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Functional Programming

Algebraic datatypes and collections

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Credit where credit is due

These slides are based on original slides by Michael R. Hansen at DTU. Thank you!

The original slides have been used for a functional programming course at DTU

Last week

We covered a substantial part of F#

- Higher-order functions
- Function composition
- Piping commands
- More lists

Folds (loops)

$$\texttt{fold f acc [}x_1\texttt{; }x_2\texttt{; …; }x_n\texttt{]}$$

returns

$$\texttt{f (… (f (f acc }x_1\texttt{) }x_2\texttt{) … }x_{n-1}\texttt{) }x_n$$

```
let acc = fold f init xs
```

vs.

```
acc = init;
for(i = 0; i < xs.Length; i++)
{
    acc = f (acc, xs[i]);
}
```

Folds (loops)

```
let rec fold f acc =
    function
    | []       -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : (‘b -> ‘a –> ‘b) -> ‘b -> ‘a list -> ‘b
```

Folds (loops)

```
let rec fold f acc =
    function
    | []        -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : ('b -> 'a -> 'b) -> 'b -> 'a list -> 'b
```

```
fold (fun acc x -> acc + x) 0 [1; 2; 3; 4] =
```

```
fold (+) 0 [1; 2; 3; 4] =
```

(+) ((+) ((+) (((+) 0 1) 2)) 3) 4 = 0 + 1 + 2 + 3 + 4 = 10

Folds (loops)

```
let rec fold f acc =
    function
    | []         -> acc
    | x :: xs -> fold f (f acc x) xs
```

```
fold : (‘b -> ‘a —> ‘b) -> ‘b -> ‘a list -> ‘b
```

```
fold (fun acc x -> acc * x] 1 [1; 2; 3; 4] =
```

```
fold (*) 1 [1; 2; 3; 4] =
```

(*) ((*) ((*) (((*) 1 1) 2)) 3) 4 = 1 * 1 * 2 * 3 * 4 = 24

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
let ??? lst =
    lst |>
    List.tail |>
    fold (fun (y, acc) z -> (z, y <= z && acc))
         (List.head lst, true) |>
    snd
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
```

```
let ??? ls
    lst |>
    List.t
    fold (
        (List.head lst, true) |>
    snd
```

List.pairwise $[x_1; \ldots; x_n]$ =
$[(x_1, x_2); (x_2, x_3); \ldots; (x_{n-1}, x_n)]$

```
let ??? lst =
    lst
    List.pairwise |>
    List.forall (fun (x, y) -> x <= y)
```

Folds (loops)

```
let ??? lst =
    snd (fold (fun (y, acc) z -> (z, y <= z && acc))
              (List.head lst, true)
              (List.tail lst))
let ??? lst =
    lst |>
    List.tail |>
    fold (fun (y, acc) z -> (z, y <= z && acc))
         (List.head lst, true) |>
    snd
let ??? lst =
    lst
    List.pairwise |>
    List.forall (fun (x, y) -> x <= y)
```

This week

- Inductively defined datatypes
- Expression trees
- Collections (sets and maps)
- Data representation
- Live coding

Questions?

Inductively defined types

- … or algebraic datatypes
- … or disjoint unions

Allow us to concisely say how members of types are created

Inductively defined types

```
type month =
| January | February | March | April
| May | June | July | August
| September | October | November
| December
```

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Inductively defined types

```
type month =
| January | February | March | April
| May
| Sep
| Dec
```

Observation 1

We use 'type', not 'let'

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Inductively defined types

```
type month =
| January | February | March | April
| May
| Sep
| Dec
```

Observation 2

Behave similarly to enum-types in Java

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday
```

Programming

```
type month =
| January | February | March | April
| May | June | July | August
| September | October | November
| December

 let numberOfDays =
 function
 | January | March | May | July
 | August | October | December -> 31
 | February -> 28
 | _ -> 30
```

Programming

```
type month =
|                                                     ..l
|
|
|
| December
```

We can pattern match on algebraic types

```
let numberOfDays =
function
| January | March | May | July
| August | October | December -> 31
| February -> 28
| _ -> 30
```

Programming

```
type weekDay =
| Monday | Tuesday | Wednesday
| Thursday | Friday |
| Saturday | Sunday

let nextWeekday =
  function
  | Monday -> Tuesday
  | Tuesday -> Wednesday
  | Wednesday -> Thursday
  | Thursday -> Friday
  | _ -> Monday
```

Arguments

Algebraic types can take arguments

```
type shape =
| Circ of float            (* radius *)
| Rect of float * float (* sides *)

let area =
  function
  | Circ r -> System.Math.PI * r * r
  | Rect (w, h) -> w * h
```

The option type

Options are used to encode
partial functions

```
type 'a option =
| None         (* No result *)
| Some of 'a (* result *)
```

You can think of options as terms that can be set to null (but nice, and type-safe, and does not cause as many bugs)

The option type

Some useful functions

```
Option.get : ‘a option -> ‘a

Option.map : (‘a -> ‘b) -> ‘a option -> ‘b option

Option.defaultValue :
        ‘a -> ‘a option -> ‘a
```

The option type

Some useful functions

```
Option.get : ‘a option -> ‘a

Option.m                                             ’b option

Option.c
        ‘a -> ‘a option -> ‘a
```

Option.get None throws an exception, similarly to taking the head of an empty list

Recursive

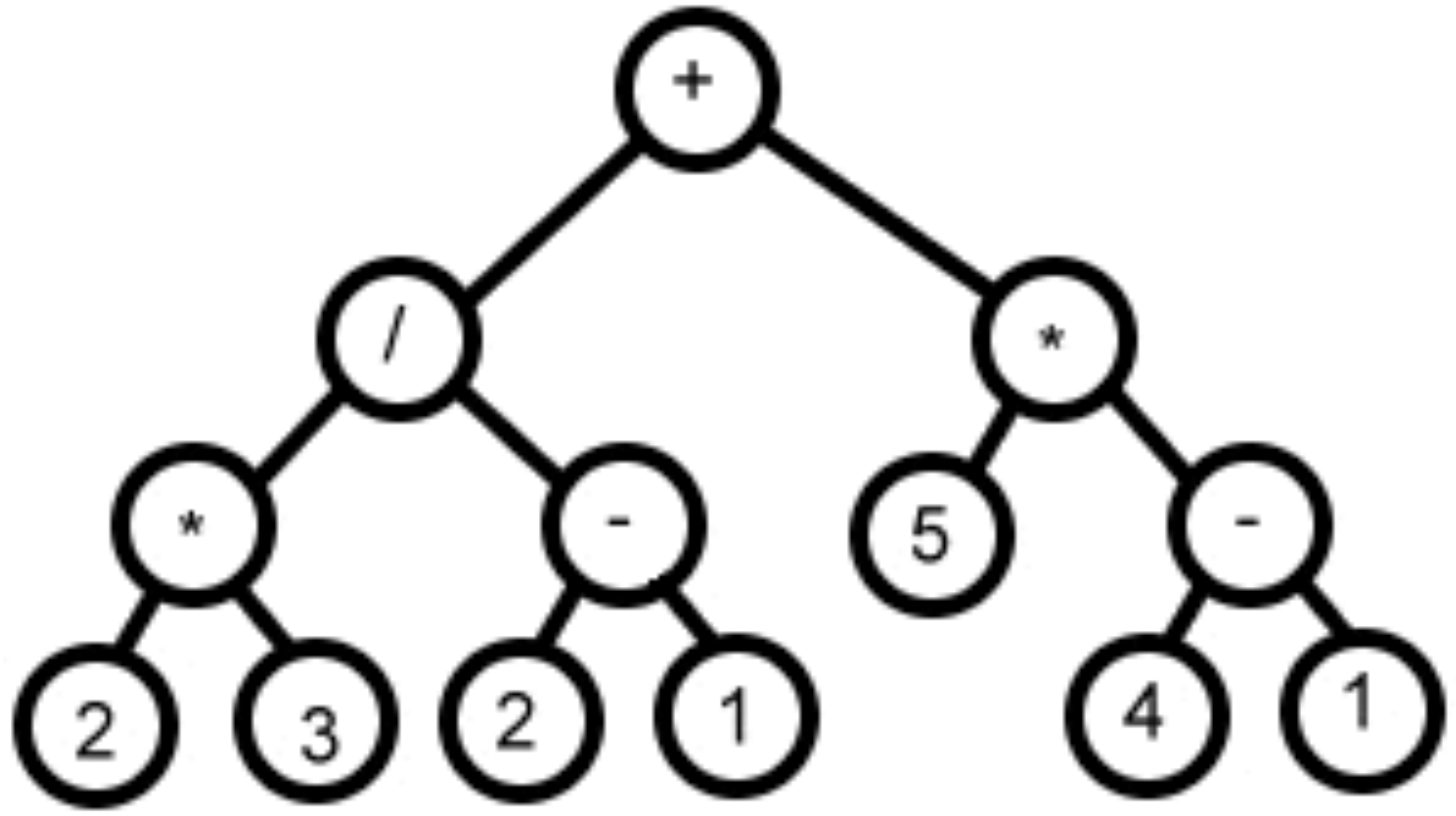
Algebraic types can be recursive and polymorphic

```
type ‘a myList =
| Nil                          (* empty list *)
| App of ‘a * ‘a myList (* cons *)

let rec length =
  function
  | Nil -> 0
  | App (_, lst) -> 1 + length lst
```

Expression trees

- Expression trees are heavily used by compilers
- Nodes contain operators
- Leaves contain values



Expression tree for 2*3/(2-1)+5*(4-1)

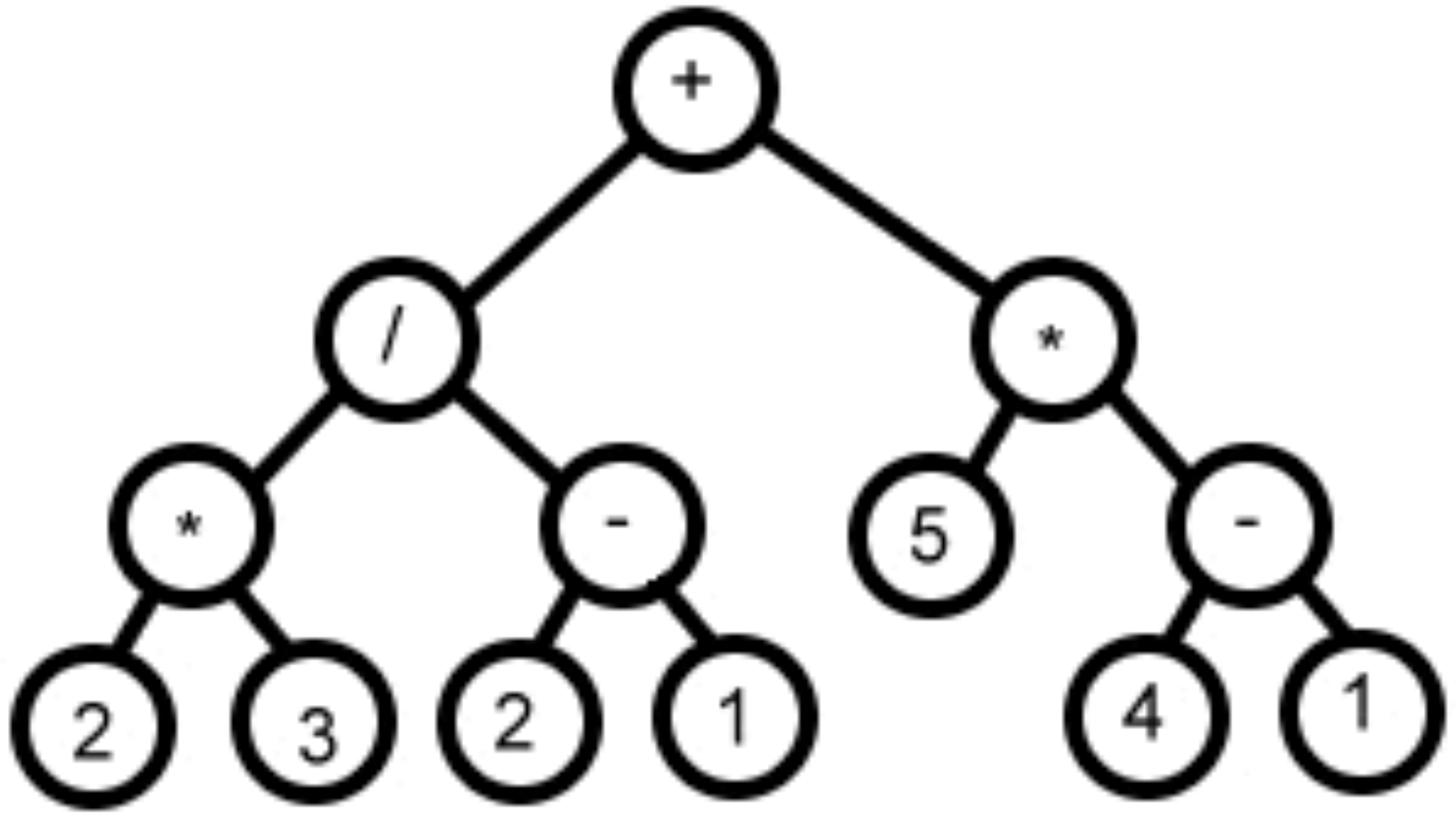
Expression trees

Expression trees are really easy to code

```
type term =
| Const of int
| Add of term * term
| Sub of term * term
| Mul of term * term
| Div of term * term
```

Expression trees

```
type term =
  | Const of int
  | Add of term * term
  | Sub of term * term
  | Mul of term * term
  | Div of term * term
```



Expression tree for 2*3/(2-1)+5*(4-1)

```
    Add
      (Div (Mul (Const 2, Const 3)
                ,Sub (Const 2, Const 1))
      ,Mul (Const 5
                ,Sub (Const 4, Const 1))
```

Exression trees

... and really easy to recurse over

```
let rec show : term -> string =
    function
    | Const f -> sprintf "%n" f
    | Add (t1, t2) ->
       "(" @ show t1 @ " + " @ show t2 @ ")"
    | Sub (t1, t2) ->
       "(" @ show t1 @ " - " @ show t2 @ ")"
    | Mul (t1, t2) ->
       "(" @ show t1 @ " * " @ show t2 @ ")"
    | Div (t1, t2) ->
       "(" @ show t1 @ " / " @ show t2 @ ")"
```

Expression Trees

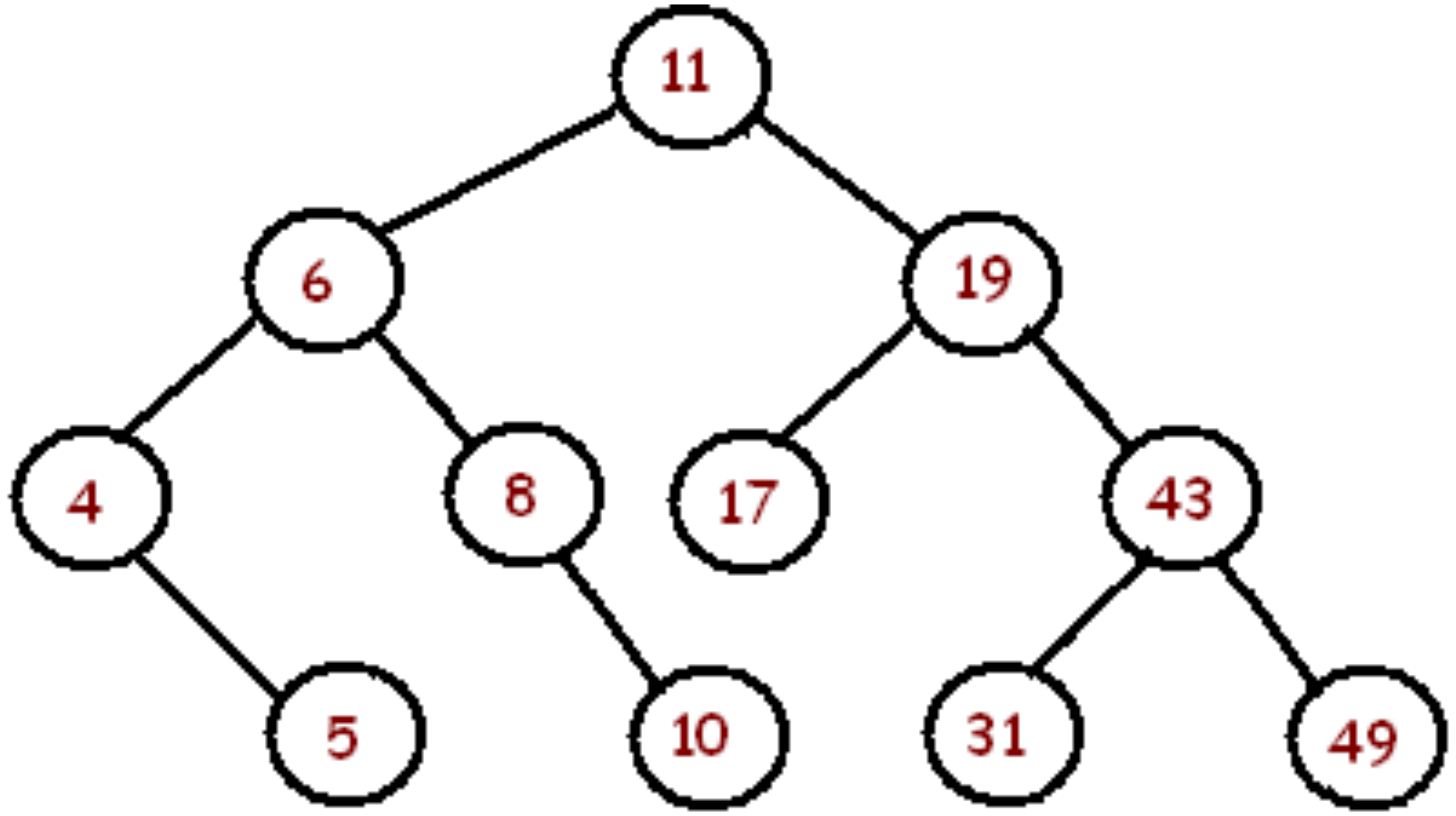
Recall the expression tree from before:

```
let t : term =
    Add
      (Div (Mul (Const 2, Const 3)
                ,Sub (Const 2, Const 1))
      ,Mul (Const 5
                ,Sub (Const 4, Const 1))
```

```
> show t;;
val it : string =
   "(((2 * 3) / (2 - 1)) + (5 * (4 - 1)))";;
```

Binary search trees

- All elements in the left subtree are smaller than or equal to the root
- All elements in the right subtree are greater than the root



Let’s code

Collections

F# has support for all collections from the .NET framework

- Lists
- Sets
- Maps
- Hash tables
- …

Sets

- Represents mathematical sets
- Intersection, union, …
- Not mutable
- Unordered
- Can only store values of comparison type
- Created inline by set [a1; a2; …; an] where all duplicates are removed

Sets (some functions)

```
Set.empty : Set<'a>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union : Set<'a> -> Set<'a> ->
            Set<'a>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union : Set<'a> -> Set<'a> ->
                Set<'a>
Set.intersect :
      Set<'a> -> Set<'a> -> Set<'a>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union : Set<'a> -> Set<'a> ->
                Set<'a>
Set.intersect :
        Set<'a> -> Set<'a> -> Set<'a>
Set.map : ('a -> 'b) -> Set<'a> ->
              Set<'b>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union : Set<'a> -> Set<'a> ->
                Set<'a>
Set.intersect :
     Set<'a> -> Set<'a> -> Set<'a>
Set.map : ('a -> 'b) -> Set<'a> ->
          Set<'b>
Set.filter : ('a -> bool) ->
                Set<'a> -> Set<'a>
```

Sets (some functions)

```
Set.empty : Set<'a>
Set.singleton : 'a -> Set<'a>
Set.union : Set<'a> -> Set<'a> ->
                Set<'a>
Set.intersect :
     Set<'a> -> Set<'a> -> Set<'a>
Set.map : ('a -> 'b) -> Set<'a> ->
            Set<'b>
Set.filter : ('a -> bool) ->
                Set<'a> -> Set<'a>
Set.fold : ('b -> 'a -> 'b) ->
               'b -> Set<'a> -> 'b
```

Maps

- Represents mathematical maps
- add, lookup, …
- Not mutable
- Unordered
- Can only have keys of comparison type

Maps (some functions)

```
Map.empty : Map<‘a, ‘b>
```

Maps (some functions)

```
Map.empty : Map<'a, 'b>
Map.add : 'a -> 'b -> Map<'a, 'b> -> Map<'a, 'b>
```

Maps (some functions)

```
Map.empty : Map<'a, 'b>
Map.add : 'a -> 'b -> Map<'a, 'b> -> Map<'a, 'b>
Map.find : 'a -> Map<'a, 'b> -> 'b
```

Maps (some functions)

```
Map.empty : Map<'a, 'b>
Map.add : 'a -> 'b -> Map<'a, 'b> -> Map<'a, 'b>
Map.find : 'a -> Map<'a, 'b> -> 'b
Map.tryFind : 'a -> Map<'a, 'b> -> 'b option
```

Maps (some functions)

```
Map.empty : Map<‘a, ‘b>
Map.add : ‘a -> ‘b -> Map<‘a, ‘b> -> Map<‘a, ‘b>
Map.find : ‘a -> Map<‘a, ‘b> -> ‘b
Map.tryFind : ‘a -> Map<‘a, ‘b> -> ‘b option

Map.map : (‘a -> ‘b -> ‘c) ->
          Map<‘a, ‘b> -> Map<‘a, ‘c>
```

Maps (some functions)

```
Map.empty : Map<'a, 'b>
Map.add : 'a -> 'b -> Map<'a, 'b> -> Map<'a, 'b>
Map.find : 'a -> Map<'a, 'b> -> 'b
Map.tryFind : 'a -> Map<'a, 'b> -> 'b option

Map.map : ('a -> 'b -> 'c) ->
          Map<'a, 'b> -> Map<'a, 'c>
Map.filter : ('a -> 'b -> bool) ->
       Map<'a, 'b> -> Map<'a, 'b>
```

Maps (some functions)

```
Map.empty : Map<‘a, ‘b>
Map.add : ‘a -> ‘b -> Map<‘a, ‘b> -> Map<‘a, ‘b>
Map.find : ‘a -> Map<‘a, ‘b> -> ‘b
Map.tryFind : ‘a -> Map<‘a, ‘b> -> ‘b option

Map.map : (‘a -> ‘b -> ‘c) ->
            Map<‘a, ‘b> -> Map<‘a, ‘c>
Map.filter : (‘a -> ‘b -> bool) ->
         Map<‘a, ‘b> -> Map<‘a, ‘b>
Map.fold : (‘c -> ‘a -> ‘b -> ‘c) ->
         ‘c -> Map<‘a, ‘b > -> ‘c
```

Higher-order functions

For lists you can potentially get away without using higher-order functions by using recursion

For sets and maps you still can (translate them to lists, work on the lists and then translate them back) but this is a **really** bad idea.

Practice using higher-order functions :)

Program interpreters

Functional languages are great for working directly on abstract syntax trees

An imperative language

```
type aExp = …
type bExp = …
```

Assuming we have types for arithmetic and boolean expressions

The abstract syntax tree of our language is defined like this

```
type stm =
  | Skip
  | Ass of string * aExp
  | Seq of stm * stm
  | ITE of bExp * stm * stm
  | While of bExp * stm
```

State

In order to keep track of program state we need a mapping from program variables to values

State

In order to keep track of program state we need a mapping from program variables to values

```
type state = Map<string, int>
```

Arithmetic expressions

```
type aExp =
  | N of int
  | V of string
  | Add of (aExp * aExp)
  | Mul of (aExp * aExp)
  | Sub of (aExp * aExp)
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
type aExp =
| N of int
| V of string
| Add of (aExp * aExp)
| Mul of (aExp * aExp)
| Sub of (aExp * aExp)
```

```
let rec evalA arith st =
  match arith with
  | N n -> n
  | V v -> Map.find v st
  | Add(a, b) ->
    let va = evalA a st
    let vb = evalA b st
    va + vb
…
```

State (some functions)

```
type state = Map<string, int>


let binop : ('a -> 'b -> 'c)
            -> (state -> 'a)
            -> (state -> 'b)
            -> state -> 'c

  = fun f x y s -> f (x s) (y s)
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
let binop f x y s = f (x s) (y s)
let rec evalA =
  function
   | N n -> fun _ -> n
   | V v -> fun s -> Map.find v s
   | Add(a, b) -> binop (+) (evalA a) (evalA b)
   | Sub(a, b) -> binop (-) (evalA a) (evalA b)
   | Mul(a, b) -> binop (*) (evalA a) (evalA b)
```

Arithmetic expressions

Arithmetic expressions can be evaluated in the context of a state

```
evalA : aExp -> state -> int
```

```
let binop f x y s = f (x s) (y s)
let rec evalA =
 function
  | N n -> fun _ -> n
  | V v -> Map.find v
  | Add(a, b) -> binop (+) (evalA a) (evalA b)
  | Sub(a, b) -> binop (-) (evalA a) (evalA b)
  | Mul(a, b) -> binop (*) (evalA a) (evalA b)
```

Boolean expressions

```
type bExp =
| TT
| FF
| Eq of (aExp * aExp)
| Lt of (aExp * aExp)
| Neg of bExp
| Con of (bExp * bExp)
```

Boolean expressions

Boolean expressions can be evaluated in the context of a state

```
evalB : bExp -> state -> bool
```

```
let rec evalB =
   <Assignment for this week>
```

Evaluating a program

```
type stm =
  | Skip
  | Ass of string * aExp
  | Seq of stm * stm
  | ITE of bExp * stm * stm
  | While of bExp * stm
```

Programs evaluation is done by updating the state

```
evalS : stm -> state -> state
```

```
let rec evalS =
   <Assignment for this week>
```

Questions?