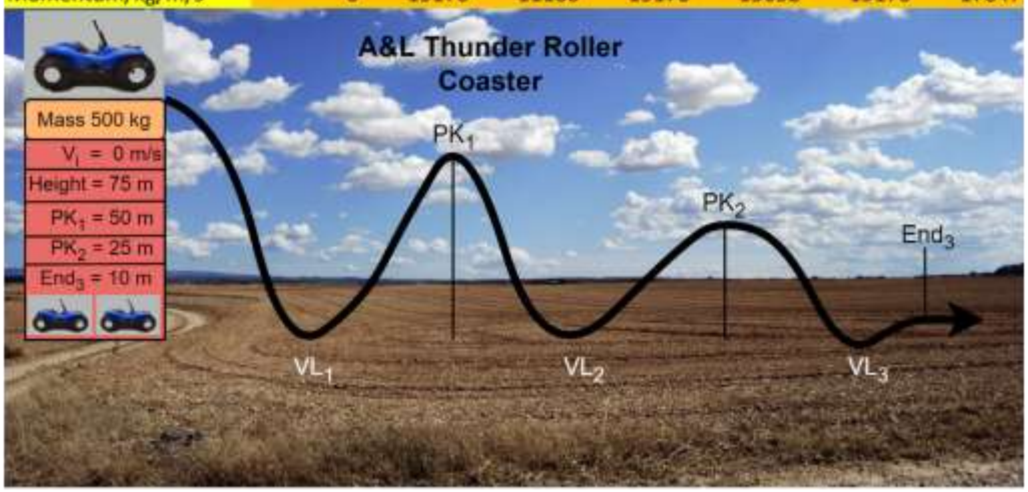
# A&L ENGINEERING
# Roller Coaster Design Report

## Coaster Design Diagram

| A & L Thunder Roller | Start | VL1 | PK1 | VL2 | PK2 | VL3 | End3 |
|---|---|---|---|---|---|---|---|
| Mass/kg | 500 | 500 | 500 | 500 | 500 | 500 | 500 |
| Height/m | 75 | 0 | 50 | 0 | 25 | 0 | 10 |
| Velocity/m/s | 0 | 38.34 | 22.14 | 38.34 | 31.30 | 38.34 | 35.69 |
| Potential Energy/J | 367500 | 0 | 245000 | 0 | 122500 | 0 | 49000 |
| Kinetic Energy/J | 0 | 367500 | 122500 | 367500 | 245000 | 367500 | 318500 |
| Momentum/kg/m/s | 0 | 19170 | 11180 | 19170 | 15652 | 19170 | 17847 |




## Kinetic Energy, Potential Energy, and Momentum Calculations

$KE = \frac{1}{2}mv^2 \rightarrow KE = \frac{1}{2} * 500 * 0^2 = 0$

$PE = mgh = 500 * 9.8 * 75 \rightarrow PE = 367500\,J$

$[\Delta p = \Delta(mv)] \rightarrow p = 500 * 0 = 0$

1. Starting Point Recap:
   a. Potential Energy: 367500 J
   b. Kinetic Energy: 0 J
   c. Momentum: 0 kg m/s

$PE = mgh = 500 * 9.8 * 0 \rightarrow PE = 0\,J$

KE + PE at the starting point will equal the KE + PE

KE is equal to 367500

$KE = \frac{1}{2}mv^2 \rightarrow 367500 = .5 * 500 * v^2 \rightarrow v = 38.34\ \frac{m}{s}$

$p = 500 * 38.34 = 19170\ kg\frac{m}{s}$

2. $VL_1$ Recap:
   a. Potential Energy: 0 $J$

b. Kinetic Energy: $367500\ J$

c. Momentum: $19170\ kg\frac{m}{s}$

$PE = mgh \ \rightarrow\ PE = 500 * 9.8 * 50 = 245000\ J$
$KE = 367500_{KE+PE_{VL_1}} - 245000_{PE_{PK_1}} = 122500\ J$
$KE = \frac{1}{2}mv^2 \ \rightarrow 122500 = .5 * 500 * v^2 \rightarrow v = 22.14\ \frac{m}{s}$
$p = 500 * 22.14 = 11180\ kg\frac{m}{s}$

3. $PK_1$ Recap:
   a. Potential Energy: 245000 J
   b. Kinetic Energy: 122500 J
   c. Momentum: 11180 kg m/s

$PE = mgh = 500 * 9.8 * 0 \ \rightarrow PE = 0\ J$
KE is equal to 367500 J (245000 + 122500)
$KE = \frac{1}{2}mv^2 \ \rightarrow 367500 = .5 * 500 * v^2 \rightarrow v = 38.34\ \frac{m}{s}$
$p = 500 * 38.34 = 19170\ kg\frac{m}{s}$

4. $VL_2$ Recap:
   a. Potential Energy: 0 J
   b. Kinetic Energy: 367500 J
   c. Momentum: 19170 kg m/s

$PE = mgh \ \rightarrow\ PE = 500 * 9.8 * 25 = 122500\ J$
$KE = 367500_{KE+PE_{VL_2}} - 122500_{PE_{PK_2}} = 245000\ J$
$KE = \frac{1}{2}mv^2 \ \rightarrow 245000 = .5 * 500 * v^2 \rightarrow v = 31.30\ \frac{m}{s}$
$p = 500 * 31.30 = 15652\ kg\frac{m}{s}$

5. $PK_2$ Recap:
   a. Potential Energy: 122500 J
   b. Kinetic Energy: 245000 J
   c. Momentum: 15652 kg m/s

$PE = mgh = 500 * 9.8 * 0 \ \rightarrow PE = 0\ J$
KE is equal to 367500 J (122500 + 245000)
$KE = \frac{1}{2}mv^2 \ \rightarrow 367500 = .5 * 500 * v^2 \rightarrow v = 38.34\ \frac{m}{s}$
$p = 500 * 38.34 = 19170\ kg\frac{m}{s}$

6. VL3 Recap:

a. Potential Energy: 0 J
b. Kinetic Energy: 367500 J
c. Momentum: 19170 kg m/s

$PE = mgh \rightarrow PE = 500 * 9.8 * 10 = 49000\,J$
$KE = 367500_{KE+PE_{VL_3}} - 49000_{PE_{End_3}} = 318500\,J$
$KE = \frac{1}{2}mv^2 \rightarrow 318500 = .5 * 500 * v^2 \rightarrow v = 35.69\,\frac{m}{s}$
$p = 500 * 35.69 = 17847\,kg\frac{m}{s}$

7. End3 Recap:
    a. Potential Energy: 49000 J.
    b. Kinetic Energy:318500 J
    c. Momentum: 17847 kg m/s

**Energy Descriptions**

We will consider the roller coaster design as a one-dimensional Kinematic application. At the initial state of the system, the coaster is stationary and located at a height of 75 meters. Kinetic is energy relative to an object's motion. The velocity of the coaster is zero. Thus, the kinetic energy will be zero.

Gravity is the attraction that increases the speed of the coaster towards the first valley. We are able to view all friction all negligible. Thus, the gravitational potential energy will be the greatest at the lowest point of the initial drop. To calculate the potential energy at the starting point, we can use the equation $PE = mgh$. At this point, we can state that KE + PE is constant for all points in a constant mass system (OpenStax, 2022).

The velocity and momentum at the starting are zero. The height at $VL_1$ is zero. The potential energy at $VL_1$ is $PE = mgh$. The KE + PE at the starting point will equal the KE + PE at $VL_1$. All of the potential energy generated from the starting point will be converted into kinetic energy at $VL_1$. We can find the final velocity at $VL_1$ by plugging in the KE into the equation $KE = \frac{1}{2}mv^2$ (OpenStax, 2022).

Now that we know the final velocity at the first drop, we can use the equation for momentum $[\Delta p = \Delta(mv)]$ becomes $p = mv$. Considering the known values, we can find the momentum at the first drop. We can see that the gravitational potential energy of the roller coaster is transformed into kinetic energy at the lowest point of the valley or $VL_1$.

The roller coaster will have an initial velocity of $38.34\ \frac{m}{s}$ in the vertical direction to $PK_1$. The next peak is located at a height of 50 meters. Thus, we have $PE = mgh \rightarrow PE = 500 * 9.8 * 50$. Again, KE + PE is constant for all points in the system. Thus, we can find Kinetic energy by subtracting the potential energy from the total energy in the system at $VL_1$, $E_{KE+PE_{VL_1}} - E_{PE_{PK_1}}$ (OpenStax, 2022).

Once again, we can find the final velocity at $PK_1$ by plugging in the KE into the KE equation. Now that we know the final velocity at $PK_1$, we can use the equation for momentum $[\Delta p = \Delta(mv)]$ becomes $p = mv$. Considering the known values, we can calculate the momentum at $PK_1$.

The height at $VL_2$ is zero. Thus, the potential energy where mass times velocity will be zero. The KE + PE at $PK_1$ will equal the KE + PE at $VL_2$. All of the energy at $PK_1$ will be converted into kinetic energy at $VL_2$. Again, we can find the final velocity at $VL_2$ by plugging in the KE into the equation $KE = \frac{1}{2}mv^2$. This is the same magnitude as the velocity at $VL_1$. Similarly, the momentum at $VL_2$ is the same as the momentum at $VL_1$ (OpenStax, 2022).

The process is repetitive and perpetual. The gravitational potential energy of the roller coaster is transformed into kinetic energy at the lowest point of the valley or $VL_2$. The roller coaster will have an initial velocity of $38.34\ \frac{m}{s}$ in the vertical direction to $PK_2$.

The next peak is located at a height of 25 meters. Thus, we have $PE = mgh \rightarrow PE = 500 * 9.8 * 2$. Again, KE + PE is constant for all points in the system. Thus, we can find Kinetic energy by subtracting the potential energy from the total energy in the system.

After finding the final velocity using the kinetic energy equation we calculated the momentum. The height at $VL_3$ is zero. Thus, the potential energy will be zero again. The KE + PE at $PK_2$ will equal the KE + PE at $VL_3$. All of the energy at $PK_2$ will be converted into kinetic energy at $VL_3$.

As stated previously, the momentum and kinetic energy of the roller coaster were the same for each lowest point in the system. The gravitational potential energy of the roller coaster is transformed into kinetic energy at the lowest point of the valley or $VL_3$. The roller coaster will have an initial velocity of $38.34\ \frac{m}{s}$ in the vertical direction to $End_3$.

The next peak is located at a height of 10 meters. Thus, we have $PE = mgh \rightarrow PE = 500 * 9.8 * 10$. Again, KE + PE is constant for all points in the system. Thus, we can find Kinetic energy by subtracting the potential energy from the total energy in the system. We can use the same process to find the final velocity and momentum of the system at the end (OpenStax, 2022).

### Collision Calculations

$$\frac{1}{2}mv^2{}_{Cart\ A} + \frac{1}{2}mv^2{}_{Cart\ B}$$

$$KE = \frac{1}{2}500 * 35.69^2{}_{Cart\ A} + \frac{1}{2} * 500 * 0^2{}_{Cart\ B} \rightarrow KE = 318500\,J$$
$$m_{A_{t_0}} v_{A_{t_0}} + m_{B_{t_0}} v_{B_{t_0}} = m_{A_{t_1}} v_{A_{t_1}}{}' + m_{B_{t_1}} v_{B_{t_1}}{}'$$
$$m_{A_{t_0}} v_{A_{t_0}} = \left(m_{A_{t_1}} + m_{B_{t_1}}\right) v'$$
$$v' = \frac{m_{A_{t_0}} v_{A_{t_0}}}{m_{A_{t_1}} + m_{B_{t_1}}} \rightarrow v' = \frac{500 * 35.69}{500 + 500} = 17.85\ \frac{m}{s}$$
$$KE = \frac{1}{2}(m_{Cart\ A} + m_{Cart\ B})v^2$$
$$KE = \frac{1}{2} * (500 + 500) * 17.85^2 = 159311\,J$$
$$159311\,J - 318500\,J = -159189\,J$$

**Collision Descriptions**

In continuation from the above, the cart came to End3 with a calculated velocity of $35.69\ \frac{m}{s}$.

The KE and PE were $318500\,J$ and $49000\,J$ respectively. We will refer to the roller coaster as Cart A and the stationary coaster as Cart B. Both carts have equal mass. “An inelastic collision is one in which the internal kinetic energy changes” (OpenStax, 2022). Under this constraint, the kinetic energy is not conserved.

The total internal kinetic energy before the collision is $\frac{1}{2}mv^2{}_{Cart\ A} + \frac{1}{2}mv^2{}_{Cart\ B}$, where the mass of Cart A and B individually is 500 kg. Cart B is stationary with zero velocity. Thus, plugging in the known values yields $KE = \frac{1}{2}500 * 35.69^2{}_{Cart\ A} + \frac{1}{2} * 500 * 0^2{}_{Cart\ B} \rightarrow KE = 318500\,J$ (OpenStax, 2022).

The kinetic energy in the system before the collision is caused by Cart A’s mass and velocity. Again, KE equals 318500. The motion of the system can be described as a one-dimensional Kinematics application along the horizontal axis. Where, the total momentum of the system before the collision is equal to the total momentum after the collision. Mathematically, we can describe this phenomenon with $m_{A_{t_0}} v_{A_{t_0}} + m_{B_{t_0}} v_{B_{t_0}} = m_{A_{t_1}} v_{A_{t_1}}{}' + m_{B_{t_1}} v_{B_{t_1}}{}'$ (OpenStax, 2022).

Cart B and B will have a velocity of zero after the inelastic collision. Thus, the above equation becomes $m_{A_{t_0}} v_{A_{t_0}} = \left(m_{A_{t_1}} + m_{B_{t_1}}\right) v'$.

We need to find the final velocity on the right side of the equation before we can calculate the Kinetic energy after the inelastic collision. Solving $v'$ yields $v' = \frac{m_{A_{t_0}} v_{A_{t_0}}}{m_{A_{t_1}} + m_{B_{t_1}}} \rightarrow v' = \frac{500*35.69}{500+500} = 17.85\ \frac{m}{s}$. Energy cannot be created nor destroyed; it can only be shared. After the collision, the two cart with equal mass will share the momentum equally (OpenStax, 2022).

The internal KE can be found by $KE = \frac{1}{2}(m_{Cart\ A} + m_{Cart\ B})v^2$ where $v'$is the final velocity that we found previously. Plugging in the known values yields $KE = \frac{1}{2} * (500 + 500) * 17.85^2 = 159311\ J$.

The change in internal kinetic energy is $159311\ J - 318500\ J = -159189$. The minus sign indicates that there was a lost of energy in the system in form of (sound) thermal energy (OpenStax, 2022).

**Friction Calculations**

*Tip*: The work-energy theorem tells us that change in kinetic energy is equal to work done on the system.

$$v' = \frac{m_{A_{t_0}} v_{A_{t_0}}}{m_{A_{t_1}} + m_{B_{t_1}}}$$
$$v' = \frac{1000 * 17.85}{1000 + 1000} = 8.9\ \frac{m}{s}$$
$$f = \frac{\frac{1}{2} m {v_i}^2}{d} \rightarrow f = \frac{.5 * 2000 * 8.9}{20} = 445\ N$$
$$W_{NC} = fd * \cos\theta \ \rightarrow \ W_{NC} = 445 * 20 * \cos 180$$
$$Work\ due\ to\ friction = -8900\ J$$

**Energy Transfers**

For the work due to friction, we will consider the work-energy theorem to determine the change in mechanical energy of the coaster system. From the theorem, we can conclude that the “net-work on a system equals the change in its kinetic energy” (OpenStax, 2022).

The net-work in a system includes conservative and nonconservative work. If we want to find the nonconservative work, then we subtract the “total work done by all conservative forces” from the change in kinetic energy (OpenStax, 2022).

Non conservative work due to friction is equal to the frictional force times the distance times the cosine of the angle between the frictional force and the direction of motion. The angle between the direction of motion and the frictional force is 180 degrees. Thus, the cosine of 180 is -1 (OpenStax, 2022).

Before we continue, we will need to find $v'$ after the collision. Earlier, we concluded that $v' = \frac{m_{A_{t_0}} v_{A_{t_0}}}{m_{A_{t_1}} + m_{B_{t_1}}}$ where in this case, $m_{A_{t_0}}$ is the combined mass of Cart A and B. Additionally, $v_{A_{t_0}}$ is the velocity of the internal kinetic energy before the collision (calculated earlier). The two carts collide with a third cart that has double the mass of Cart A (OpenStax, 2022).

Plugging in the known values yields, $v' = \frac{1000*17.85}{1000+1000} = 8.9\ \frac{m}{s}$. Note that Cart A had an initial velocity 35.69 before colliding with Cart B. The final velocity of the internal system was cut in half after the collision. Similarly, When Cart A and B collided with an object of equal mass, the velocity of the internal system was cut in half (17.85 $\frac{m}{s}$ before, 8.9 $\frac{m}{s}$ after).

Now that we know the final velocity of the internal kinetic system, we can solve the work-energy theorem for frictional force and plug in the known values, $f = \frac{\frac{1}{2}m{v_i}^2}{d} \rightarrow f = \frac{.5*2000*8.9}{20} = 445\ N.$

Picking back up on nonconservative work, we now know the frictional force over 20 meters. We can use the equation $W_{NC} = fd * \cos\theta \ \rightarrow \ W_{NC} = 445 * 20 * \cos 180$ to find the work due to friction. We can conclude that the nonconservative work due to friction is $-\ 8900\ J$ (OpenStax, 2022).

Thank you for your time. Please feel free to follow up with me with any questions, comments, or concerns. With that being said. It has been a pleasure ladies and gentlemen.

**References**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