e-01 6.93400000e-01 3.23450000e+02]
 [4.36620548e-02 5.10230000e-01 4.59930000e+02]
 [3.86479806e-01 4.90950000e-01 3.69010000e+02]
 [0.00000000e+00 7.50000000e-01 3.00000000e+02]
 [5.44444444e-02 7.50000000e-01 3.00000000e+02]
 [1.08888889e-01 7.50000000e-01 3.00000000e+02]
 [1.63333333e-01 7.50000000e-01 3.00000000e+02]
 [2.17777778e-01 7.50000000e-01 3.00000000e+02]
 [2.72222222e-01 7.50000000e-01 3.00000000e+02]
 [3.26666667e-01 7.50000000e-01 3.00000000e+02]
 [3.81111111e-01 7.50000000e-01 3.00000000e+02]
 [4.35555556e-01 7.50000000e-01 3.00000000e+02]
 [4.90000000e-01 7.50000000e-01 3.00000000e+02]
 [5.00000000e-01 0.00000000e+00 3.00000000e+02]
 [5.00000000e-01 8.33333333e-02 3.00000000e+02]
 [5.00000000e-01 1.66666667e-01 3.00000000e+02]
 [5.00000000e-01 2.50000000e-01 3.00000000e+02]
 [5.00000000e-01 3.33333333e-01 3.00000000e+02]
 [5.00000000e-01 4.16666667e-01 3.00000000e+02]
 [5.00000000e-01 5.00000000e-01 3.00000000e+02]
 [5.00000000e-01 5.83333333e-01 3.00000000e+02]
 [5.00000000e-01 6.66666667e-01 3.00000000e+02]
 [5.00000000e-01 7.50000000e-01 3.00000000e+02]]
```

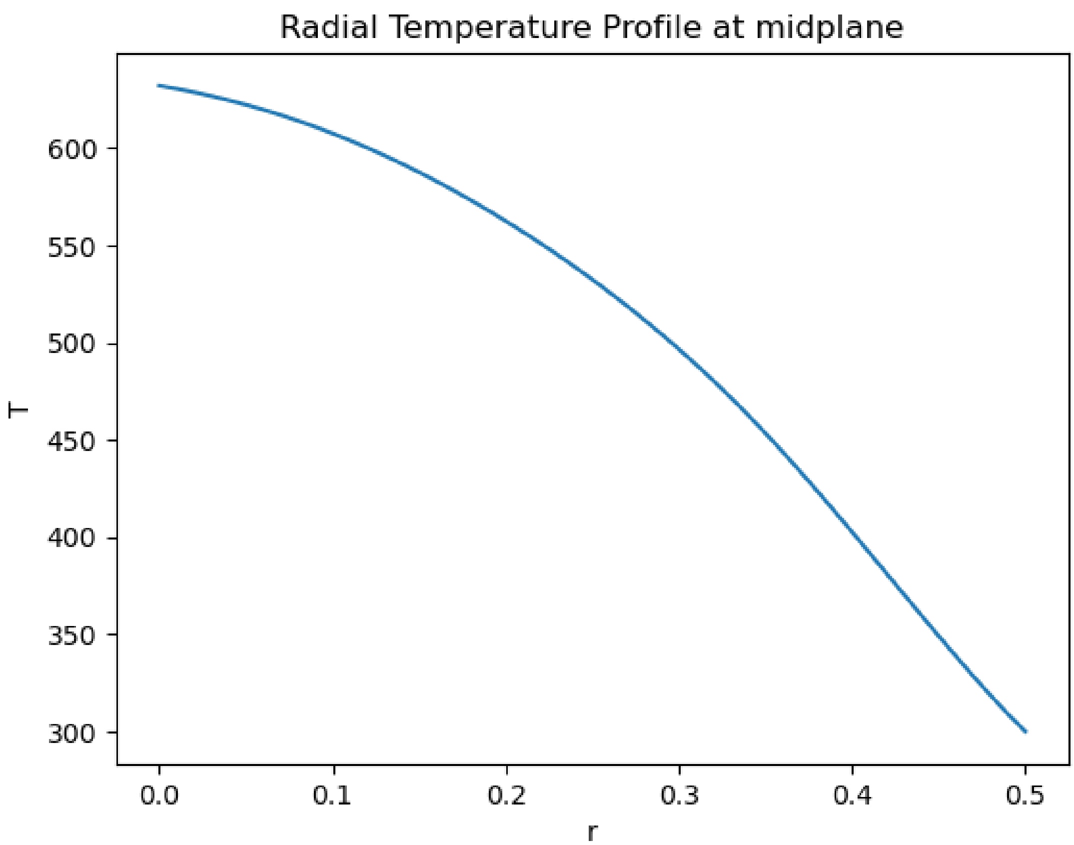
[9]:

```
# prompt: plot rbf over dataR from 0 to .5, and dataZ from 0 to .75

import numpy as np
import matplotlib.pyplot as plt
```

```python
from mpl_toolkits.mplot3d import Axes3D
from scipy.interpolate import RBFInterpolator

# Assuming dataR and dataZ are defined as in the previous code

r_interp = np.linspace(0, 0.5, 50)
z_interp = np.linspace(0, 0.75, 50)
R_interp, Z_interp = np.meshgrid(r_interp, z_interp)

T_interp = rbf(np.column_stack((R_interp.flatten(), Z_interp.flatten()))).
    ↪reshape(R_interp.shape)

# Create a 3D plot of the interpolated temperature profile
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(R_interp, Z_interp, T_interp)
ax.set_xlabel('r')
ax.set_ylabel('z')
ax.set_zlabel('T')
ax.set_title('Interpolated Temperature Profile')

plt.show()
```

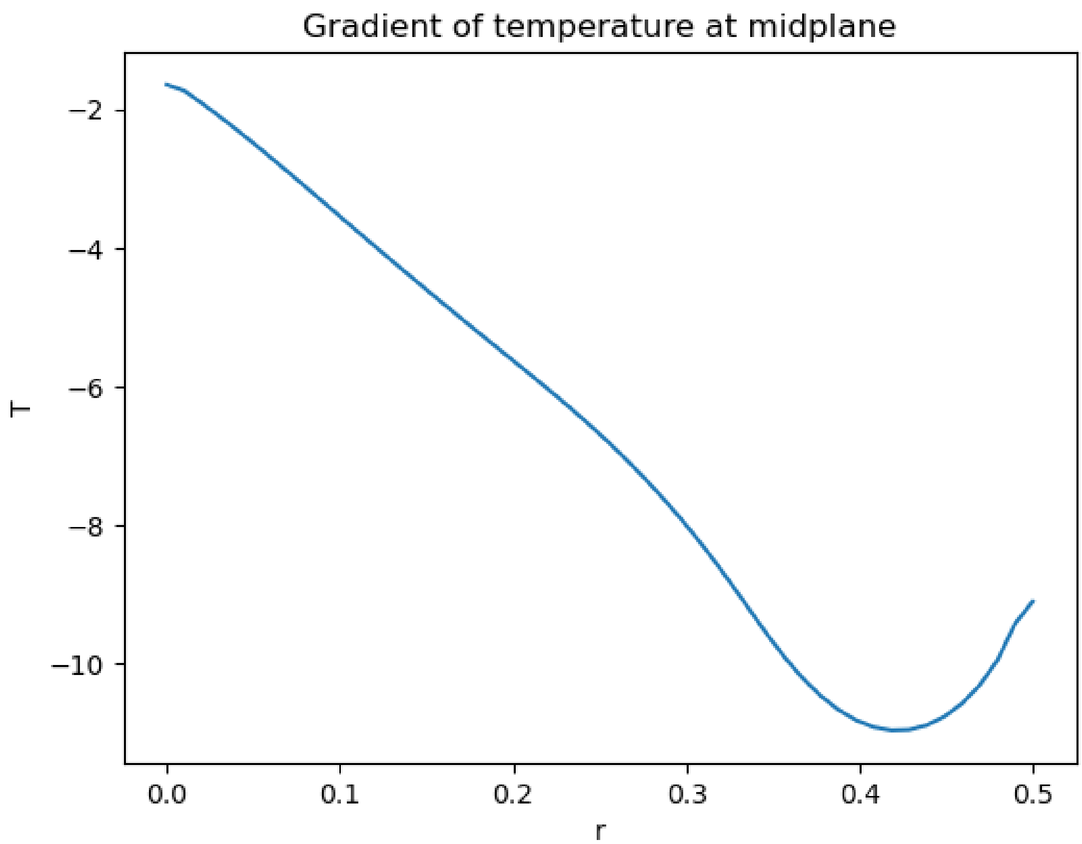


[10]:
```python
# prompt: Plot the temperature profile over R from 0 to .5 at z=0

import numpy as np
import matplotlib.pyplot as plt

# Assuming r_interp, z_interp, and T_interp are defined as in the previous code

# Find the index of z=0 in z_interp
z_0_index = np.argmin(np.abs(z_interp - 0))

# Extract the temperature profile at z=0
T_interp_z0 = T_interp[:, z_0_index]

# Plot the temperature profile
plt.plot(r_interp, T_interp_z0)
plt.xlabel('r')
plt.ylabel('T')
plt.title('Radial Temperature Profile at midplane')
plt.show()
```



The gradient is:

[12]:
```
plt.plot(r_interp, np.gradient(T_interp_z0))
plt.xlabel('r')
plt.ylabel('T')
plt.title('Gradient of temperature at midplane')
plt.show()
```



1 Question 2

You run an experiment and obtain the following data:

x	y1	y2	y3	y4	y5
0.00	-29.49	-2.14	15.88	22.69	28.53
1.11	2.83	18.02	-25.45	-32.45	7.50
2.22	1.97	-10.49	-0.18	-32.10	-40.31
3.33	-38.09	-46.16	-7.87	-33.97	-38.39
4.44	-3.97	-32.22	-33.95	-11.07	-32.47

x	y1	y2	y3	y4	y5
5.56	4.45	-10.88	20.43	6.57	-8.49
6.67	50.22	51.29	80.02	66.15	84.90
7.78	164.11	190.26	160.94	182.35	163.18
8.89	331.75	306.51	278.40	302.13	335.44
10.00	517.06	483.20	476.73	512.16	500.64

[6]:

```
import numpy as np

# Define the table as a list of lists
d = np.array([
    [0.00, -29.49, -2.14, 15.88, 22.69, 28.53],
    [1.11, 2.83, 18.02, -25.45, -32.45, 7.50],
    [2.22, 1.97, -10.49, -0.18, -32.10, -40.31],
    [3.33, -38.09, -46.16, -7.87, -33.97, -38.39],
    [4.44, -3.97, -32.22, -33.95, -11.07, -32.47],
    [5.56, 4.45, -10.88, 20.43, 6.57, -8.49],
    [6.67, 50.22, 51.29, 80.02, 66.15, 84.90],
    [7.78, 164.11, 190.26, 160.94, 182.35, 163.18],
    [8.89, 331.75, 306.51, 278.40, 302.13, 335.44],
    [10.00, 517.06, 483.20, 476.73, 512.16, 500.64]
])
```

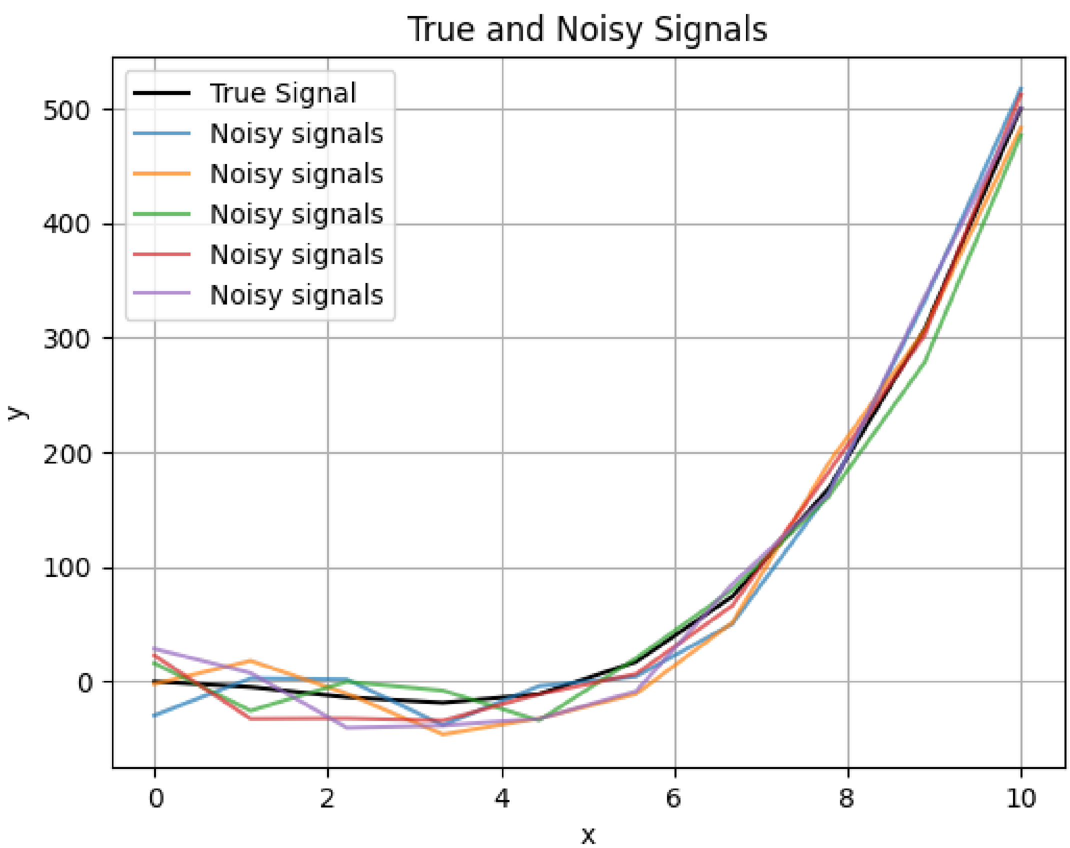
1.1 a) Determine the best cubic polynomial fit to this data with the uncertainty

{method, implementation, answer}

[7]:

```
# prompt: plot the curve x^3-5x^2 from 0 to 10. For every step, sample it 5
 ↪times with uniform error. Plot the true and noisy signal. Assemble the x,
 ↪and 5 noisy signals together

import numpy as np
import matplotlib.pyplot as plt

# Define the true function
def true_function(x):
  return x**3 - 5*x**2

# Generate x values
x = np.linspace(0, 10, 10)

# Calculate the true y values
y_true = true_function(x)

# # Create noisy signals
# noisy_signals = []
```

```python
# for i in range(5):
#   noise = np.random.uniform(-30, 30, size=len(x))
#   y_noisy = y_true + noise
#   noisy_signals.append(y_noisy)

x, noisy_signals = d[:,0], d[:,1:]


# Plot the true signal
plt.plot(x, y_true, label='True Signal', color='black')


plt.plot(x,noisy_signals, label=f'Noisy signals', alpha = 0.7)

plt.xlabel('x')
plt.ylabel('y')
plt.title('True and Noisy Signals')
plt.legend()
plt.grid(True)
plt.show()
```

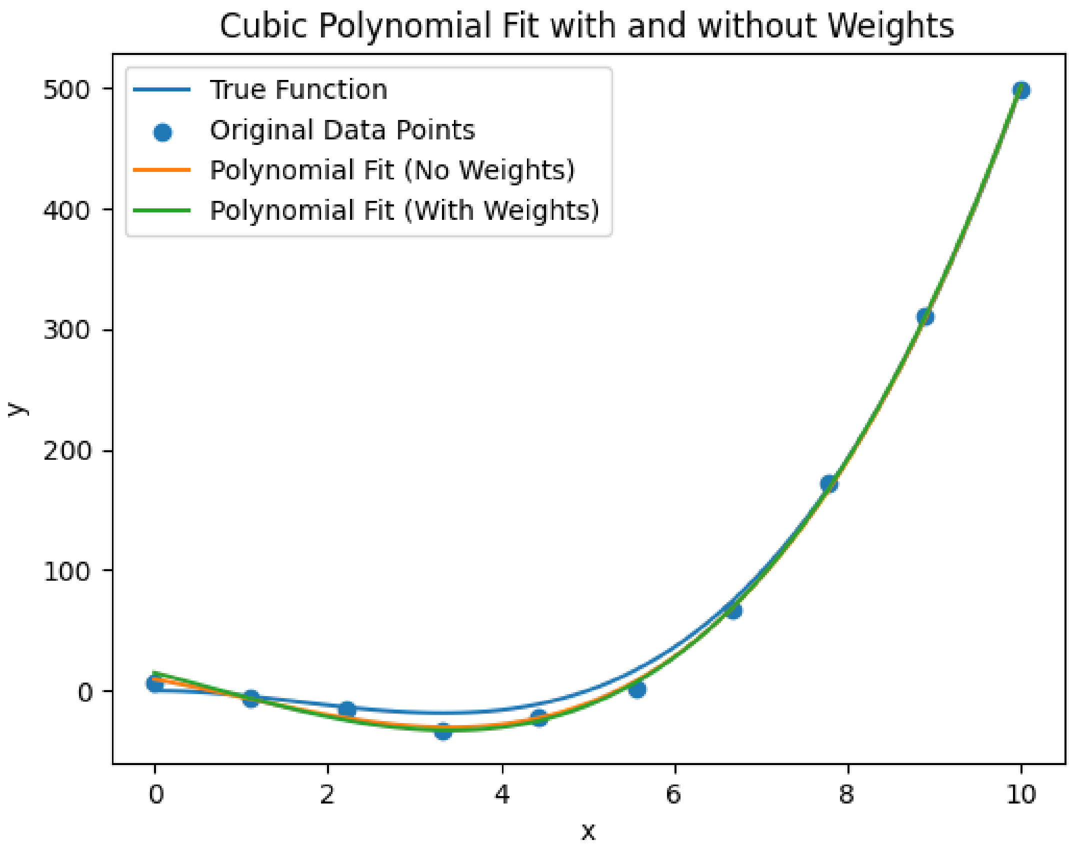


[8]:
```
import numpy as np

# Assuming 'data' is defined as in the preceding code

# Calculate the mean of the last 5 columns
mean_last_5 = np.mean(d[:, -5:], axis=1)

# Calculate the variance of the last 5 columns
variance_last_5 = np.var(d[:, -5:], axis=1)
print(d[:,0])
print(mean_last_5)
print(variance_last_5)

# Create a table using the first column, mean, and variance
table_data = np.column_stack([d[:,0], mean_last_5, variance_last_5])
```

```
[ 0.    1.11  2.22  3.33  4.44  5.56  6.67  7.78  8.89 10.  ]
[  7.094  -5.91  -16.222 -32.896 -22.736   2.416  66.516 172.168 310.846
 497.958]
[440.717384 383.00876  290.698616 172.109904 159.741824 128.324304
 203.570824 140.555416 437.870904 248.440576]
```

[9]:
```
import numpy as np
import matplotlib.pyplot as plt

x = table_data[:, 0]
y_mean = table_data[:, 1]
y_std = table_data[:, 2]

# Cubic polynomial fit without weights
coeffs_no_weights = np.polyfit(x, y_mean, 3)
polynomial_no_weights = np.poly1d(coeffs_no_weights)

# Cubic polynomial fit with weights (using 1/std_dev as weight)
weights = 1 / y_std
coeffs_with_weights = np.polyfit(x, y_mean, 3, w=weights)
polynomial_with_weights = np.poly1d(coeffs_with_weights)

# Define the true function
def true_function(x):
  return x**3 - 5*x**2

# Generate x values for plotting
x_plot = np.linspace(0, 10, 100)

# Calculate the true function values
y_true = true_function(x_plot)
```

```python
# Calculate the polynomial values for plotting
y_no_weights = polynomial_no_weights(x_plot)
y_with_weights = polynomial_with_weights(x_plot)

# Plot the true function, original points, and both polynomials
plt.plot(x_plot, y_true, label='True Function')
plt.scatter(x, y_mean, label='Original Data Points')
plt.plot(x_plot, y_no_weights, label='Polynomial Fit (No Weights)')
plt.plot(x_plot, y_with_weights, label='Polynomial Fit (With Weights)')

plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Polynomial Fit with and without Weights')
plt.legend()
plt.show()
```

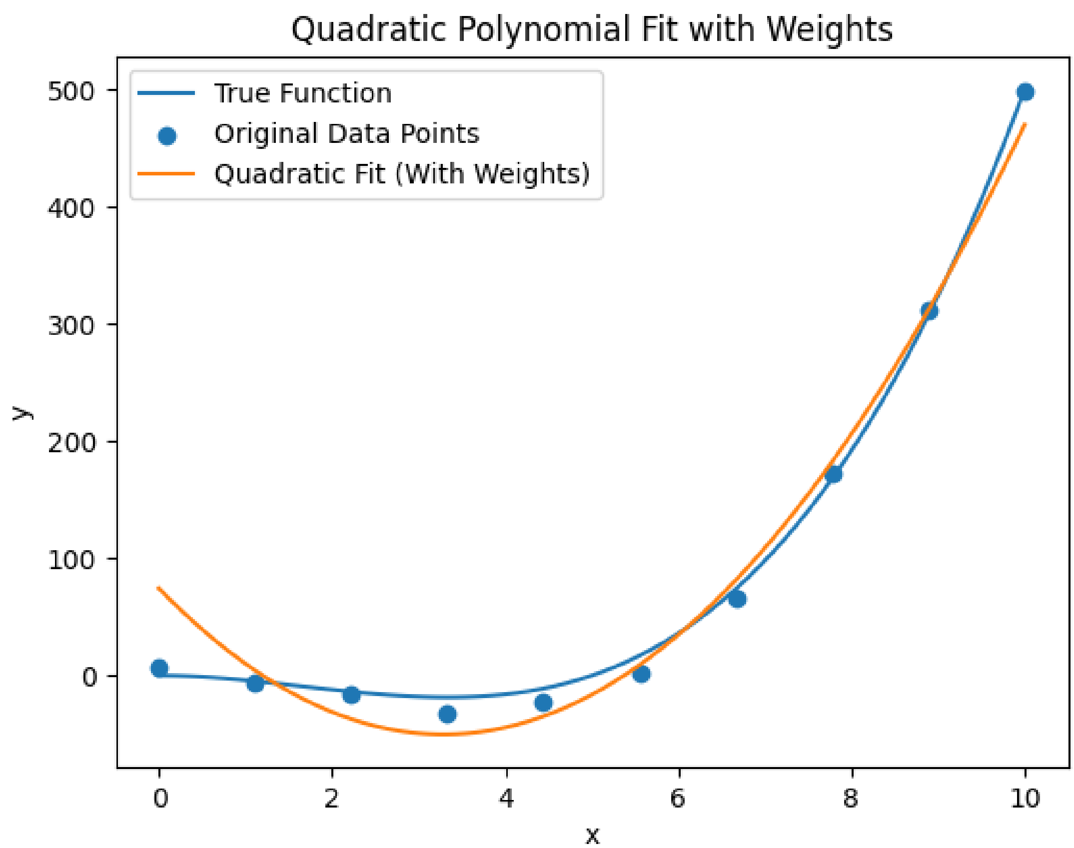


1.2 b) Your manager thinks this should be a quadratic. Which do you think it should be and why?

{Answer}

[10]:

```
# prompt: repeat the polyfit with a quadratic using the weights

import matplotlib.pyplot as plt
import numpy as np
# Assuming 'data' and 'table_data' are defined as in the preceding code

x = table_data[:, 0]
y_mean = table_data[:, 1]
y_std = table_data[:, 2]

# Quadratic polynomial fit with weights (using 1/std_dev as weight)
weights = 1 / y_std
coeffs_with_weights = np.polyfit(x, y_mean, 2, w=weights)
polynomial_with_weights = np.poly1d(coeffs_with_weights)

# Define the true function
def true_function(x):
  return x**3 - 5*x**2

# Generate x values for plotting
x_plot = np.linspace(0, 10, 100)

# Calculate the true function values
y_true = true_function(x_plot)

# Calculate the polynomial values for plotting
y_with_weights = polynomial_with_weights(x_plot)

# Plot the true function, original points, and both polynomials
plt.plot(x_plot, y_true, label='True Function')
plt.scatter(x, y_mean, label='Original Data Points')
plt.plot(x_plot, y_with_weights, label='Quadratic Fit (With Weights)')

plt.xlabel('x')
plt.ylabel('y')
plt.title('Quadratic Polynomial Fit with Weights')
plt.legend()
plt.show()
```


Quadratic Polynomial Fit with Weights
True Function
Original Data Points
Quadratic Fit (With Weights)
500
400
300
200
100
0
y
0
2
4
6
8
10
x

[10]: