Claim 12 (Counter ADT increments are not $\mathcal{I}$-confluent with respect to less-than constraints)**.** Consider the following transactions:

$$T_{1i} := inc_1(c);\ commit$$
$$T_{2i} := inc_2(c);\ commit$$

and the less-than inequality constraint:

$$I_i(D) = \{val(c, D) < 2\}$$

I_i holds over the empty database state ($I_i(\{\}) \rightarrow true$) and when T_a and T_b are independently executed:

$$T_{1i}(\{\}) = \{inc_1(c) = 1\},\ I_i(\{inc_1(c) = 1\}) \rightarrow true$$
$$T_{2i}(\{\}) = \{inc_2(c)\},\ I_i(\{inc_2(c)\}) \rightarrow true$$

However, merging these states results in invalid state:

$$I_u(\{inc_1(c)\} \sqcup \{inc_2(c)\}) \rightarrow false$$

Therefore, $\{T_{1i}, T_{2i}\}$ is not $\mathcal{I}$-confluent under I_u. □