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Role of Collective Effects in Systematics of Level Density of Nuclei

Rol' kollektivnykh effektov pri sistematike plotnosti urovnei yader.

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Systematics of experimental data on the neutron resonance density, taking into account the influence of pair correlations and collective effects on the level densities of excited nuclei, is considered. Consistency of the obtained results with available theoretical and experimental information on the level density parameter is shown.

At present the description of a wide range of phenomena connected with the formation and decay of a compound nucleus is carried out on the basis of the statistical theory of nuclear reactions. Within the framework of this approach the main characteristic of the nucleus is the density of excited states for a given set of constants of motion which determine the process under consideration. In analyzing and systematizing the corresponding experimental data for level density calculations the Fermi-gas model equations are used in many articles:

$$\rho(U,J)=\frac{2J+1}{24\sqrt{2}\,\sigma^3 a^{1/4}(U-\delta)^{5/4}}\exp\left\{2\sqrt{a(U-\delta)}-\frac{(J+1/2)^2}{2\sigma^2}\right\}$$
$$\sigma^2=\frac{6}{\pi^2}\overline{m^2}\sqrt{a(U-\delta)}. \qquad (1)$$

These expressions are rather simple and contain only three parameters: the level density parameter $a = \pi^2 g/6$, proportional to the single-particle state density g at the Fermi surface, the mean-square value of the angular momentum projection of these states $\overline{m^2}$ and the correction δ for the even-odd differences in level density. The value $\overline{m^2}$ is

selected, as a rule, on the basis of the quasi-classical estimate $m^2 \approx 0{,}22 \div 0{,}25\ A^{2/3}$; the correction δ is identified with the analogous mass formula correction and the parameter a is determined by fitting (1) to the observed neutron resonance density [1].

The expressions of the traditional Fermi-gas model do not take into consideration a whole number of extremely important properties of nuclei, which are determined, on the one hand, by the existence of considerable shell inhomogeneities in the single-particle level scheme and, on the other hand, by rather strong influence of correlation effects of the superconducting type and of coherent effects of collective nature on the statistical characteristics of nuclei. The level density calculation methods developed in the last few years, which are based on the microscopic Hamiltonian of the superfluidity model of the nucleus make it possible to obtain a consistent and coherent description of all these effects [2, 3]. Up to the present a lot of examples of experimental manifestation of these effects in the cross-sections of various nuclear reactions has accumulated [3]. Unfortunately, sufficiently rigorous methods for calculating the level density turn out to be extremely time-consuming, and this strongly limits the possibilities of their practical application to experimental data analysis. Therefore, it appears to be of essential importance to search for a

level density description which should take into account the principal concepts of the theory of structure of highly excited nuclei to the necessary extent, but at the same time should be sufficiently simple and convenient for practical application. The present article discusses the main expressions necessary for constructing such a phenomenological description and considers the most characteristic features of the experimental data systematics obtained within the framework of the proposed model.

1. Main Expressions of the Model

When pair correlations and collective effects are to be taken into account the equation

$$\rho(U, J) = \rho_{BCS}(U, J) K_{vib}(U) K_{rot}(U) \qquad (2)$$

for the total level density has to be used instead of (1). A rigorous determination of the level density enhancement coefficients K_{vib} and K_{rot}, resulting from the collective modes in highly excited nuclei, was considered within the framework of a microscopic approach in the articles [2, 3]. In constructing a phenomenological description in the region of excitation energies near the neutron binding energy it is possible to use an adiabatic estimate of these coefficients [4]:

$$K_{rot} = \begin{cases} \sigma_\perp^2 & \text{for deformed nuclei} \\ 1 & \text{for spherical nuclei} \end{cases} \qquad (3)$$

$$K_{vib} \simeq \exp\left\{1.7\left(\frac{3m_0 A}{4\pi\sigma_{kin}^2}\frac{C_{kin}}{C}\right)^{2/3} U^{2/3}\right\}. \qquad (4)$$

where $\mathcal{J}_\perp$ is the perpendicular moment of inertia and t the temperature of the excited nucleus, σ_{ldm} the surface tension coefficient in the liquid-drop model, and the ratio C/C_{ldm} characterizes the difference of the stiffness coefficients of the excited nucleus from the corresponding coefficients of the liquid-drop model [5]. The density of the non-collective excited nuclear states can be written in a form equivalent to (1):

$$\rho_{BCS}(U,J)=\frac{2J+1}{2\sqrt{2\pi}\,\sigma_{eff}^3\,Det}\exp\left\{S-\frac{(J+1/2)^2}{2\sigma_{eff}^2}\right\}. \qquad (5)$$

$$\sigma_{eff}^2=\begin{cases}\mathcal{J}_\perp\mathcal{J}_\parallel/\mathcal{J} & \text{for deformed nuclei}\\ \mathcal{J}\cdot t & \text{for spherical nuclei.}\end{cases}$$

The interconnection between the temperature t and the entropy S and the excitation energy U as well as the excitation energy dependence of the preexponential factor Det, of the parallel and the perpendicular moments of inertia $\mathcal{J}_\parallel$ and $\mathcal{J}_\perp$ are determined by the corresponding equations of state of the superfluidity model of the nucleus [3].

One of the principal parameters of this model is the correlation function Δ_o; the critical temperature t_{cr} of the phase transition from the superconducting to the normal state is directly connected with this quantity:

$$t_{cr}=0{,}567\,\Delta_o. \qquad (6)$$

Above the critical temperature the equations of state of the superfluidity model of the nucleus differ from the Fermi-gas state equations only by an excitation energy shift:

$$U = at^2 + E_{cond},$$
$$S = 2at = 2\sqrt{a(U - E_{cond})}, \quad \mathrm{Del} = \frac{144}{\pi} a^{1/2},$$
$$\mathcal{J}_{\parallel} = \frac{6}{\pi^2} \overline{am^2}(1 - {}^2/_3\varepsilon), \quad \mathcal{J}_{\perp} = {}^2/_5 m_0 r_0^2 A^{5/3}(1 + {}^1/_3\varepsilon) \quad /\hbar^2 \text{*)} \qquad (7)$$

In equations (7) for the moments of inertia the dependence of the quadrupole deformation of the nucleus on the parameter ε is displayed explicitly.

In order to include shell effects in the discussion one must use in equations (7) a value of the parameter a which depends in a specific way on the excitation energy [3]. For this purpose the following equations may be used:

$$a(U, Z, A) = \tilde{a}(A)\{1 + f(U)\delta W(Z, A) / (U - E_{cond})\},$$
$$\tilde{a}(A) = \alpha A + \beta A^{2/3}, \quad f(U) = 1 - \exp\{-\gamma(U - E_{cond})\}, \qquad (8)$$

which had been successfully applied before to the systematics of the level density parameter within the framework of the Fermi-gas model [6]. In equations (8) $\tilde{a}$ is the asymptotic value of the level density parameter at high excitation energies, δW is the shell correction to the nuclear binding energies (nuclear masses) and f(U) is a dimensionless "universal"

*) A factor $1/\hbar^2$ has been introduced explicitly into the last equation of (7) in order to get the correct dimension for all moments of inertia (translators' comment).

function which determines the energy dependence of the level density parameter.

Below the phase transition point, one can describe the thermodynamic functions of interest by means of a comparatively simple parametrization of the energy dependence of the corresponding functions, as considered in [7],

$$U=U_{cr}(1-\varphi^2),\quad S=S_{cr}\frac{t_{cr}}{t}(1-\varphi^2),$$
$$\mathrm{Det}=\mathrm{Det}_{cr}(1-\varphi^2)(1+\varphi^2)^2,$$
$$\mathcal{F}_{\parallel}=\mathcal{F}_{\parallel}^{cr}\frac{t_{cr}}{t}(1-\varphi^2),\quad \mathcal{F}_{\perp}=\mathcal{F}_{\perp}^{0}+(\mathcal{F}_{\perp}^{cr}-\mathcal{F}_{\perp}^{0})\frac{t_{cr}}{t}(1-\varphi^2). \tag{9}$$

In the equations (9) the values of the corresponding thermodynamic functions at the critical point are marked by the index "cr", i.e.

$$U_{cr}=c_{cr}t_{cr}^2+E_{cond},\quad S_{cr}=2a_{cr}t_{cr},$$
$$\mathrm{Det}_{cr}=\frac{144}{\pi}a_{cr}^3t_{cr}^5,\quad \mathcal{F}_{\parallel}^{cr}=\frac{6}{\pi^2}a_{cr}\overline{m^2}(1-{}^2/_3\epsilon), \tag{10}$$

$\mathcal{F}_{\perp}$ is the perpendicular moment of inertia in the ground state, which can be taken directly from experimental data or by using the rough empirical estimate $\mathcal{F}_{\perp}^{0}\approx\mathcal{F}_{\perp}^{cr}/3$ [3]. The relationship between the function $\varphi=(1-U/U_{cr})^{1/2}$ and the temperature is defined by the equation [7]:

$$\varphi=\mathrm{th}\left(\frac{t_{cr}}{t}\varphi\right). \tag{11}$$

This equation can be solved for the temperature corresponding to a given excitation energy or, on the other hand, one can

find U for a given temperature $t < t_{cr}$ and then determine the other thermodynamic functions by means of equations (9). The condensation energy, which characterizes the lowering of the ground state energy of the Fermi-gas resulting from the correlation interaction, is described by the expression:

$$E_{cond}=g_{cr}\Delta_0^2/4=3a_{cr}\Delta_0^2/2\pi^2. \qquad (12)$$

The influence of shell effects on the behavior of the thermodynamic functions in the superconducting phase (9) is given by the value of the level density parameter at the critical point a_{cr} which is to be obtained as the solution of the equation

$$a_{cr}=\tilde{a}\{1+\delta W[1-\exp(-\gamma a_{cr}t_{cr}^2)]/a_{cr}t_{cr}^2\}. \qquad (13)$$

As is shown in [8], the even-odd differences of the thermodynamic functions in the model of the superfluid nucleus are determined by the ground state energy shift. They can be reproduced, if in equations (7) - (9) the value

$$U-\begin{cases}0 & \text{for even-even nuclei}\\ \Delta_0 & \text{for odd nuclei}\\ 2\Delta_0 & \text{for odd-odd nuclei}\end{cases} \qquad (14)$$

is used as formal excitation energy.

The equations (2) - (14) necessary for calculating the level density with account of collective and correlation effects

turn out to be more complex, of course, than the simple equations (1) of the Fermi-gas model. But this complication of the model is inevitable, if we want to obtain a uniform and consistent description of the level density and other statistical characteristics of nuclei in a wide excitation energy range. The number of parameters, which characterize the excited nucleus, in this model remains practically the same as in the phenomenological level density description of the Fermi-gas model; therefore the equations (2) - (14) can be successfully used for the analysis and systematics of experimental data [9].

2. Systematics of Neutron Resonance Parameters

The analysis of averaged neutron resonance parameters offers the most direct information on the level density of excited nuclei, which will also be the basis for selecting the parameters of the phenomenological description.

The procedure of experimental data analysis consisted of the following steps.

1. We have confined ourselves to the discussion of the region of heavy nuclei with $A \geq 150$, for which the neutron resonance density data $\rho(B_n, J)$ are well systematized and the results of the various experimental works show a comparatively small scatter [1, 10]. For the following analysis we have used the data compilation presented in [10].

2. On the basis of equations (2) - (14) the values of the asymptotic level density parameter $\tilde{a}$ have been derived from the experimental data on $\rho(B_n,J)$. For this purpose, experimental values of the shell correction to the mass formula

$$\delta W = M_{exp}(Z,A) - M_{ldm}(Z,A,\epsilon) \qquad (15)$$

were used for describing the energy dependence of the level density parameter (8); M_{exp} is the experimental value of the mass defect and M_{ldm} its liquid-drop component, calculated for the equilibrium deformation ϵ [11]. The correlation function was also taken from the systematics of even-odd mass differences of nuclei as $\Delta_o = 12A^{-1/2}$. In the calculations of the K_{vib} coefficients (4) the liquid drop value of the stiffness coefficient $C = C_{ldm}$ was used, and in the calculations of the moments of inertia of the excited nuclei the value $\overline{m^2} = 0.24\ A^{2/3}$ and the deformation parameters $\varepsilon = 0,3$ for nuclei with $150 \leq A \leq 185$, $\varepsilon = 0,2$ for nuclei with $185 < A < 204$ and $\varepsilon = 0,24$ for $A \geq 230$ were taken. In the region of spherical nuclei with $204 \leq A \leq 210$ only the coefficient of vibrational level density enhancement was taken into account in the calculation of $\rho(B_n,J)$.

3. By minimization of the experimental point scatter in the mass number dependence of $\tilde{a}$ (8) by the least-squares method the value of the parameter γ was determined.

The values of $\tilde{a}/A$ obtained in this analysis are shown in Fig. 1.

For comparison the values $\tilde{a}/A$ derived from the experimental data with the equations of the Fermi-gas model (1) [6] and of the superfluidity model of the nucleus (5) without account of collective effects are presented in the same figure. The latter type of treatment produces strong point scatter, which can be reduced to a considerable degree by introducing the coefficients of collective level density enhancement. Thereby the values obtained for $\tilde{a}/A$ are grouped near a straight line parallel to the abscissa for the nuclei in the region $150 \leq A \leq 180$ and $A \geq 230$ having pronounced rotational properties and for the spherical nuclei with $A \approx 204 \div 210$, which are near the doubly magic lead isotope. In the region of transition nuclei, $185 \leq A \leq 204$, the points fall below the expected asymptotic curve. Although these nuclei belong to the spherical type according to the data on the equilibrium deformation of ground states, in highly excited states their properties are apparently nearer to those of deformed nuclei, i.e., it can be expected that with the increase in excitation energy the difference between the static and dynamic deformations of the nucleus vanishes, and this is reflected in a less pronounced difference in the properties of nuclei in the deformed and transition region. However, for transition nuclei the adiabatic estimate of rotational effects as well as the liquid drop estimate of the stiffness coefficients can turn out to be excessively rough. Therefore, this region of nuclei as well as the region of lighter nuclei with $A < 150$, not analyzed

in this article, will have to be investigated more thoroughly, and the results of the proposed phenomenological model must be applied to these nuclei with a certain precaution.

The values $\tilde{a}_j$ derived from the analysis of experimental data can be used to determine the coefficients α and β, which characterize the contribution of the volume and surface region of the nucleus to the level density. If only the set of data presented in Fig. 1 is considered, it is not possible to separate the contribution of the volume and surface components in a statistically significant way by describing them directly by means of equation (8). We verified that approximately the same description of points in the lower part of Fig. 1 can be obtained with the following parameter values (in units of MeV^{-1}):

$$\left.\begin{array}{ll} \alpha=0{,}0931, & \beta=0 \quad (16a) \\ \alpha=\beta=0{,}0792 & \quad (16b) \\ \alpha=0{,}0730, & \beta=0{,}1147 \quad (16c) \end{array}\right\} \gamma=0{,}064.$$

The first of these sets corresponds to the simplest one-parameter description of the asymptotic level density parameter, the second to the simple assumption of an identical value of the coefficients of the volume and surface components of $\tilde{a}$, and the third set to a theoretical evaluation of the parameter α for a square well potential [12]. The small differences in the values of the root-mean-square deviation of the considered points from the fitted curves do not permit to give preference to any of these sets (or to many equivalent

sets with other interrelated choices of α and β) on the basis of the criteria of the least-squares method. Therefore, for the sake of simplicity, in the following systematic investigation we have used the one-parameter description of the asymptotic level density parameter (16a).

The values of the level density parameter obtained in this analysis agree very well with the results of theoretical calculations of the parameter $\tilde{a}$ carried out for the Woods-Saxon potential [3]. Fig. 2 shows a comparison of the results of the theoretical calculations of $\tilde{a}$ and $\tilde{a}$ (B_n) with the values obtained within the framework of the phenomenological description. The good agreement of the values $\tilde{a}(B_n)$ indicates that the function f(U) in (8), used by us for approximating the energy dependence of the level density parameter, reproduces sufficiently correctly the main features of the theoretical calculations of the behaviour of $\tilde{a}(U)$. In Fig. 2 there are presented also values of the parameter $\tilde{a}$, calculated for the Nilsson oscillator potential [3]. The discrepancy between the latter values and the results obtained for the Woods-Saxon potential shows the dependence of the results of theoretical calculations of the level density of excited nuclei on the choice of the single-particle potential; this dependence is to be taken into account when one compares purely theoretical level density calculations with experimental data. This ambiguity of the theoretical calculations is an additional

argument for the necessity of constructing phenomenological systematics for the level densities of excited nuclei.

Fig. 3 shows the systematics of the values of the parameter $\tilde{a}(B_n)$ obtained in the present analysis together with the results of the equivalent systematics based on the Fermi-gas model equations [6]. At first glance it might appear that both resonance parameter systematics presented in Fig. 3 are equivalent, as they give an approximately identical level density description at excitation energies near the neutron binding energy. However, these descriptions have been obtained at different absolute values of the level density parameter, as the consideration of the collective effects noticeably diminishes the value $\tilde{a}$. The values $\tilde{a}$ obtained in this way agree well with the experimental data derived from the analysis of the spectra of inelastically scattered neutrons with energies up to 7 MeV [13]. This agreement of the data seems very important, as the evaporation spectra are sensitive just to the value of the level density parameter and not to the absolute value of the level density, and because within the framework of the Fermi-gas model it is impossible to explain the discrepancy between the parameters a derived from the resonance data and from the evaporation spectra.

A more precise definition of the asymptotic values of the level density parameter is very important, too, for a consistent

analysis of the fission cross sections of heavy nuclei [14], and proper account of the pair correlations is important for the description of the level density in the excitation energy region below the neutron binding energy [14, 15]. It would be possible to cite quite a number of other examples to demonstrate the deviation of the experimental data from the predictions of the Fermi-gas model [3]. Therefore, it seems almost obvious that more rigorous, but inevitably also more complicated models will have to be used for the analysis and systematics of experimental data. These could be consistent microscopic models [2, 3], but in most cases an almost equivalent description can be obtained also by the phenomenological approach which has been discussed in the present article. A justification for the complication of the analysis is the consistency of the derived parameters characterizing quite diverse experimental information on the statistical properties of nuclei in a wide range of excitation energies, and the close interconnection of the applied theoretical concepts with the models used to describe the properties of low-lying nuclear states.

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3. Ergebnisse für die Verbindungen der Gruppe II

Diese Ergebnisse sind in Tabelle II aufgeführt. Wie bereits erwähnt, enthalten die Verbindungen der Gruppe II außer einem Phenylkern eine Arylgruppe, die aus einem aromatischen Polycyclus (Fluoren, Phenantren, usw., Tabelle II) besteht.

Alle diese Arylgruppen zeichnen sich durch eine spektroskopische Besonderheit aus: Ihr erster Elektronenabsorptionsübergang ($S_o \rightarrow S_1$) ist vom Typ $^1A_1 \rightarrow {}^1L_b$ (Bande α in der Bezeichnung von Clar). Die Oszillatorenstärke dieses Übergangs ist aber gering, und daraus ergibt sich die Unmöglichkeit, mit diesen "Arylverbindungen" den Laser-Effekt zu bewirken. Dagegen zeigt Tabelle II, daß dieselben Arylgruppen, mit einer Oxazol-Phenyl-Gruppe verbunden, einen Lasereffekt bewirken können. In den von uns in einer früheren Veröffentlichung vorgetragenen Überlegungen wird dieses Phänomen berücksichtigt.

Bei diesen Arylgruppen ist der zweite Absorptionsübergang $S_o \rightarrow S_1$ vom Typ $^1A \rightarrow {}^1L_a$ (Bande p in der Bezeichnung von Clar), und seine Oszillatorenstärke ist groß. Außerdem ist die Polarisation der Übergänge $S_o \rightarrow {}^1L_b$ und $S_o \rightarrow {}^1L_a$ unterschiedlich und entlang der Symmetrieachsen des Moleküls gerichtet. In Tabelle II wurden diese Polarisationsrichtungen angegeben Durch Hinzufügen eines Auxochrom-Substituenten in der Polarisationsrichtung eines dieser Übergänge wird seine Energie geringer, ohne daß der andere Übergang durch diese Störung wesentlich beeinträchtigt wird.

Es wurde gezeigt, daß sich durch diese Eigenschaft die Fähigkeit einer Verbindung verstärken ließe, einen Lasereffekt zu bewirken (4,8). Substituiert man nämlich eine der untersuchten Arylgruppen in der Polarisationsrichtung des Übergangs $^1A \rightarrow {}^1L_a$, dann wird die Energie des Zustands ^1L_b kleiner, während der Zustand ^1L_a praktisch nicht gestört wird.

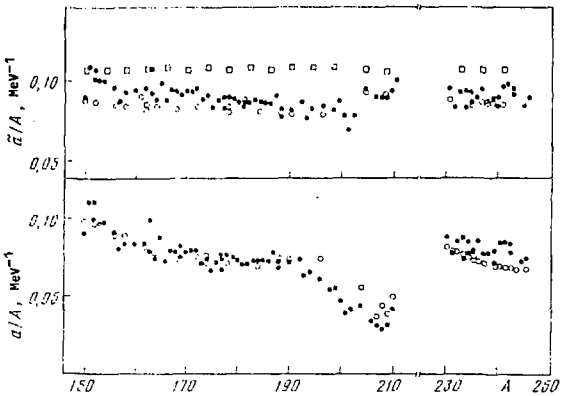

Fig. 2: Comparison of the experimental values of the ratio a/A (●) with the results of the theoretical calculations for the Woods-Saxon potential (O) and for the Nilsson oscillator potential (□). Above: asymptotic value of $\tilde{a}/A$, below: value of a/A at $U = B_n$.

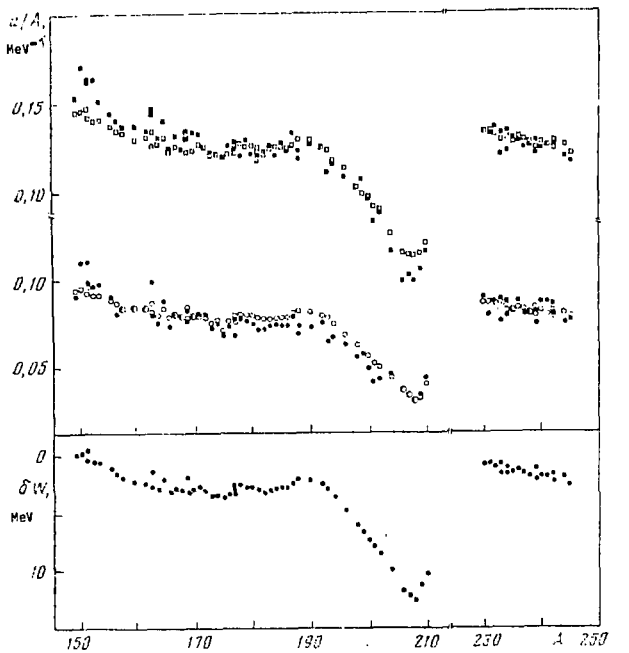

Fig. 3: Experimental values of a/A, obtained on the basis of the equations of the Fermi-gas model (■) and of the superfluidity model of the nucleus with account of the collective effect (●). The open circles indicate the results of the phenomenological description of the a/A ratio for the corresponding models. In case of coincidence of the experimental and phenomenological values the symbol is half-blackened. In the lower part of the figure the values of the shell corrections δW are shown [11].

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