Complementary Filter

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1

Limitation of Wiener Filter

- Signal and noise must both be random.
- Many applications have a deterministic signal and random noise.
- Extend Wiener filter (or Phillips approach) to allow deterministic signal.

2

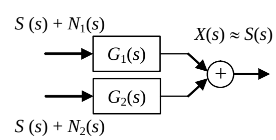
Example: Aircraft Position

- $s(t)$ = position of aircraft in flight (scalar)
- Use sensor to determine the position.
- Position is deterministic.
- Measurement includes random errors.
- Need second sensor to use Wiener filter.

3

Instrumentation Application

- Use two measurements of the same signal.
- Use a different filter for each signal.
- Add the two filtered signals.
- Select filter transfer functions to minimize mean square error.



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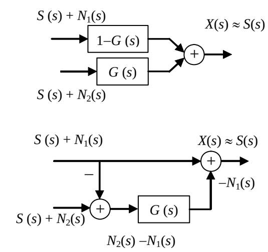
Selection of Two Filters

Attempt to make overall output approximately equal to the signal.

$$
\begin{aligned}
X(s) &= G_1(s)[S(s)+N_1(s)] + G_2(s)[S(s)+N_2(s)] \\
&= [G_1(s)+G_2(s)]S(s) + G_1(s)N_1(s) + G_2(s)N_2(s) \\
&= S(s) + [1-G(s)]N_1(s) + G(s)N_2(s) \\
&= [S(s)+N_1(s)] + G(s)[N_2(s)-N_1(s)]
\end{aligned}
$$

Block Diagrams



Complementary Filter Properties

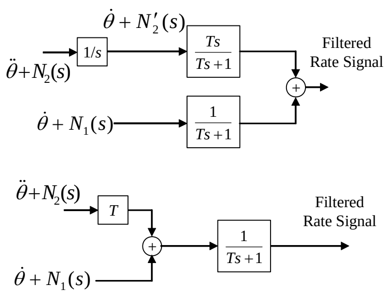
- Signal unaffected by choice of filter $G(s)$
- Noise affected by choice of filter.
- If $N_1(s)$ low frequency and $N_2(s)$ high frequency, use a low pass filter $G(s)$ then $[1–G(s)]$ is a high pass filter.
- Input to $G(s)$ is a purely random signal
  - Estimate – $N_1(s)$ using a Wiener filter
  - $N_2(s)$ noise

Example: Position Servo

- Tachometer provides noisy velocity measurement.
- Noisy accelerometer measurement.
- Assume: need a LPF for the tachometer signal.

$$
G(s) = \frac{1}{Ts+1} \qquad 1-G(s) = \frac{Ts}{Ts+1}
$$

Block Diagram



9

Example (Cont.)

- Minimize error to obtain the optimum filter.
- Optimum linear filter of selected form.
- Use (causal) Wiener filter for the optimum linear filter.

10