Claim 17 (Arbitrary modifications to a list ADT are not $\mathcal{I}$-confluent with respect to equality constraints on the size of the list)**.** Consider the following transactions:

$$T_{1l} := del(x_i, l);\ add(x_a, l);\ commit$$
$$T_{2l} := del(x_i, l);\ add(x_b, l);\ commit$$

and the list size invariant:

$$I_l(D) = \{size(l) = 1\}$$

Now, the size invariant holds on a list of size one ($I_u(\{add(x_i, l)\}) \rightarrow true$) and on independent state modifications:

$$T_{1l}(\{add(x_i, l)\}) = \{add(x_i, l),\ del(x_i, l),\ add(x_a, l)\}$$
$$T_{2l}(\{add(x_i, l)\}) = \{add(x_i, l),\ del(x_i, l),\ add(x_b, 1)\}$$

However, merging these states result in an invalid state:

$$I_l(\{add(x_i, l),\ del(x_i, l),\ add(x_a, l)\}$$
$$\sqcup \{add(x_i, l),\ del(x_i, l),\ add(x_b, l)\}) \rightarrow false$$

Therefore, $\{T_{1l}, T_{2l}\}$ is not $\mathcal{I}$-confluent under I_u. □