A&L ENGINEERING
Case Material Evaluation Report

Newton's Laws of Motion

Greetings ladies and gentlemen. Thank you for joining me this afternoon for the A&L Engineering case material evaluation presentation. We have a lot of information to cover and it is critical to define some baseline concepts before we dive in.

Problem Statement: Which phone material will provide the most protection at the point of impact when drop from a height of 2 meters.

Success Criteria: Approximate the objects motion using diagrams, equations, and statistics.

Force: The commonsensical definition of force would be the push and pulling action that is placed on another object. In the game tug of war, two teams on opposite ends try to move a flag on the middle of the rope to one side. One team has to pull more than the other for the flag to move. This is the basic concept of force (OpenStax, 2022).

Newton's First Law: Also known as the law of inertia (measured by mass), Newton's first law is defined as "there must be a cause for there to be any change in velocity". Restated for clarity, an object will stay at rest or in motion unless acted upon by another object. An athlete can start jogging on a treadmill at a specific rate. However, they will not be able to maintain a constant rate (velocity) forever because of the energy expended during the jog (OpenStax, 2022).

We will attempt to model Newton's first law by hanging a cell phone above the ground. Additionally, we will observe the phone's attempt to stay in motion, when dropped from the ceiling, by measuring its mass and interaction with the ground.

Newton's Second Law: Newton tried to approximate an object's motion at speeds much less than light. A close sibling to Newton's first law is the mathematical relationship between mass and a change in speed. We discovered earlier that an object's inertia is measured by mass. A change in speed is a change in velocity which equates to an acceleration (OpenStax, 2022).

A cell phone will be the object that is measured to determine the mass of the system. The initial state of the system will be at rest. The net external forces are in equilibrium. When the phone is released, the system is no longer in equilibrium.

Gravity is the force that attracts objects to the earth's surface. The acceleration of earth's gravity is more than the upward force holding the phone in the air. The phone will continue to stay in motion until there is a change in velocity or until it hits the ground.

Newton's Third Law: Representing the symmetry in nature, Newton's third law states that "one body cannot exert a force on another without experiencing a force itself". The phone accelerates towards the ground. We are able to determine the force of the phone from the inertia (mass) and earth's gravity (acceleration) before the point of impact (OpenStax, 2022).

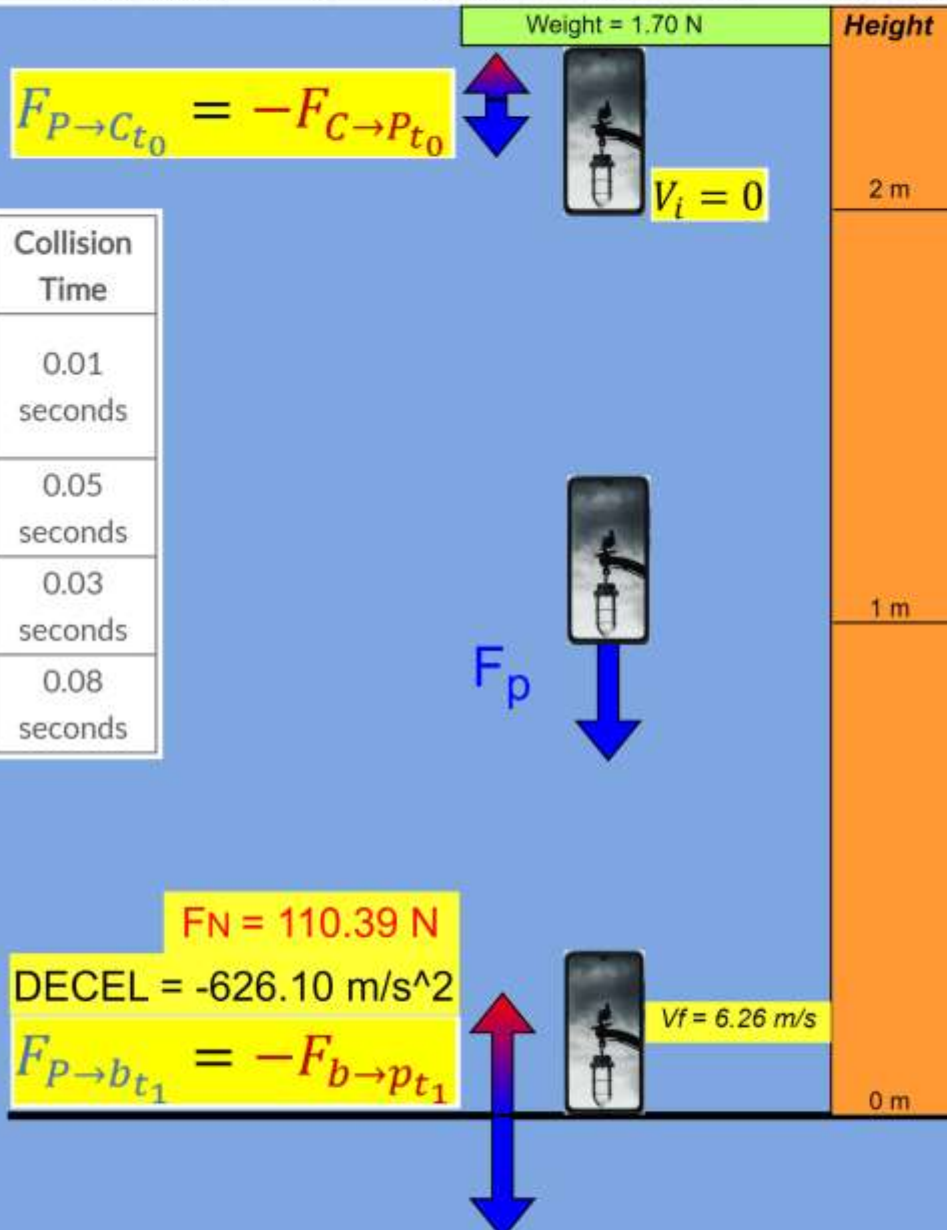
A derivation of Newton's second law represents the phone's final velocity before the collision. At the point of impact, the distance traveled and final velocity is zero. Newton's third law approximates this motion with the sum of the forces equaling the inertia times the acceleration in the vertical direction. Using this relationship and the final velocity from Newton's second law, we are able to determine the deceleration and the force of the impact (OpenStax, 2022).

We will document the findings and perform a comparative analysis between the materials in question. Afterwards, we will make a recommendation on which case material performed the best. Please refer to the force diagrams below.

Force Diagrams

Phone Material Drop Test: Phone Only

	Material/g	Phone + Material/kg	Weight/N	Distance/m	Accel/m/s^2	Time/s	Velocity/m/s	Collision/s	Decel/m/s^2	FORCE/N
Phone Only	6.2	0.1736	1.70128	2	9.8	0.20	6.26	0.01	-626.10	110.39
Hard Plastic	1.1	0.2044	2.00312	2	9.8	0.20	6.26	0.03	-208.70	44.66
Silicone	1.7	0.2212	2.16776	2	9.8	0.20	6.26	0.05	-125.22	29.87
Rubber	3.2	0.2632	2.57936	2	9.8	0.20	6.26	0.08	-78.26	23.18

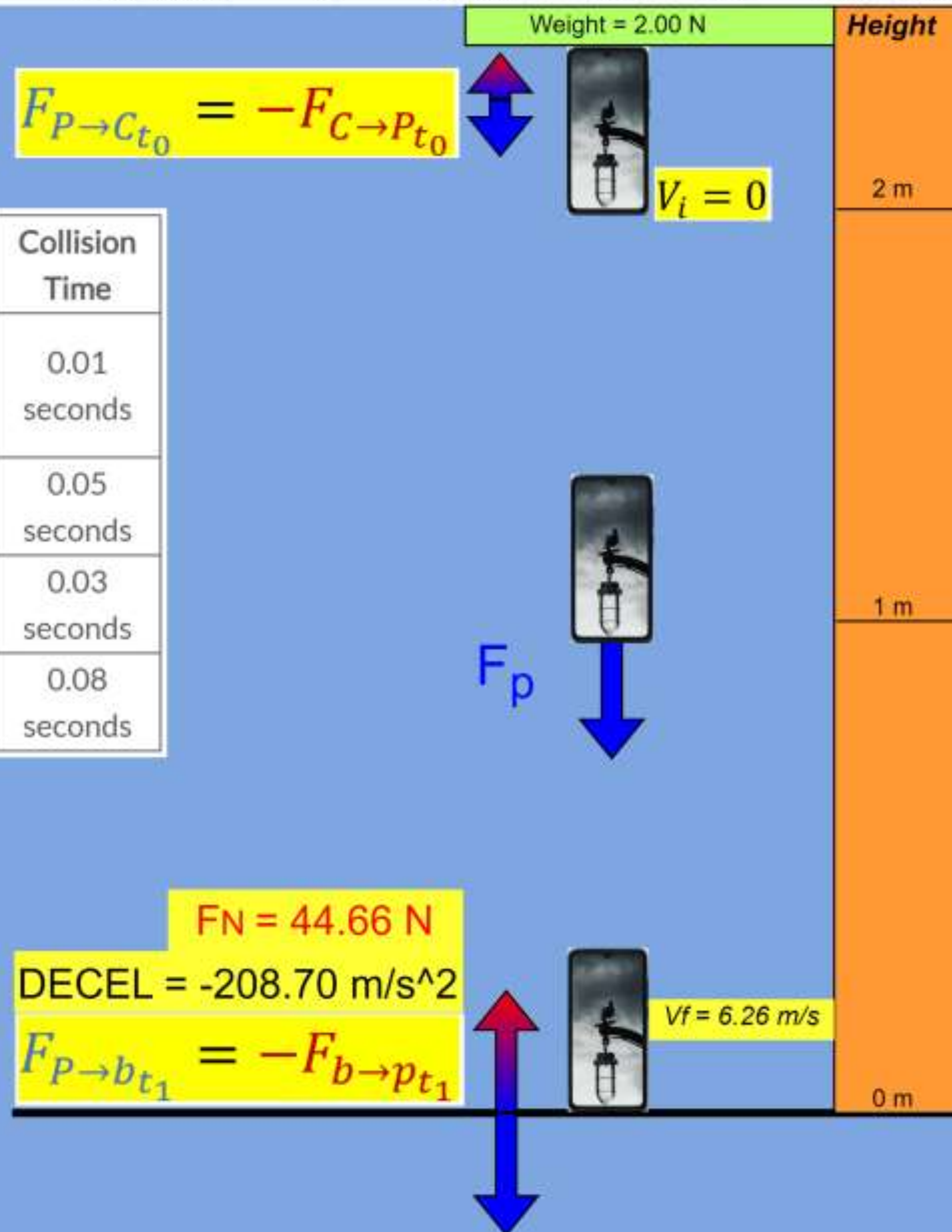


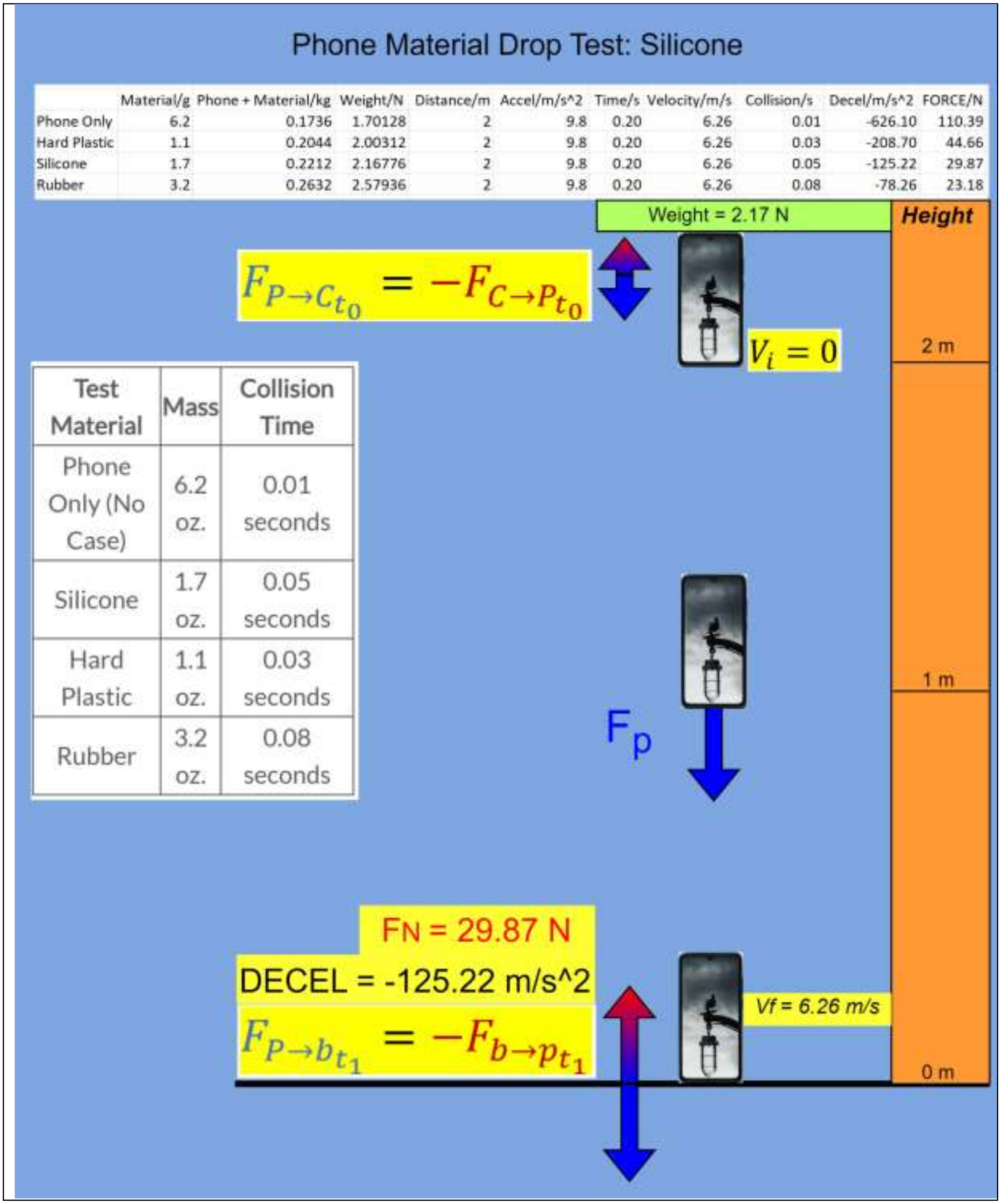
Test Material	Mass	Collision Time
Phone Only (No Case)	6.2 oz.	0.01 seconds
Silicone	1.7 oz.	0.05 seconds
Hard Plastic	1.1 oz.	0.03 seconds
Rubber	3.2 oz.	0.08 seconds

Phone Material Drop Test: Hard Plastic

	Material/g	Phone + Material/kg	Weight/N	Distance/m	Accel/m/s^2	Time/s	Velocity/m/s	Collision/s	Decel/m/s^2	FORCE/N
Phone Only	6.2	0.1736	1.70128	2	9.8	0.20	6.26	0.01	-626.10	110.39
Hard Plastic	1.1	0.2044	2.00312	2	9.8	0.20	6.26	0.03	-208.70	44.66
Silicone	1.7	0.2212	2.16776	2	9.8	0.20	6.26	0.05	-125.22	29.87
Rubber	3.2	0.2632	2.57936	2	9.8	0.20	6.26	0.08	-78.26	23.18

Test Material	Mass	Collision Time
Phone Only (No Case)	6.2 oz.	0.01 seconds
Silicone	1.7 oz.	0.05 seconds
Hard Plastic	1.1 oz.	0.03 seconds
Rubber	3.2 oz.	0.08 seconds





Phone Material Drop Test: Silicone

	Material/g	Phone + Material/kg	Weight/N	Distance/m	Accel/m/s^2	Time/s	Velocity/m/s	Collision/s	Decel/m/s^2	FORCE/N
Phone Only	6.2	0.1736	1.70128	2	9.8	0.20	6.26	0.01	-626.10	110.39
Hard Plastic	1.1	0.2044	2.00312	2	9.8	0.20	6.26	0.03	-208.70	44.66
Silicone	1.7	0.2212	2.16776	2	9.8	0.20	6.26	0.05	-125.22	29.87
Rubber	3.2	0.2632	2.57936	2	9.8	0.20	6.26	0.08	-78.26	23.18

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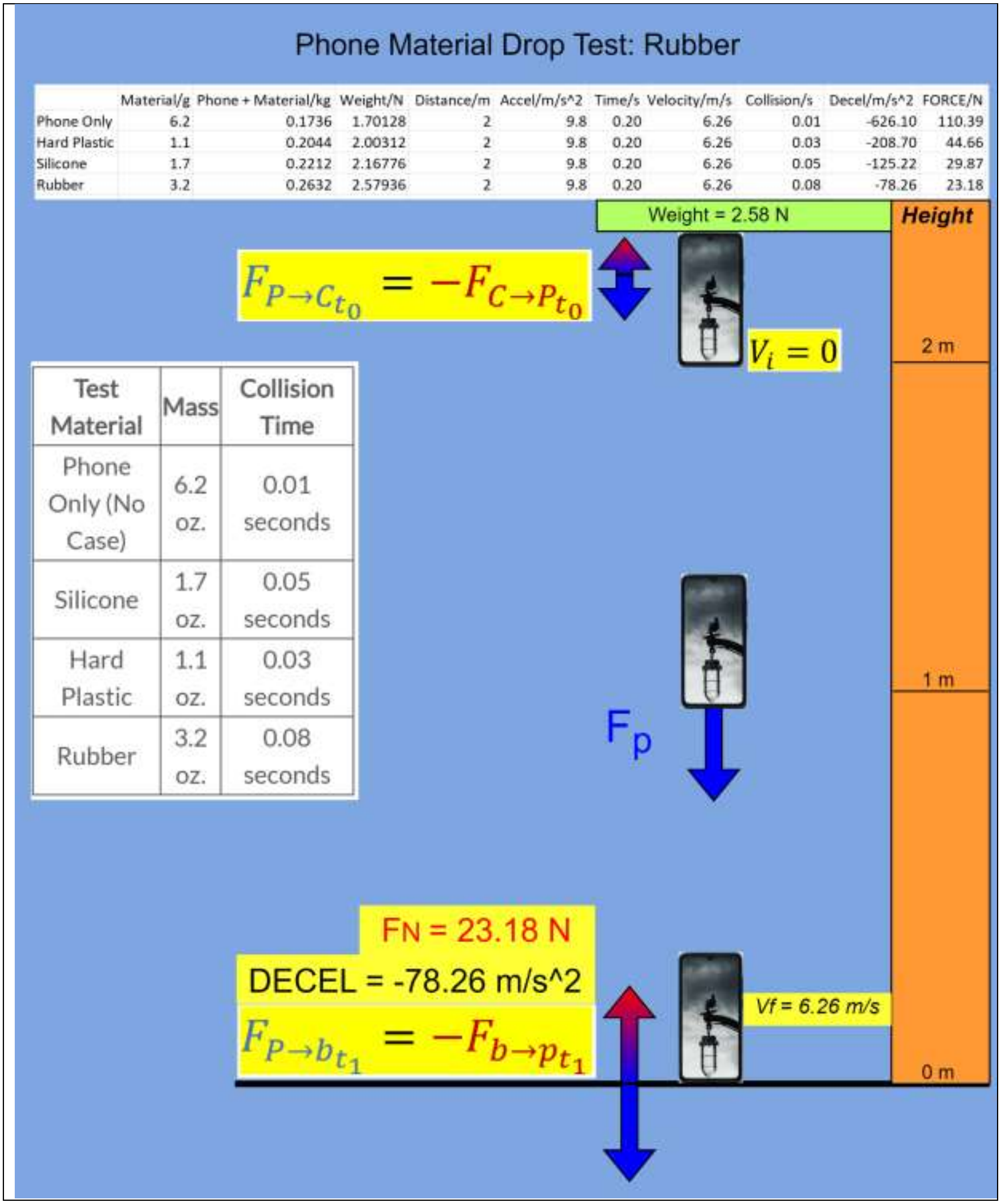

Velocity Calculations

Cell phones have become an intricate part in human activity and communication. The higher powers at A&L Engineering instructed me to evaluate the durability of three materials commonly used for constructing cell phone cases. During testing, I determined how hard the phone will hit the ground, with each material, when dropped from a set height.

I began my observation by determining the height I will be using for each drop. My choices were between 1 and 2 meters. Consumers would be highly interested in the material drop results from about six feet. Thus, I chose 2 meters. Each test required the phone to move from one point to another. After we have changed the phone's position, then we can note that we have displaced the phone over a distance of 2 meters.

The baseline mass of the phone (6.2 oz) was provided to me by a third party. Before I could continue, I needed to convert the mass of the phone into kilograms. We all have heard the term, "what goes up, must come down". This is due to the acceleration of earth's gravity. When an object is dropped, gravity is the force of attraction that accelerates the object towards the ground. The weight for each material was found by $(mass + material) * \frac{28\ g}{1\ oz} * \frac{1\ kg}{1000\ g}$.

The drop was conducted in a free fall environment. The effects of friction (air buoyancy) and drag (force opposite the direction of motion) were ignored. I displaced the phone over a distance of $2m$ with the acceleration of earth's gravity at 9.8 $\frac{m}{s^2}$.

We are able to calculate the time of the fall by using the equation $\left(distance = \frac{1}{2}at^2 \ \rightarrow t = \sqrt{\frac{2 * 2m}{9.8\frac{m}{s}}} = \ .639\ seconds\right)$. The phone is speeding up as it falls towards the ground. We can calculate how fast the phone is traveling before impact by obtaining the product of the acceleration and time $\left(V_f = a * t = 9.8\ \frac{m}{s^2} *\ .639\ s = 6.26\ \frac{m}{s}\right)$. Additionally, we could use the time independent equation ${V_f}^2 = {V_i}^2 + 2ad$. Where initial velocity at the time of the drop is zero. Thus, the equation simplifies to $V_f = \sqrt{2 * 9.8 * 2} \rightarrow V_f = 6.26\ \frac{m}{s}$. (OpenStax, 2022).

Final velocity describes how fast the phone is traveling before it collides with the ground. Additionally, final velocity is defined as a vector quantity which means it has an associated number and direction. I was able to determine the final velocity of the phone without any material to be 6.26 $\frac{m}{s}$. The phone drop was conducted in a simulated free fall environment. Thus, the acceleration will remain the same while the weight of the phone increases. Referencing the table on all the diagrams, I was able to state a final velocity of 6.26 $\frac{m}{s}$ for all the materials.

Relative to Newton's second law and the known equation $F = m * a$, we can use the maximum displacement equation $\boldsymbol{s} = \frac{1}{2}\ \boldsymbol{a}t^2$ to find the deceleration during impact. At the point of impact, the displacement is zero and the forces are in equilibrium or we can assume there are not any oscillations (OpenStax, 2022).

The phone drop is a one-dimensional Kinetics variation, $y_f = y_i + v_i t + \frac{1}{2}at^2$. The Kinetics equation attempts to model motion when the forces are not in equilibrium. At the point of contact, all forces are equal and the $\frac{1}{2}$ drops from the equation. We derive the simple but elegant equation $a = \frac{V_f - V_i}{t_f - t_i}$. Stated, the deceleration is equal to the change in velocity over the change in time (OpenStax, 2022).

Earlier, I calculated the final velocity for the phone by itself when drop from a height of 2 meters. Using the same equation $\left(V_f = a * t\right)$, we can rearrange the equation to find the deceleration during the collision time $\left(a = \frac{\Delta v}{\Delta t} = \frac{-6.26}{.01} = -626\ \frac{m}{s^2}\right)$. Performing the calculation yields a negative acceleration in the vertical direction.

To calculate the deceleration for each material, the same final velocity was divided by the associated collision time. The deceleration of the phone decreased as the weight of the phone and the collision time increased.

1. Phone by itself: $a = \frac{-6.26}{.01} = -626\ \frac{m}{s^2}$
2. Hard Plastic: $a = \frac{-6.26}{.03} = -208.67.\frac{m}{s^2}$
3. Silicone: $a = \frac{-6.26}{.05} = -125.20\ \frac{m}{s^2}$
4. Rubber: $a = \frac{-6.26}{.08} = -78.25\ \frac{m}{s^2}$

Force Calculations

We applied Newton's second law to find the final velocity of the phone and the deceleration at the point of impact. Under Newton's third law, for every action there is an equal and opposite reaction. The phone's force increases proportionately to the increase in acceleration. We determined the deceleration for the phone by itself (-626). Dividing that result by 9.8 will give us the ratio relative to earth's gravity (64:1) (OpenStax, 2022).

At the point of impact, the phone exerts a force on the floor $(F = m * a)$ that is equal and opposite to the force that the floor exerts the phone $(F_{p \to b_{t_1}} = -F_{b \to p_{t_1}})$. Moreover, the sum of the forces in the vertical direction is equal to the mass of the phone times the acceleration in the vertical direction (OpenStax, 2022).

Written mathematically, $\Sigma F = ma_y$. To determine the force of the impact, after plugging in the known values for a phone by itself, yields $F - (.1736 * 9.8) = (.1736 * 626)$. The force of the impact when a phone is dropped from a 2-meter height was 110.39 N. The subsequent observations followed the same format. The calculations are shown below (OpenStax, 2022).

1. Phone by itself: $F - (.1736 * 9.8) = (.1736 * 626) = 110.39\ N$
2. Hard Plastic: $F - (.2044 * 9.8) = (.2044 * 208.67) = 44.66\ N$
3. Silicone: $F - (.2212 * 9.8) = (.2212 * 125.20) = 29.87\ N$
4. Rubber: $F - (.2632 * 9.8) = (.2632 * 78.25) = 23.18\ N$

Modeling Motion

A & L engineering decided to focus on three main areas when describing how the force equations model the phone drop test. Covered in more detail below, I will address the velocity before impact, acceleration, and force.

Newton's first law describes an objects motion and its tendency to resist changes to its velocity. Similarly, an object will stay at rest unless acted upon by a net external force. At the initial state of the test, a phone is suspended 2 meters above the ground. While suspended, the forces acting on the phone are equal. Stated for clarity, the weight of the phone pulling it down is equal and opposite to the reactionary force holding it to the ceiling (also known as a force pair) (OpenStax, 2022).

When the phone was released, the forces were no longer in equilibrium. The force (weight) of the phone causes it to change position. As described earlier, $(F = m * a)$. The phone will change its position at an acceleration rate of 9.8 $\frac{m}{s^2}$. Time of the drop was found by using $distance = \frac{1}{2}at^2$ and solving for t. After the time of the drop was found, the product of earth's gravity and time was used to calculate the final velocity at the point of impact (OpenStax, 2022).

We were able to assume that after the point of impact, the final velocity and distance traveled was zero. Also, we described the phone drop test to be a one-dimensional Kinematics application of the maximum vertical displacement equation $y_f = y_i + v_i t + \frac{1}{2}at^2$ which, after considering the state of equilibrium, reduces to $deceleration = \frac{\Delta v}{\Delta t}$ (OpenStax, 2022). A negative difference in velocity divided by the change in time yielded a vector quantity in the upward direction. We noticed that as the weight of the material increased, the deceleration during the collision time decreased. We were able to keep the phone stationary at the top with Newton's first law. Next, we were able to drop the phone with Newton's second law. Lastly, we can determine the force of the impact with Newton's third law.

Under the third law, we were able to plug in the know values of mass, acceleration, and deceleration to the equation $\Sigma\text{F} = ma_y$. The force of the impact is the sum of the mass times the deceleration plus the mass times the acceleration of gravity (kinetic plus potential) (OpenStax, 2022).

We discussed earlier that force is proportionate to acceleration through the multiplicative rule. Similarly, the data shows that as the collision time increases, the force of the impact and deceleration decreases (OpenStax, 2022).

Material Recommendation

A cell phone, with different materials, was dropped from a height of 2 meters. I observed the final velocity before the point of impact, the deceleration at the point of impact, and the force of the impact. Below are the force numbers for each material tested.

1. Phone by itself: $110.39\ N$
2. Hard Plastic: $44.66\ N$
3. Silicone: $29.87\ N$
4. Rubber: $23.18\ N$

From the above, we can see that the force of the impact on a phone by itself is five times the amount of the heaviest material. Some protection is better than none. Relative to a recommendation, throughout this presentation, I have noted that the increase in weight led to a decrease in the deceleration at the point of impact.

The rubber material, with the lowest deceleration and force, outperformed the others at the point of impact. I am confident in recommending rubber for the case material.

Other physics considerations would encompass friction. The observation was conducted using a standard free fall. We did not account for air buoyance or the force of drag in the opposite direction of the phone's motion. In practice, these forces would slow the phone down or decrease the final velocity before impact (OpenStax, 2022).

When final velocity is decreased, then the deceleration and force of the impact will also decrease. However, if we were to use a linear interpretation to develop a positive correlation, then rubber would still be the material of choice.

Thank you for your time and please feel free to follow up with me about any comments, questions, or concerns.

References

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