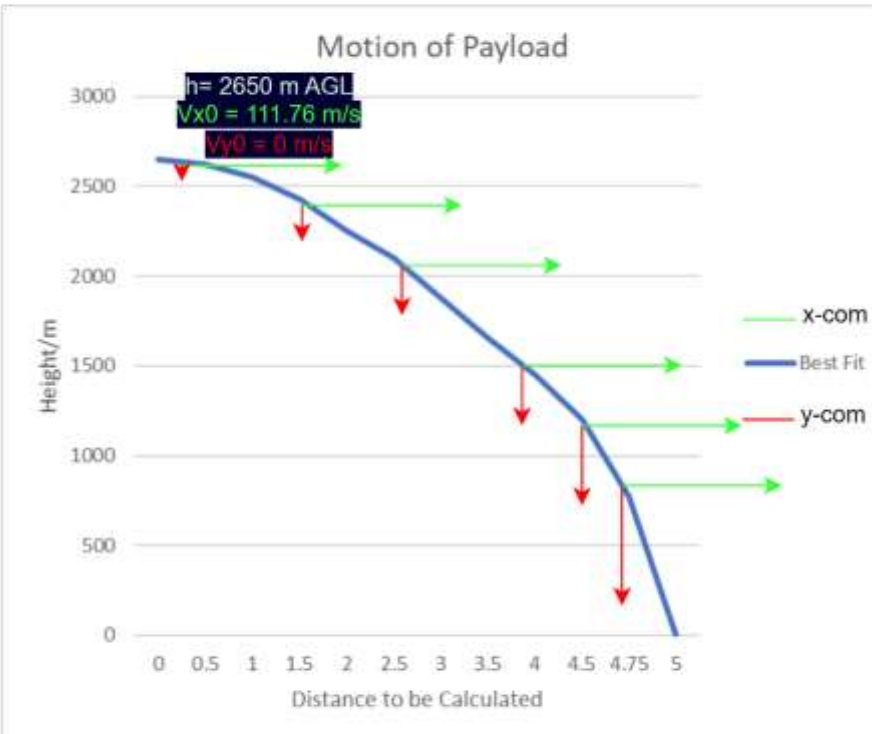
A&L ENGINEERING
Supply Drop Plan by Maurice Wesley

Horizontal and Vertical Motion

Diagram



Kinematics Calculations

I needed to calculate the x/y component and the final velocity of a payload dropped from a height of 2650 meters with an initial velocity of 111.76 m/s. Using my understanding of kinematic equations and the given variables, I calculated the horizontal and vertical motion of the payload to ensure it arrives at the drop site.

a. Below, I determined the initial velocity of the payload when launched, I calculated the velocity of the payload when it hits the ground, and located the distance to drop from the target (OpenStax, 2022).

  i. Horizontal component: $x = V * \cos(0) = 111.76 \frac{m}{s}$

  ii. Vertical component: $y = V * \sin(0) = 0 \frac{m}{s}$

  iii. $y_f = y_0 + V_0 t - \frac{1}{2} * 9.8 * t^2$

  iv. $0 = 2650 + \sin(0) * t - .5 * 9.8 * t^2$

  v. $t = \sqrt{\frac{2650}{.5*9.8}} = 23.26\ s$

  vi. $V_f = V_0 + at \rightarrow 0 \frac{m}{s} - 9.8 * 23.26 = 227.95 \frac{m}{s}$

vii. $x_f = x_0 + V_{x_0}t - \frac{1}{2} * 0 * t^2 \rightarrow 111.76\ \frac{m}{s} * 23.26\ s = 2599.54\ m$

viii. $Initial\ Velocity: 0\ \frac{m}{s}$

ix. $Final\ Velocity: 227.95\ \frac{m}{s}$

x. $Distance: 2599.54\ m$

Equation Descriptions

a. We use arrows to describe a vector's direction and magnitude. When considering a two-dimensional projectile in motion, we can graph the path using three vectors. The horizontal and vertical components are represented on the x and y axis respectively.

b. The third vector signifies the straight-line distance between the starting and finishing points of the object's motion. Referencing the resource material provided, "motions along perpendicular axis can be analyzed separately". Restated for clarity, the x-component does not affect the y-component (OpenStax, 2022).

c. The aircraft is traveling parallel to the x-axis when the payload is dropped. Thus, the angle of the drop is zero degrees.

d. It is best practices to consider the y-axis as vertical. First, the acceleration due to gravity occurs in the vertical direction. To find the vertical component, we use the equation $A_y = A \sin\theta$ where A is the initial velocity times the sin of the trajectory. Earlier, we determined the trajectory angle to be zero (OpenStax, 2022).

e. Now that we have the vertical component, we can use the maximum vertical displacement equation, $y = y_0 + V_{y_0}t - \frac{1}{2} * 9.8 * t^2$, to find the time for the payload to fall from 2650 meters to the ground (OpenStax, 2022).

f. After we have the time, then we can use the equation to find final velocity $V_f = V_0 + at$.

g. It is best practices to consider the x-axis as horizontal. First, the acceleration due to gravity does not occur in the horizontal direction. We can assume there is no acceleration in the horizontal direction. To find the horizontal component, we use the equation $A_x = A\cos\theta$ where A is the initial velocity times the cosine of the trajectory. Similarly, the trajectory angle is zero (OpenStax, 2022).

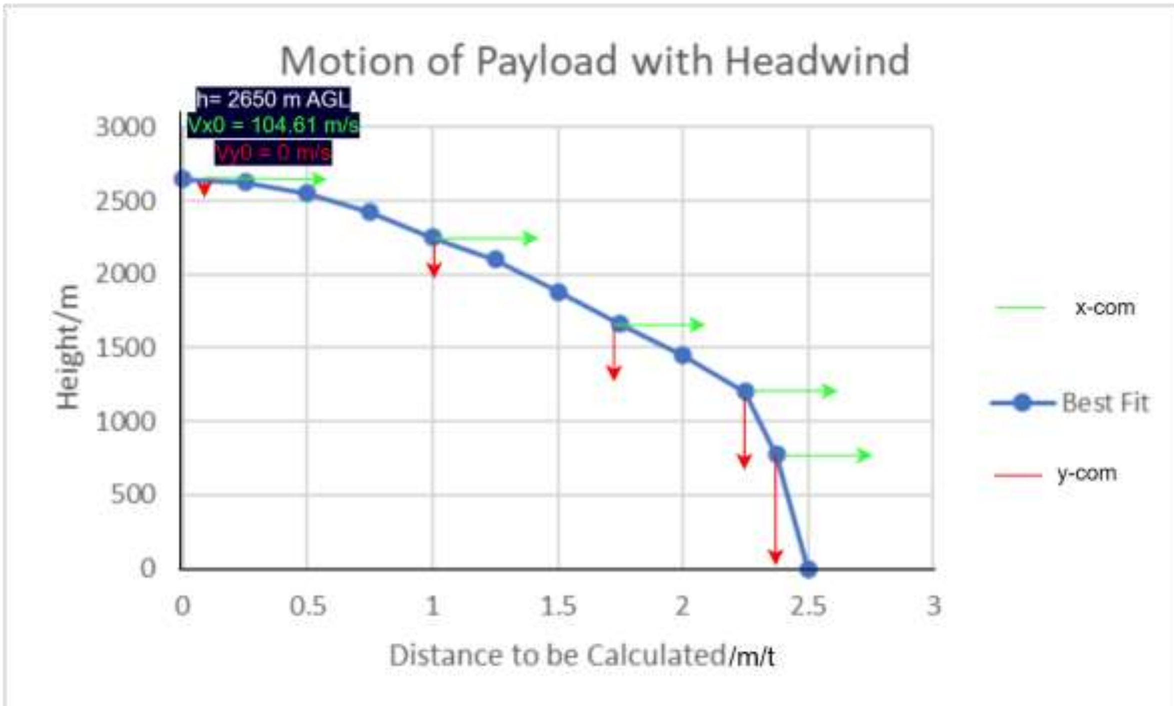
h. Now that we have the horizontal component and the time it will take for the payload to hit the ground, we can use the maximum horizontal displacement equation, $x = x_0 + V_{x_0}t - \frac{1}{2} * 0 * t^2$, to calculate the distance from the drop. The initial position is zero and there is zero acceleration in the horizontal direction. Thus, we simply multiply the initial velocity times the calculated time it takes for the payload to hit the ground (OpenStax, 2022).

i. The payload drop involved considering a two-dimensional projectile in motion. There are three major factors to consider when analyzing motion using the kinematics principles (velocity, acceleration, displacement). An object that is displaced through the air will be subjected to four forces. One constraint for the payload was to assume that all forces were negligible except for gravity (OpenStax, 2022).

j. There are two independent one-dimensional motions that sum to the two-dimensional curved path of the horizontally dropped payload. Gravity was defined as $a_y = -g = -9.8\frac{m}{s^2}$. Thus, the acceleration due to gravity in the downward direction was a positive value (OpenStax, 2022).

Adaptations for Scenario One

Diagram



Adaptation Description

a. By definition, a headwind is in the opposite direction of the projectile's motion. The drop angle is zero degrees. We can use vector subtraction to find the total velocity of the x-component along the horizontal axis. We discussed earlier that we can independently analyze motion along a perpendicular axis. Thus, a headwind will not have an impact on the y-component of the projectile's motion (OpenStax, 2022).

b. In Chicago, at the time of this writing, the wind velocity was 16 mph (7.15 $\frac{m}{s}$). As noted earlier, the headwind is opposite of the projectile's direction. Using vector subtraction:

c. $111.76\ \frac{m}{s} - 7.15\frac{m}{s} = 104.61\ \frac{m}{s}$

d. Now, we can plug in the adjusted x-component velocity to find total distance. The headwind does not affect the vertical component and we determined the time aloft to be 23.26 seconds.

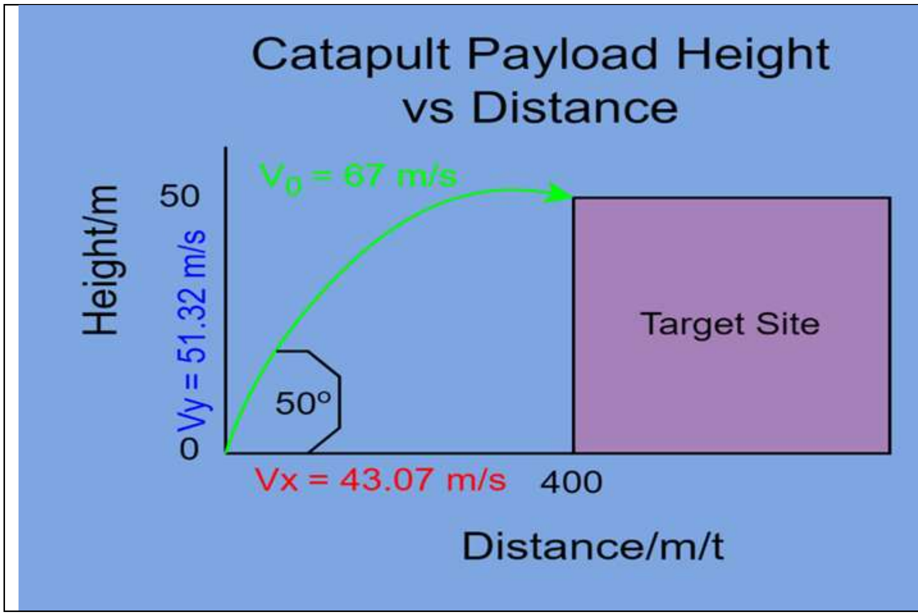
e. $x_f = x_0 + V_{x_0}t - \frac{1}{2} * 0 * t^2 \rightarrow 104.61\ \frac{m}{s} * 23.26\ s = 2433.23\ m$

f. The first alternative required us to consider dropping the payload with a headwind. The kinematics principles allow us to analyze motion independently on perpendicular axis. Thus, using vector subtraction, we calculated the change in the x-component when accounting for a headwind (motion in the opposite direction). The distance for the drop decreased from 2599.54 meters to 2433.23 meters (OpenStax, 2022).

g. First, the airplane carrying the payload could drop the load at a faster initial velocity to compensate for the headwind. An increase in ground speed equal to the increase in headwind would net the same distance from the drop.
h. Second, the airplane carrying the payload could drop closer to the target site. The distance from the target at the time of the drop would decrease from 2599.54 meters to 2433.23 meters.

Adaptations for Scenario Two

Diagram



Adaptation Description

a. First, we find the horizontal and vertical components (OpenStax, 2022).
b. Horizontal component: $x = V * \cos(\theta) \rightarrow\ 67\ \frac{m}{s} * \cos 50 = 43.07$
c. Vertical component: $y = V * \sin(\theta) = 67\ \frac{m}{s} * \sin 50 = 51.32$
d. Now that we have the horizontal and vertical components, we can determine the maximum vertical and horizontal displacement (OpenStax, 2022).
e. $V_{y_f} = V_{y_0} - 9.8 * t\ \rightarrow 51.32 - 9.8t\ \rightarrow t = \frac{51.32}{9.8} = 5.24\ s$
f. Now that we have the time in the vertical direction, we can calculate y_f (OpenStax, 2022).
g. $y_f = \frac{{V_0}_y{}^2}{2g} \rightarrow \frac{51.32^2}{2*9.8} = 134.37\ m$
h. We can calculate the maximum horizontal displacement by multiplying the time times the horizontal component (OpenStax, 2022).
i. $x_f = x_0 + V_{x_0}2t - \frac{1}{2} * 0 * t^2\ \rightarrow 43.07\ \frac{m}{s} * 2 * 5.24\ s = 451.37\ m$
j. In the above equation, acceleration of gravity along the horizontal axis is zero.

k. We were able to determine that the maximum vertical displacement for payload when using a catapult was 134.37 meters and the maximum horizontal displacement was 451.37 meters.
l. We know the maximum horizontal displacement when the surface is flat. However, the payload has to clear a 50-meter cliff. We can calculate the time for the payload to fall 84.37 meters (134.37 - 50) by dividing the maximum vertical displacement by the time (134.37 / 5.24). Then, we divide 84.37 by the result and add the quotient to the vertical time aloft. Please see below.
m. $\frac{134.37}{5.24} = 25.64\ \frac{m}{s}\ \rightarrow\ \frac{84.37}{25.64} = 3.29\ s\ \rightarrow 5.24 + 3.29 = 8.53\ s$
n. Now that we know the time aloft to 50ft above the ground, we multiply the horizontal component times the time aloft to determine if the payload will clear the 50-meter cliff.
o. $8.53\ s * 43.07\ \frac{m}{s} = 367.39\ meters$
p. We have determined that the payload will be at a distance of 367.39 meters when it is 50 meters above the ground. The payload will not make it over the 50-meter cliff.
q. The catapult is located at a fixed position. The optimal trajectory angle for a projectile is 45 degrees. At the same initial velocity, a five percent decrease in the trajectory angle would net a 10-meter loss in the previous distance at 50 meters above the ground.
r. I would recommend decreasing the launch angle to 45 degrees and increasing the initial velocity. Adjustments to these variables would increase the maximum horizontal displacement.

References

Open Stax College. (2022). *College physics 2e*. OpenStax. Retrieved March 20, 2024 from 3.1 Kinematics in Two Dimensions: An Introduction - College Physics 2e | OpenStax

Open Stax College. (2022). *College physics 2e*. OpenStax. Retrieved March 21, 2024 from 3.2 Vector Addition and Subtraction: Graphical Methods - College Physics 2e | OpenStax

Open Stax College. (2022). *College physics 2e*. OpenStax. Retrieved March 23, 2024 from 3.3 Vector Addition and Subtraction: Analytical Methods - College Physics 2e | OpenStax

Open Stax College. (2022). *College physics 2e*. OpenStax. Retrieved March 23, 2024 from 3.4 Projectile Motion - College Physics 2e | OpenStax