TOWARDS GENERAL EQUILIBRIUM MODELS OF URBAN LAND USE

INTRODUCTION
THE TASK OT A GENERAL EQUILIBRIUM MODEL

After important attempts by Wingo and Beckmann, the von Thünen's theory was first successfully extended in an urban context by Alonso [1964]. Since then, the subject has advanced considerably, inspiring a great deal of theoretical work. Although it has vastly improved our understanding of the city, most of the work thus far undertaken is intrincially limited because it assumes a monocentric urban configuration, that is a city with a prespecified center, the CBD, which employs the entire urban labor force [1]. Though this assumption greatly simplifies all analytical problems encountered, it has major drawbacks, both theoretically and empirically.

From a theoretical viewpoint, it is clear that whether a city is monocentric or not should be explained within the framework of a theoretical construct, or model, and not specified beforehand as is so often done in making the monocentricity assumption.

A theoretically complete model would yield a spatial structure of the city in which locations of households and firms are endogenously determined, without assuming the location of either *a priori*. In addition, empirical work suggests that the monocentric configuration is but one of many spatial patterns a city can assume. Studies (e.g., Kemper and Schmenner [1974] and Mills [1972]) have shown the increasing tendency for both households and firms to locate away from the city center, a phenomenon which diminishes the role of the CBD.

1. Another common limitation is the static nature of most models. In this paper, too, we restrict our attention on static urban land use theory. In Section 4 we briefly mention to dynamic models of urban land use.

A statistical study and test by Odland [1978] shows that a hypothesis of monocentricity may not be tenable. With all this research, it can therefore convincingly be claimed that cities are not always monocentric, and therefore that monocentricity is at best a dubious assumption. If the theory of urban land use is to be made theoretically sound, practically useful and empirically accurate, models without a *prior* assumption of monocentricity must be developed.

At least two approches to relaxing the monocentric assumption are conceiveable. One is to equip a model with prespecified multiple centers (e.g., Papageorgeou and Casetti [1971], Romanos [1977] and White [1976]). These *multicentric models.* however, are really just as flawed as monocentric models in that spatial structure is assumed beforehand and not allowed to emerge endogenously. The second approach, which is theoretically more pleasing, is to develop a *general equilibrium model* of urban land use which does not assume an *a priori* location of either employment or households.

The fundamental task of a general equilibrium model of urban land use is to generate a *structured space* from a *void space.* Without assuming any *a priori* locations, the model must spatially organize all economic activities simultaneously. To be sure, such a model might well yield a monocentric configuration given a particular parameter set. But it could also yield other configurations, and herein lies the potential explanatory power of such a model. Under various scenarios and parameter sets, the model would produce different spatial patterns, thereby shedding light on why certain configurations are observed in the real world.

If the model succeeds in clarifying the conditions for the existence of the CBD, we will then be able to judge the adequancy of the monocentric assumption adopted by current urban land use models. We will also be able to step foward to a more fruitful theory, one capable of explaining various kinds of non-monocentric phenomena such as employment suburbanization and the formation of subcenters. The purpose of this paper is to suggest that the development of general equilibrium models of urban land use is possible and that these models will open new perspectives to urban economics. The author also believes that the theoretical work involved offers a unique challenge and opportunity to location theorists.

In Section 2, we discuss alternative approaches to developing meaningful models of general equilibrium urban land use. By considering the *Spatial Impossibility Theorem* of Starrett, we conclude that three prototypes of general equilibrium models are conceivable : Type

A — the port city model, Type B — the competitive equilibrium model with spatial externalities, and Type C — the noncompetitive equilibrium model with scale economies. In Section 3, we present a simple Type B model of general equilibrium urban land use. We show that model is capable of yielding a multicentric equilibrium pattern as well as monocentric and dispersed patterns, and that the model generally yields multiple equilibria under each fixed set of parameters. Finally, in Section 4, we suggest future research directions.

THREE PROTOTYPES OF THE GENERAL EQUILIBRIUM MODEL

We say a general equilibrium model of urban land use is *meaningful* when it *has a solution which is nontrivial.*

An urban configuration is called *trivial* if all activities locate at one point or if they are uniformly distributed over the urban area. Clearly, trivial urban configurations are rarely observable, and they are not interesting from a theoretical point of view. Hence, an important initial goal is that of constructing a meaningful model. To achieve this goal, it is worthwhile recalling the following theorem by Starrett [1978]. Suppose that we have a closed economy in which the following assumptions are satisfied.

A 1 (*No Relocation Costs*). All agents are free to choose locations.
A 2 (*Perfect Markets*). There exist complete markets for all goods and services at all locations [2].
A 3 (*Homogeneous Space*). Utiliy functions of households and technologies of firms are independent of location, and initial endowments of resources are equal over the space [3].

Then, we can conclude as follows.

2. This implies there is no price discrimination ; that is, market equilibrium prices must be the same for all agents at each location. This also implies that there are no (technological) externalities in consumption and production and that all goods and services are transacted through markets. Note also that assumption *does not* require that transactions actually take place for each commodity at each location, since some markets may be inactive.

3. That is, households and firms have no *a priori* preference for locations, and the initial resource base is the same for all locations. However, this assumption of homegeneous space *does not imply* a uniform transport plain. There may exist some non-uniform transport networks, and hence transport costs may depend on directions of movement.

Spatial Impossibility Theorem (*Starrett, 1978*) [4]. Under assumptions A 1 to A 3, there is no competitive equilibrium with a positive total transport cost. That is, under A 1 to A 3, the only possible competitive equilibrium is the one in which no good or person is transported (with positive cost).

Here, by competitive equilibrium, we mean the market equilibrium in which all agents take prices as given. In the context of urban land use, an urban configuration requires no transport cost only when all activites are concentrated at one point or when they are uniformly distributed over the urban area.

Therefore, this theorem says that, if assumptions A 1 to A 3 are upheld, there exists either a trivial solution or no competitive equilibrium [5]. As the base of a general equilibrium model of land use in *cities,* A 3 is not a bad assumption. In other words, it is quite natural to require that a meaningful general equilibrium model of urban land use should be capable of generating a nontrivial solution even when assumption A 3 is satisfied. Then, spatial impossibility theorem above suggests that the following three prototypes of meaningful general equilibrium models are conceivable.

4. This statement of the theorem summarizes only part of the result due to Starrett [1978]. For proof of the theorem, see Starrett [1978].

5. For example, recall the famous *assignment problem* formulated by Koopmens and Beckmann [1957], in which n agents are assigned to n plots. It is assumed that each agent is indivisable, that no plot can be occupied by more than one agent, and that each agent requires transactions of some positive amounts of intermediate goods with other agents. Assumptions Al to A 3 are also satisfied in this problem. Since any feasible assignment requires a positive transport cost, it immediately follows from Starrett's spatial impossibility theorem that the optimal spatial assignment for the Koopmans - Beckmann problem cannot be sustained by competitive market mechanism. Actually, in the Koopmans - Beckmann problem, no feasible assignment can be sustained by competitive market mechanism.

For another example, consider the general equilibrium model proposed by Schweizer, Varaiya and Hartwick [1976]. This model is an adaptation of the Arrow-Debreu model, in which a (discrete) location index is introduced and each household is allowed to choose only one residential location. The model satisfies assumption Al. And if we assume that the set of market places is identical to the set of market places for immobile commodities, then the model satisfies assumption A 2 as well. Therefore, from the spatial impossibility theorem, it follows that, if we also assume A 3, the Schweizer-Varaiya - Hartwick model has only a trivial solution (i.e., uniform distribution of all activities among all locations).

This latter example suggests that directly extending the Arrow - Debreu model to a spatial economy is not an appropriate way to develop *meaningful models* of general equilibrium urban land use. However, it must be noted that if we appropriately modify their model by introducing exports and imports, and if we assume these exports and imports occur only at a prespecified location (or, locations), it becomes a general equilibrium model of type A (defined immediately below).

Port City Model

In this model, we retain assumptions A 2 and A 3. However, the city by itself does not constitute a closed economy, but it is connected with the rest of the world at a port (or, station), whose location is fixed *a priori*. We assume that all agents in the rest of the world are fixed (i.e., they cannot move into the city in question), and that all agents in the city are free to move within a given (political) boundary. Examples are Mills [1970], Goldstein and Moses [1975], Schweizer and Varaiya [1976, 1977], Schweizer [1978], and Solow [1973]. Though this is a good starting point, it is not completely general since the location of final demands for the city (i.e., exports and imports) is *a priori* fixed at one location. Besides, the land use pattern generated as the solution of this model type is essentially monocentric; the land rent curve monotorically decreases in distance from the port, and there is no local agglomeration of activities such as subcenters. Moreover, if final demands were 0 (i.e., no exports and imports), the model would generate only a trivial solution (i.e., a uniform distribution of activites). For yet another reason, this model is not sufficiently general in characterizing modern cities; namely, few modern cities export and import most goods exclusively from their centers.

Competitive Equilibrium Model With Spatial Externalities

Suppose we want to construct a competitive equilibrium model of urban land use which is sufficiently general so that it is capable of generating a nontrivial solution even when assumptions A 1 and A 3 are ratisfied. Then, it follows from the spatial impossibility theorem that assumption A 2 must be abandoned, and the model must contain certain market imperfections. In the context of urban land use, market imperfections which are compatible with competitive equilibrium are non-price interactions among agents (e.g., information exchange among business firms, social interactions among households) and external effects (e.g., various kinds of technological spillover effects and congestion effects, and increase in locational identity through agglomeration). We call non-price interactions and external effect *spatial externalities* [6]. Local public goods can be considered another source of spatial externalities.

6. Most external effects, in the context of urban land use, are influenced by distances among agents, and costs and benefits of non-price interactions are

Noncompetitive Equilibrium Model With Scale Economies

Due to scale economies (including indivisibility), if the number of a type of agent is sufficiently small relative to the overall urban economy, then these agents may behave non-competitively (e.g., perceived market power effects on prices and market areas of their outputs and inputs, in particular, labor and land). A *non-competitive equilibrium model* may be capable of generating a nontrivial solution even when assumption A 1 to A 3 are satisfied.

The simplest class of models in this category is the « factory town model » which involves a city with a single monopolistic firm and workers (e.g., Mills [1967] and Kanemato [1980, Chapter II]). A more general approach would be to combine the Lösch-Christaller — Hotelling typed spatial monopolistic model with the Alonso-Muth land use model.

Of course, by combining the three prototypes above, we can develop a variety of general equilibrium models. However, before combining them, it is advantageous to first fully develop each prototype. Since Type A model has been well studied, we deal primarily with Type B model in this paper. The Type C model is briefly discussed in Section 4.

An important conclusion follows from the above discussion. Observe that, except for the Type A model, a meaningful general equilibrium model of urban land use can be obtained only when market imperfections are introduced. Since the equilibrium state in a imperfect market is generally inefficient, normative considerations necessarily become very important. We must consider the corrective interventions that will restore efficiency in the urban land market.

A SIMPLE TWO-SECTOR MODEL

A seminal work by Beckmann [1976] contains the first development of a Type B general equilibrium model (competitive equilibrium with spatial externalities). The model focuses on the role of social interactions among households in shaping equilibrium patterns of residential

generally affected by distances among agents. Moreover, if external effects and costs and benefits of non-price interactions are independent of distances among agents, they do not play a role as factors in generating nontrivial solution (i.e., non-uniform distribution of activities). Due to these reasons, we adopt the term of spatial externalities.

living. The utility of each household is assumed to depend on the amount of space occupied and on the *average distance* to all other households in the city. For simplicity and clarity, work and shopping trips are ignored. It is shown that, at equilibrium, this residential city exhibits a dispersed population distribution with a single peak.

Although Beckmann's model is ingeneous, it contains one-sector only and is therefore of limited practical applicability. The purpose of this section is to present a two-sector, Type B model, which is an extension of Beckmann's work [7]. Before giving the precise description, we will provide a brief sketch.

We consider a one-dimensional city which consists of two sectors, *households* and *business firms*. Unlike the Beckmann model, we introduce spatial externalities among business firms [8]. Namely, business firms are assumed to need non-price interactions among themselves (e.g., information exchange by face-to-face communication), and hence the productivity of each business firm climbs with increasing accessibility to other business firms. Households supply labor to business firms; hence the distribution of households follows closely the distribution of jobs (commuting from residence to the job site is not costless). The equilibrium configuration of the city is obtained as the outcome of these interactions among business firms and households. By considering two different types of accessibility measures among business firms, *linear accessibility measure* and *spatially discounted accessibility measure*, we show that the character of the equilibrium urban configuration is critically affected by the type of measure involved.

Recall that, one of the major contribution of Alonso [1964] is the clear definition of *bid rent* functions in the context of urban land use; the explicit use of bid rent curves enabled him to graphically analyze the equilibrium configuration of the city.

We too follow his approach in this paper. However, it must be recalled that Alonso [1964] defines bid rent as a function of the distance from the *city center*. Obviously, this definition of bid rent is useful only in monocentric models, where, the argument, « distance from the city

7. The model presented below is based on Ogawa [1980], Ogawa and Fujita [1980], and Fujita and Ogawa [1982]. Borukhov and Hachman [1977] developed a model similar to Bechmann [1976]. The model by Capozza [1976] is similar to ours, but his treatment of distance between firms is less explicit. The models formulated by Odland [1976, 1978] are more general, but the analysis is not fully developed.

8. Of course, taking into account the spatial externalities of both household and business sectors (and possibly between them) makes the model more general, but the analysis is further complicated. We will come back to this point in Section 4.

center », has a definite meaning. For general equilibrium models, something new is required, way to make bid rent conditional on the urban configuration determined by the model. Thus, the generalization of the theory of land use requires a conceptual change in the notion of bid rent, one that leads to greater generality.

Model Formulation

Suppose a city develops on a long, narrow strip of homogeneous agricultural land of width 1 (unit distance). Since the width of the land is sufficiently small, the city may be treated as *linear city*. Each urban location can then be reprensented by a point, x, on the line.

Suppose also that there are N identical households in the city, and that the utility function of each household is given by u (Z, S), where Z and S are the amount of composite commodity consumed and the amount of land occupied by the household, respectively. For simplicity, we assume that lot size, S, is fixed for all households at some positive constant size S_h. We also assume that each household is home to one worker which supplies labor to a business firm ; travel cost per household consists solely of its journey to work. If we assert that the composite commodity is imported at constant price 1 (numeraire) [9], the budget constraint of a household residing at x and working at x_w is given by

$$Z + R(x) S_h + t\,|x - x_w| = W(x_w) \tag{1}$$

where R (x) is land rent at x, t is commuting cost per unit distance, $|x - x_w|$ is commuting distance, and W (x_w) is the wage paid by business firms at x_w. Here we are assuming that all lands are owned by absentee landlords [10].

Since lot size is fixed to a constant S_h, the objective of each household is to maximize the consumption level of composite good Z, subject to its budget constrain (1), by appropriately choosing residential location x and working location x_w :

$$\max_{x, x_w} Z = W(x_w) - R(x) S_h - t\,|x - x_w| \tag{2}$$

9. Assuming that urban transport cost for the composite commodity is zero, we can consider that firm ouptut constitutes a part of the composite commodity consumed by the households in the city.

10. This is just for simplicity of expression. Instead, we can assume that the income of each household consists of wage income W (x_w) and non-wage income Y_0, which includes an equal share of total land rent revenues. We see later that, in the context of our model (in particular, since lot size is fixed), the equilibrium configuration is independent of Y_0 and hence of any assumption about land ownership.

In the model, we allow for M identical business firms [11], each producing some kind of service or information which is exported from the city at a constant price P_0 [12]. The production technology of each firm is assumed to be of fixed-coefficient type. The production activity of a business firm requires S_b units of land and L_b units of labor. However, due to the external effects among business firms, productivity is also affected by the accessibility to the rest of business firms. Denoting by $\pi(x)$ the profit of a business firm at location x, we can express this idea in two different forms : either in a multiplicative production equation

$$\pi(x) = P_0 \bar{Q} A(x) - R(x) S_b - W(x) L_b \tag{3}$$

or in an additive equation

$$\pi(x) = P_0 \bar{Q} + A(x) - R(x) S_b - W(x) L_b \tag{4}$$

where $A(x)$ represents the degree of spatial accessibility of location x, the functional form of which is specified later. In equation (3), $\bar{Q}$ represents the output level of a business firm under the standard condition, $A(x) = 1$; the output of the firm is assumed to increase proportionally to $A(x)$. In equation (4), $P_o \bar{Q}$ represents the gross revenue at the standard condition, $A(x) = 0$, and $A(x)$ represents the savings in interaction costs relative to the standard condition. Given either equation, the locational choice behavior of a business firm can be commonly expressed as followed.

$$\max_x \pi(x) = a + k A(x) - R(x) S_b - W(x) L_b \tag{5}$$

where, $a = 0$ and $k = P_o \bar{Q}$ for equation (3), and $a = P_o \bar{Q}$ and $k = 1$ for equation (4).

We consider two different specifications in describing the functional form of spatial accessibility, $A(x)$:

$$A(x) = \int b(y) e^{-\alpha|x-y|} dy \tag{6}$$

or

$$A(x) = \int b(y) (K - \alpha |x - y|) dy. \tag{7}$$

In both equation, α is a positive constant and $b(y)$ is the density of business firms at y (the number of business firms per unit distance at y);

11. Note that, though all business firms are assumed to be identical *from the view point of location behavior*, they may be different in some aspects (e.g., the actual services they provide, or the information they produce).

12. As noted in footnote 9, assuming that urban transport cost for output in zero, we can assume that a part of the output produced by business firms is consumed by households. In this case $P_0 = 1$ (the price of composite commodity).

in equation (7), K is a non-negative constant. We call A (x), defined by (6), *spatially-discounted accessibility measure.* When defined by (7), we call A (x) a *linear accessibility measure* [13].

Assuming that the locational choice behaviors of households and firms are reprenseted, respectively, by (2) and (5), our task is to analyze the equilibrium spatial configuration of the city. The market for land and labor is assumed to be perfectly competitive everywhere. We denote with a non-negative constant, R_a, the agricultural land rent which is assumed to be exogenously given. As noted before we assume that the population of the city is fixed to N ; however firms are free to enter or leave. Then, at equilibrium, competition drives the profit level of each firm to 0, and the total number of urban firms, M, is given by

$$M = N/L_b \tag{8}$$

The spatial structure of the city now be described by the following system,

$$\{h\ (x),\ b\ (x),\ R\ (x),\ W\ (x),\ c\ (x),\ U\}$$

where $h\ (x)$ is the household density function (the number of households per unit of distance at x), $b\ (x)$ is the business firm density function, R (x) is the land rent profile, W (x) is the wage profile, $c\ (x)$ is the communting pattern, and U is the household utility level. To define function $c\ (x)$ precisely, let us introduce the following terminology and notation.

$$\text{Residential Area}: h_+ = \{x \mid h\ (x) > 0\}$$

$$\text{Business Area}: b_+ = \{x \mid b\ (x) > 0\}\ .$$

A commuting pattern, $c\ (\cdot)$, is a right-continuous function from h_+ onto b_+ ; $x_w = c\ (x)$ means that households at x commute to firms at x_w [14].

To state the equilibrium conditions for the problem we introduce the following functions :

13. For example, if we combine (4) and (7) with K = 0, then we have

$$\pi\ (x) = P_0\ Q - \alpha \int b\ (y) \mid x - y \mid dy - R\ (x)\ S_b - W\ (x)\ L_b$$

In this equation, the term, $\int b\ (y) \mid x - y \mid dy$, represents the *average distance* from a firm at x to all other firms. This accessibility measure was first used by Beckmann [1976]. In Ogawa and Fujita [1980], we exclusively studied the case of linear accessibility measure ; and in Fujita and Ogawa [1982], we studied the case of spatially-discounted accessibility measure. The present paper summarizes the results of these two papers in a unified form.

14. Each of $h(\cdot)$, $b(\cdot)$, $W(\cdot)$ and $R(\cdot)$ is assumed to be a non-negative function with a positive finite Reimann integral on R. Both residential area h_+ and business area b_+ are assumed to be bounded in R.

$$(9)\qquad \Psi(x, x_w, U) \equiv \Psi(x, x_w, U \mid W(x_w)) = (W(x_w) - Z(U) - t\,|x - x_w|)/S_h$$

$$(10)\qquad \Psi(x, U) \equiv \Psi(x, U \mid W(\cdot)) = \max_{x_w} \Psi(x, x_w, U \mid W(x_w))$$

$$(11)\qquad \Phi(x, \pi) \equiv \Phi(x, \pi \mid A(x), W(x)) = (a + k\,A(x) - W(x)\,L_b - \pi)/S_b$$

In (9), $Z(U)$ is the inverse of $U = u(S_h, Z)$; it represents the amount of Z necessary to derive utility level U. Function $\Psi(x, x_w, U)$ gives the maximum land rent that can be paid by a household, at location x, which derives utility level U and whose resident worker commutes to x_w. Function $\Psi(x, U)$ represents the maximum value of $\Psi(x, x_w, U)$, which is obtained by optimally choosing job location x_w. We call both $\Psi(x, x_w, U)$ and $\Psi(x, U)$ the *bid rent function of the household*; however, we must note that $\Psi(x, x_w, U)$ is a function of both residential location x and job location x_w, while $\Psi(x, U)$ is the function of only residential location x. In (11), function $\Phi(x, \pi)$ gives the maximum land rent which a business firm could pay at location x while deriving profit level π. We call $\Phi(x, \pi)$ the *bid rent function of the business firm.* These functions are generalized forms of the bid rent function originally defined by Alonso [1964] in the context of monocentric urban land use [15].

Since $\pi = 0$ at equilibrium, for the sake of simplicity we put

$$(12)\qquad \Phi(x) \equiv \Phi(x, 0).$$

We assume that the agricultural bid rent is exogenously given by a positive constant R_a. Using the bid rent functions so defined, we can summarize as follows the necessary and sufficient conditions for an equilibrium configuration of a system, $\{h(x), b(x), R(x), W(x), c(x), U\}$ [16]:

15. Recall that Alonso [1964] defines bid rent for a household (firm) as a function of the distance from the city center and of the utility (profit) level of the household (firm). That is, in the Alonso model, bid rents are not functions of spatial distributions of urban activities. Notice that, this simple definition of the bid rent function is meaningful only in the context of a monocentric city model. In our general equilibrium model, each bid rent function is defined in a form which is *conditional on the urban configuration* to be determined. Namely, bid rent $\Psi(x, U)$ depends not only on U but also on the wage profile, $W(\cdot)$, over all x; bid rent $\Phi(x, \pi)$ depends not only on π but also on wage $W(x)$ and accessibility $A(x)$ at x, where $A(x)$ in turn, depends on the business firm density, $b(\cdot)$, over all x.
16. Of course, by definition, $h(x) \geqq 0$, $b(x) \geqq 0$, $W(x) \geqq 0$, $R(x) \geqq 0$ at each x (recall footnote 14).

(i) land market equilibrium conditions : at each x,

$$
\begin{array}{ll}
\text{(i.1)} & \mathrm{R}(x) = \max\{\Psi(x, \mathrm{U}), \Phi(x), \mathrm{R}_a\} \\
\text{(i.2)} & \mathrm{R}(x) = \Psi(x, \mathrm{U}) \quad \textit{if } h(x) > 0 \\
\text{(i.3)} & \mathrm{R}(x) = \Phi(x) \quad \textit{if } b(x) > 0 \\
\text{(i.4)} & \mathrm{S}_h\, h(x) + \mathrm{S}_b\, b(x) \leqq 1 \\
\text{(i.5)} & \mathrm{S}_h\, h(x) + \mathrm{S}_b\, b(x) = 1 \textit{ if } \mathrm{R}(x) > \mathrm{R}_a
\end{array}
$$

(ii) communting pattern and labor market equilibrium conditions [17] :

(ii.1) $\Psi(x, c(x), \mathrm{U}) = \Psi(x, \mathrm{U})$ for sach $x \in h_+$,

(ii.2) $\int_{\mathrm{I} \cap h_+} h(x)\, dx = \int_{c(\mathrm{I} \cap h_+)} \mathrm{L}_b\, b(x_w)\, dx_w$

for every interval $\mathrm{I} = (a, b) \in (-\infty, \infty)$,

(ii.3) $\int h(x)\, dx = \mathrm{N}, \ \int b(x)\, dx = \mathrm{N}/\mathrm{L}_b$.

Condition (i.1) indicates that the equilibrium land rent curve is the upper envelope of three equilibrium bid rent curves. Conditions (i) altogether ensure that, at each location, land is occupied by the maximum bidder (under equilibrium U or $\pi = 0$) for the land. By definitions (9) and (10), condition (ii.1) is equivalent to

$$\mathrm{W}(c(x)) - t\,|\,x - c(x)\,| = \max_{x_w} \mathrm{W}(x_w) - t\,|\,x - x_w\,| \text{ for each } x \in h_+ .$$

That is, (ii.1) ensures that each household chooses the optimal job location so that disposable income is maximized. Finally, conditions (ii.2) and (ii.3) together guarantee that each household resides somewhere and works at some business office and that the labor supply is equal to the labor demand at each location in the city.

The character of the equilibrium urban configuration is very different depending on whether the spatial accessibility measure, A (x), is linear or spatially discounted. Therefore, we must discuss each case separately in the following two sub-sections. But first we will introduce some useful terminology and notation.

(exclusive) residential district	$\mathrm{RD} = \{x \mid h(x) > 0,\ b(x) = 0\}$
(exclusive) business district	$\mathrm{BD} = \{x \mid h(x) = 0,\ b(x) > 0\}$
integrated district	$\mathrm{ID} = \{x \mid h(x) > 0,\ b(x) > 0\}$

Note the difference betwen residential area and residential district, and between business area and business district. By definition, $h_+ = \mathrm{RD} \cup \mathrm{ID}$ and $b_+ = \mathrm{BD} \cup \mathrm{ID}$.

17. Recall that $h_+ = \{x \mid h(x) > 0\}$, and $c(\mathrm{I} \cap h_+) = \{x_w \mid x_w = c(x), x \in \mathrm{I} \cap h_+\}$.

Equilibrium Urban Configuration - Linear Accessibility Measure

Suppose that accessibility measure A (x) is given by equation (7). Then, we can easily confirm that A (x) is concave in x (strictly concave inside b_+, linear outside b_+). From this, we can show that (i) an equilibrium urban configuration uniquely exists under each set of parameters and (ii) each equilibrium urban configuration belongs to one of the following three categories [18] :

u_1 : monocentric configuration

u_0 : completely mixed configuration

u_* : incompletely mixed configuration

Figures 1 to 3 depict these three urban configurations.

The upper part of Figure 1 shows the land use pattern of a monocentric urban configuration. The business district is surrounded by two residential districts of equal size. The business district at the center may be called the CBD. By taking the origin at the center of the business district, the equilibrium wage profile is given by

$$\text{(13)} \qquad W(x) = W(0) - t\,|x|\,.$$

The lower part of Figure 1 shows bid rent curves, $\Psi(x, U)$ and $\Phi(x)$. The equilibrium land rent curve in given by $R(x) = \max\{\Psi(x, U), \Phi(x), R_a\}$ as mentioned earlier [19]. Most urban land use literature focuses exclusively on this form of urban land use.

Figure 2 depicts a completely mixed urban configuration in which households and business firms locate everywhere in an equal proportion. Each location is self-contained with respect to the supply and demand of labor ; hence, no commuting occurs in the city. The land rent curve is strictly concave.

18. This subsection is based on Ogawa and Fujita [1980], and details of the analysis can be found there.

19. At the central area of the business district, we can observe a « positive rent gradient ».

This phenomenon is due to the concavity of accessibility measure $A(x)$ and to linearity of wage curve $W(x)$. That is, in bid rent function (11), the benefit of assessibility, $kA(x)$, increases at a decreasing rate toward the center, and the $kA(x)$ curve is flat at the center ; while, from (13), labor cost $W(x)\ L_b$ increases linearly toward the center. Hence, at the central area of the business district, marginal benefit from accessibility increase is dominated by marginal cost from wage increase. Consequently, bid rent $\Phi(x, \pi = 0)$ by business firm declines toward the center. Though this result in partly due to the assumption of a fixed coefficient production function, it can also happen in a more general model. In particular as long as the function of accessibility benefit is smooth and concave near the city center and commuting cost is strictly increasing toward the center, this phenomenon of positive rent gradient is always possible.

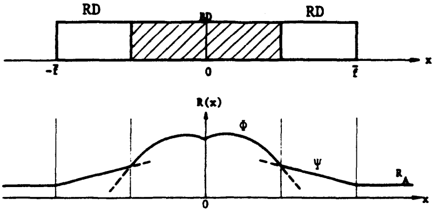

Figure 1.
Monocentric configuration (u_1)

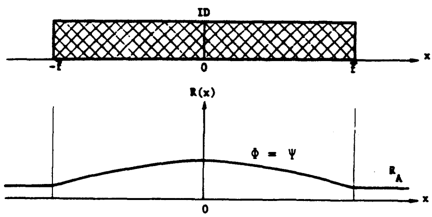

Figure 2.
Completely mixed configuration (u_0)

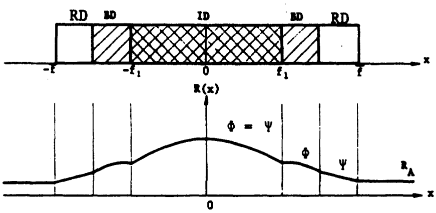

Figure 3.
Incompletely mixed configuration (u_*)

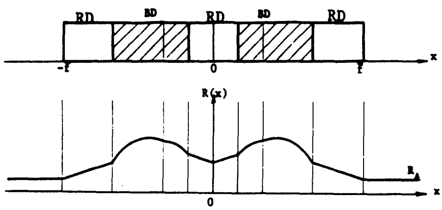

Figure 4.
Duocentric configuration (u_2)

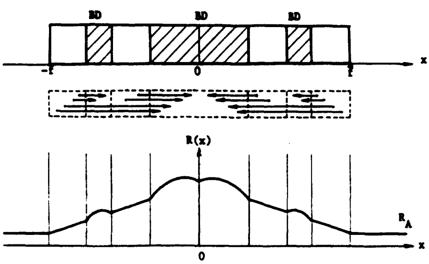

Figure 5.
Tricentric configuration of type A (u_{3A})

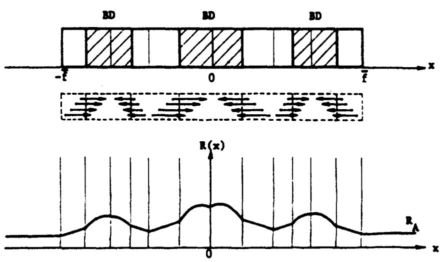

Figure 6.
Tricentric configuration of type B (u_{3B})

The incompletely mixed urban configuration depicted in Figure 3 is a mixture of the above two extremes. Two business districts surround the integrated district at the center, and a residential district constitutes the suburb at each side. The integrated district is, again, self-contained with respect to labor. Obviously, this pattern approaches the monocentric configuration as f_1 (the fringe distance of the integrated district) goes to 0, and it approaches the completely mixed configuration as f_1 goes to $\bar{f}$ (see Figure 3).

Figure 7 describes the parameter-range for each category of equilibrium urban configurations, where

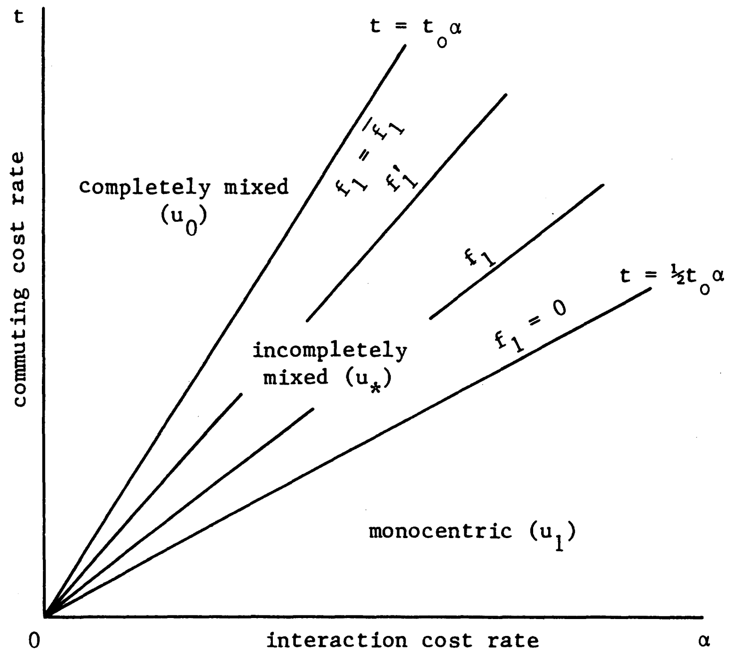

Figure 7. Parameter ranges for three categories of urban configuration — linear accessibility measure

$$t_0 = M\ S_h/(S_b + S_h\ L_b) = N\ S_h/L_b\ (S_b + S_h\ L_b).$$

Let us call parameter α in (7) the *interaction cost rate*. Then, from the figure we see that the city assumes a monocentric configuration when interaction cost rate α is sufficiently large relative to commuting cost rate t. Conversely, when α is small relative to t, the city assumes a completely mixed configuration. Between these two extremes, an incompletely mixed configuration is the equilibrium solution. Note that, in the case of the incompletely mixed configuration, the size of the integrated district, f_1, increases as the ratio, t/α, increases.

Equilibrium Urban Configuration - Spatially Discounted Accessibility Measure

The more interesting case is with spatially discounted accessibility measure [20]. Suppose accessiblity measure A (x) is given by equation (6). Then, A (x) is not everywhere concave in x (concave inside each business district, convex inside each residential district). From this, the next results follows.

(i) In addition to the three urban configurations (u_1, u_*, u_0) just examined, there exist many other equilibrium configurations (potentially, infinitely many), including those with subcenters.

(ii) The solution is not always unique (under each fixed set of parameters). That is, *multiple equilibria* occur in a wide range of parameter values.

(iii) Because of the existence of multiple equilibria, the change of urban configuration (in the sense of comparative statics) with respect to parameter change is not always smooth ; *catastrophic transition* in configuration can occur as parameters take certain critical values.

We have identified the existence of the following three additional categories of equilibrium configuration [21].

u_2 : duocentric configuration ;
u_{3A} : tricentric configuration of Type A ;
u_{3B} : tricentric configuration of Type B.

Figures 4 to 6 present each type of new urban configuration. The duocentric pattern described in Figure 4 can be interpretated either

20. For details about the analysis in this section, see Fujita and Ogawa [1982].
21. All other urban configurations with more than three centers are left for future investigation. We have also limited our analysis to the case of symmetric urban configurations. The existence of each type of urban configuration was confirmed by combining an algebraic analysis and numerical analysis. Namely, because of the complicated functional form of function A(x) (although it seems simple at first glance), it is quite difficult to manipulate our model in a purely analytical fashion. Hence, after obtaining the necessary and sufficient conditions for each possible urban configuration, numerical explorations were carried out to determine the range of parameters for each configuration.

as one city with two business districts or as two adjoining cities creating external economies for one another and enjoying agglomeration economies within a system of cities. As depicted in Figure 5, all workers commute inwardly in the case of Type A tricentric configuration. On the other hand, as depicted in Figure 6, the city of Type B is divided into three parts with respect to the demand and supply of labor. Another difference between Type A and Type B is that, though the size of the business district at the center overwhelms the size of the two fringe business districts in Type A, these sizes are approximately equal in Type B. Therfore, the tricentric pattern of Type A can be considered as the spatial configuration of one city with a central business district and two subcenters [22]. On the other hand, the tricentric pattern of Type B may be regarded either as a system of three cities, each with a CBD, or as one city with three subcenters.

In order to investigate the parameter-range for each category of equilibrium urban configuration, recall that the parameters in our models are [23]

$$\{k,\ t,\ \alpha,\ \mathrm{N},\ \mathrm{S}_h,\ \mathrm{S}_b,\ \mathrm{L}_b\} \tag{14}$$

where, $k = \partial\, \pi / \partial\, \mathrm{A}\,(x)$, t is commuting cost rate, α is the spatial discounting rate of external effects, N is the population, S_h (S_b) is the lot size for a household (business firm), and L_b is the labor input coefficient for a business firm. Among these, the important parameters are k, t, and α. Thus, let us fix the other parameters arbitrarily as follows :

$$\{\mathrm{N},\ \mathrm{S}_h,\ \mathrm{S}_b,\ \mathrm{L}_b\} = \{1000,\ 0.1,\ 1,\ 10\} \tag{15}$$

Figure 8 describes the parameter-range of each urban configurations on $\{t/k,\ \alpha\}$ — space [24]. It is readily observed that the ratio, t/k, figures in the solution, and not the individual values of k and t. Hence, the results of the analysis are shown on $\{t/k,\ \alpha\}$ — space. The parameter-range of the monocentric (completely mixed) configuration is below curve u_1 (above curve u_0). The imcompletely mixed urban configuration occurs in the area between two curves, u_* and u_0. The area for the

22. In his 1972 book, Mills suggested the development of formal models which can generate subcenters. Our model here might be the first of such models.

23. The maximization of profit $\pi\ (x)$ is equivalent to the maximization of $\pi\ (x) - a$; hence parameter a does not affect the equilibrium urban configuration. Thus, parameter a is not included in (14).

24. For a more precise description of parameter-ranges for each urban configuration, see Figure 14 of Fujita an Ogawa [1982].

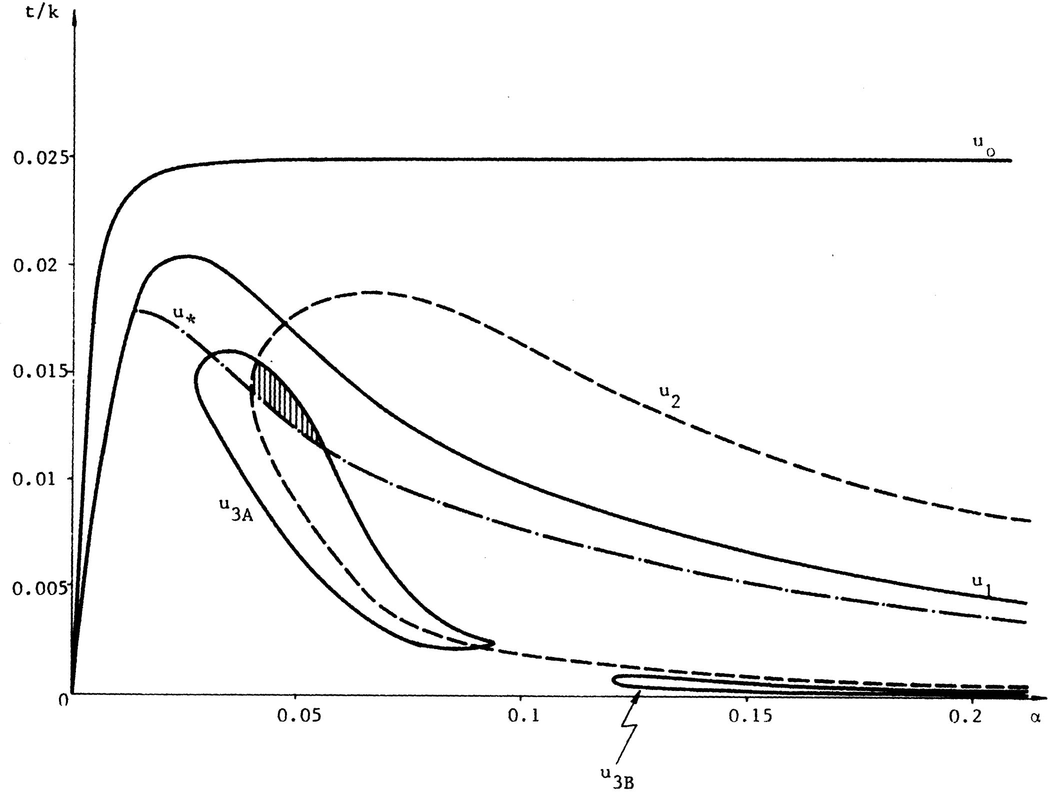

Figure 8. Parameter ranges for six categories of urban configuration — Spatialy discounted accessibility measure

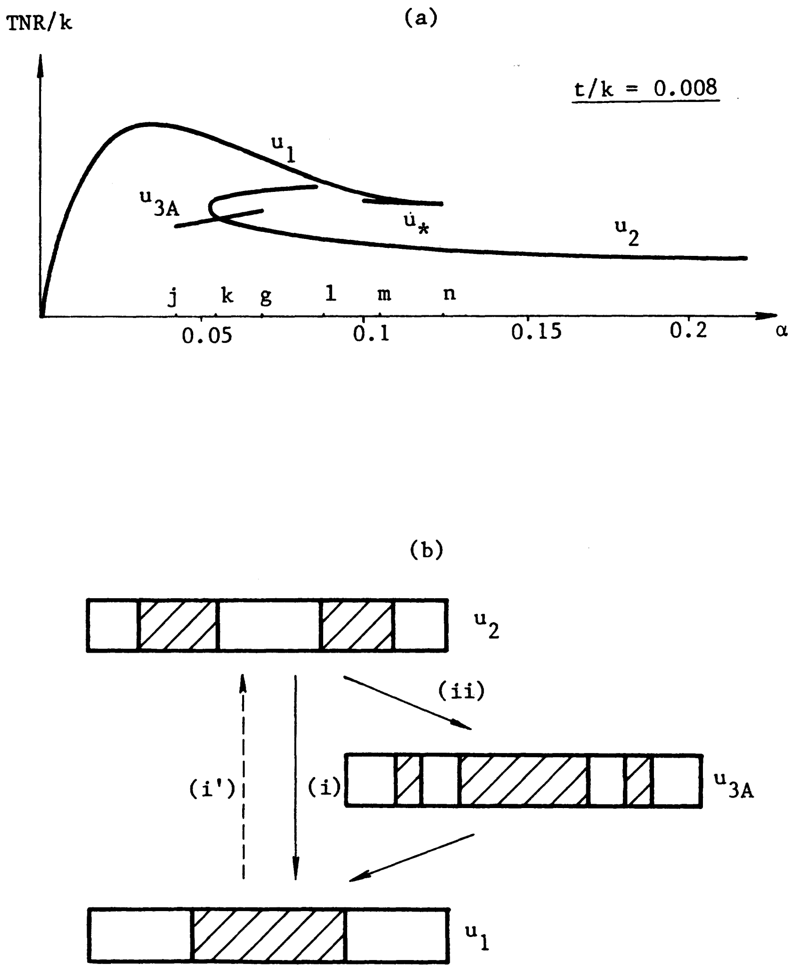

Figure 9. Structural transition of urban configuration

duocentric configuration (tricentric of Type A, tricentric of Type B) is encircled by curve u_2 (curve u_{3A}, curve u_{3B}).

We can see from Figure 8 that multiple equilibria occur in many regions of parameter-space. For example, when parameters fall in the shaded region, the city can assume at least four different equilibrium configurations (u_1, u_*, u_2, u_{3A}). Because of this, catrastrophic (i.e., discontinuous) transitions of urban configuration often occur with changes in parameter values.

Figure 9 depicts an example of catastrophic transition. Here, TNR represents the *total net land rent* (net of agricultural land rent) when the value of t/k is kept at 0.008. Suppose the initial urban configuration is duocentric (pattern u_2) and that the value of α continuously decreases. The equilibrium urban configuration remains u_2 until α reaches point k in Figure 9 (a). However, when α decreases below point k, the same u_2 cannot be the equilibrium configuration any longer. If the city is to romain in equilibrium, it must assume either configuration u_1 or u_{3A}. In either case, as depicted in Figure 9 (b), the change in spatial structure is discontinuous, To further exemplify such transitions, we note that if α continuously increases while $t/k = 0.008$, the urban configuration discontinuously changes from u_1 to u_2 at point n. It is important to observe that these structural transitions are not reversible ; namely, transition of u_2 to u_1 occurs at point k, while transition of u_1 to u_2 occurs at point n.

Figure 10 illustrates the effects of population change on the equilibrium urban configuration in the case of the monocentric pattern, where values of other parameters are kept at those in (15). We see from this figure that the larger N is, the smaller t/k must be at each α in order that the monocentric urban configuration remains in equilibrium. This implies that, as population increases, the city less likely to exhibit a monocentric pattern.

Two tasks are left for further research (in the case of spatially discounted accessibility measure) :

(I) We have so far limited our analysis to the case of symmetric urban configurations with less than four centers. All other configurations (asymmetric or with more than three centers) are left for further investigation. Hence a future task is to obtain *all* the equilibrium urban configurations under each set of parameters.

(II) In the case of multiple equilibria, the question of which configuration will actually be realized cannot be answered within the framework of our static model. Similarly, the comparative static approach to the study of the change in urban spatial structure is not satisfactory. For both purposes, we need a *stability analysis* of urban configurations.

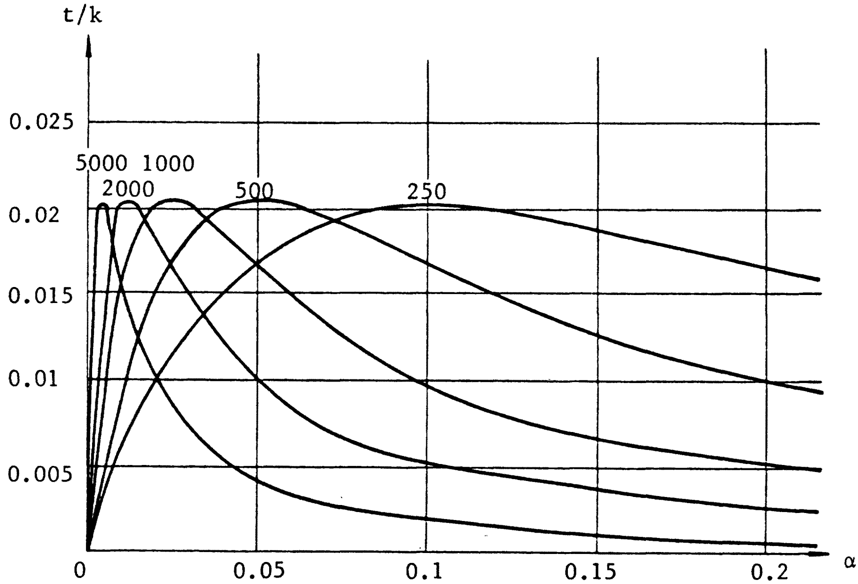

Figure 10. Effect of population change on the parameter-range of a monocentric configuration

Optimum Urban Configuration

As noted in Section 2, the normative study of each general equilibrium model of urban land use is very important for the following reasons :

(i) With the execption of Type A (Port City) case, a meaningful general equilibrium model is based on some kind of market imperfections ; hence, the market equilibrium is generally inefficient.

(ii) In the case of multiple equilibria, the relative superiority among equilibria becomes a very import issue.

(iii) To date, most normative models of urban land use have investiaged the optimality given a monocentric urban configuration.

However, the optimal land use under a monocentric configuration may be sub-optimal when considered against other configurations. An important question naturally arises : when is a monocentric urban configuration optimal among all possible configurations ? In this subsection, we study these issues within the context of our two-sector model [25].

An urban configuration is called *optimal* (i.e., efficient, or Pareto-optimal) if it maximizes the net revenue, B, subject to the constraint that each household attains a prespecified utility level. To be precise, let $\{h(x), b(x)\}$ be an arbitrary pair of distributions of households and firms, and choose an arbitrary utility level, U. Define the following,

$$B(U) = \text{revenues - costs}$$

where

$$\text{revenues} = \int (a + k\,A(x))\,b(x)\,dx$$

$$\text{costs} = \text{commuting costs}^{26} + \text{household consumption} + \text{agricultural rents}$$

$$= \int t \mid D(x) - S(x) \mid dx + Z(U)\,N + R_a\,(S_h\,N + S_b\,(N/L_b))$$

Here, $A(x)$ is given by equation (6) or (7) ; $Z(U)$ is the solution of $u(Z, S_h) = U$; and

$$D(x) = \int_{-\infty}^{x} L_b\,b(y)\,dy$$

= total labor demanded by business firms locating to the left of x,

$$S(x) = \int_{-\infty}^{x} h(x)\,dy$$

= total labor supply by household locating to the left of x.

The optimal land use problem may now be formulated as follows :

Choose non-negative distribution functions $h(x)$ and $b(x)$ so as to maximize

$$B(U) = \int (a + k\,A(x))\,b(x)\,dx - \left(\int t \mid D(x) - S(x) \mid dx + Z(U)\,N + R_a \cdot (S_h\,N + S_b\,N/L_b)\right) \tag{16}$$

subject to

(17) (a) land constraint $S_h\,h(x) + S_b\,b(x) \leqq 1$

(18) (b) activity-unit number constraint $\int h(x)\,dx = N$,

$$\int b(x)\,dx = N/L_b.$$

25. For details about this part of study, see Ogawa [1980].

26. The difference, $D(x) - S(x)$, representsthe excess demand or supply of labor in that part of the city to the left of x. There must be a flow of labor as large as $\mid D(x) - S(x) \mid$ passing though point x so as to meet the labor requirement in the right-hand side of the city (given that, $M = N/L_b$).

Hence, assuming no cross-commuting in the city (which is clearly not efficient), the total commuting cost is given as $\int t \mid D(x) - S(x) \mid dx$.

Note that the last two terms of the objective function are constant. The solution, then, is independent of utility level U, and we can conclude that :

Property 1. The optimal urban configuration is independent of utility level U [27].

The optimality conditions (necessary conditions) of the problem are summarized in the appendix. It is not difficult to observe that the only difference between the equilibrium and optimality conditions concerns the bid rent function of the business firm :

$$\text{(19)} \qquad \text{in equilibrium,} \qquad \Phi(x) = (a + k\,A(x) - W(x)\,L_b)/S_b$$

$$\text{at optimal,} \qquad \hat{\Phi}(x) = \Phi(x) + k\,A(x)/S_b$$

$$\text{(20)} \qquad = (a + 2\,k\,A(x) - W(x)\,L_b)/S_b$$

Namely, under the optimal land use, each business firm locating at x receives a *location subsidy,* $k\,A(x)$; hence, its bid rent $\hat{\Phi}(x)$ is the sum of the original bid rent, $\Phi(x)$, and the location subsidy divided by lot size S_b. We call $\hat{\Phi}(x)$ the *business firm bid rent with location subsidy.* The interpretation of location subsidy is as follows. Suppose an additional business firm locates at x. Then, existing business firms get additional spatial externalities from the presence of the new firm, which are given, for a firm locating at y, by $e^{-\alpha|x-y|}$ or $(K - \alpha\,|x - y|)$. Accordingly, one additional business firm increases the total productivity of existing firms by $\int b(y)\,k\,e^{-\alpha|x-y|}\,dy$ or $\int b(y)\,k\,(K - \alpha\,|x - y|)\,dy$, which are both exactly equal to $k\,A(x)$.

Hence, in order to correctly count the *social* marginal productivity of firms at location x, each firm at x should *be* subsidized by $k\,A(x)$. Comparing (19) and (20), we can immediately conclude the following :

Property 2. The optimality conditions under parameter set $\{k, t, \alpha, N, S_h, S_b, L_b\}$ coincides with the equilibrium conditions under parameter set $\{2k, t, \alpha, N, S_h, S_b, L_b\}$. Equivalently, the equilibrium conditions under parameter set $\{k, t, \alpha, N, S_h, S_b, L_b\}$ coincides with optimality conditions under parameter set $\{k, 1/2\,t, \alpha, N, S_h, S_b, L_b\}$.

From this property, we can see that the optimal city is more tightly concentrated than the equilibrium city. Due to location subsidy

27. Note that this strong conclusion follows from the assumption of fixed lot size ($S = S_h$) for households. That is, when lot size S is variable, this conclusion does not always hold true.

k A (x), firms in the optimal city are encouraged to locate in the central area where A (x) is large. In other words, when a location subsidy policy is in effect, all firms should concentrate until the commuting rate, t, becomes an overwhelming factor of total costs for the city.

In the case of linear accessibility measure, the equilibrium urban configuration is unique under each set of parameters. Hence, by using Proprety 2, we can immediately obtain the optimal urban configurations from the equilibrium urban configurations. From Figure 7 and Property 2, we can arrive at Figure 11, which depicts the relationship between equilibrium and optimal urban configurations. For each value of α, the range of t in which a monocentric configuration is optimal is twice the range in which a monocentric configuration represents the equilibrium state.

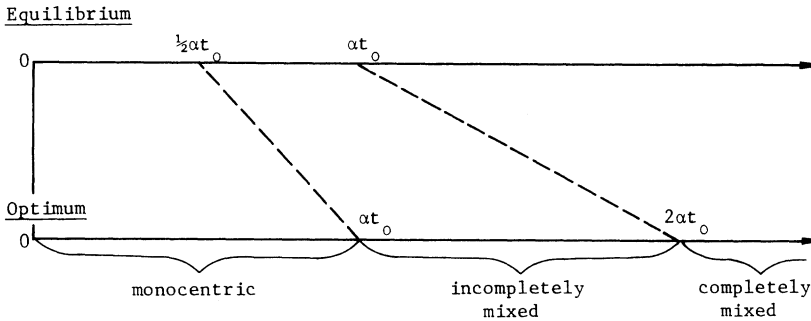

Figure 11. Relationship between equilibrium and optimal urban configurations

In the case of spatially descounted accessibility measure, the equilibrium urban configuration is not always unique under each set of parameters. To obtain the optimal urban configuration under a parameter set, $\{k, t, \alpha, N, S_h, S_b, L_b\}$, we must first obtain all the equilibrium configurations, $i = 1, 2, ...$, under the parameter set $\{2k, t, \alpha, N, S_h, S_b, L_b\}$. Next, we must calculate the total net rent,

$$\mathrm{TNR}_i = \underset{\text{in the city}}{\text{total land rents}} - \underset{\text{for the city,}}{\text{agricultural opportunity rents}}$$

for each equilibrium configuration i. Then, the optimal urban configuration can be determined by choosing the one which has the maximum value of

$$\mathrm{TNR}_i + \mathrm{N}\,\mathcal{Z}\,(\mathrm{U}_i), \tag{21}$$

where U_i is the equilibrium utility level in configuration i[28]. However, since we have not yet confirmed *all* the equilibrium urban configurations, the determination of the optimal urban configurations is left for the future. Note also that, in the case of spatially discounted accessibility measure the relative superiority among multiple equilibria can be determined by comparing the measure, (21). This work is left for the future, too.

SUGGESTIONS FOR FURTHER RESEARCH

In the previous section, we presented a simple two-sector model of general equilibrium urban land use with spatial externalities (Type B model). In this section, we first discuss how this model can be generalized in various useful ways. We then discuss how various models of Type C (non-competitive urban land use with scale economies) might be developed.

Type B (Competitive Equilibrium with Spatial Externalities)

More general utility and production functions

In our model, the lot size for each household is fixed to an arbitrary number.

Instead, we could use a more general utility fuction, $U(Z, S)$, in which both Z and S are variable. As long as we do not consider spatial

28. The justification of this procedure is as follows. In equilibrium, $W(c(x)) - t \mid x - c(x) \mid - R(x) S_h = Z(U)$ for $x \in h_+$, $\pi(x) = a + k A(x) - R(x) S_b - W(x) L_b = 0$ for $x \in b_+$, $\int h(x)\, dx = N$, and $\int b(x)\, dx = N/1_b$. Hence, if we denote by U_i the (equilibrium) utility level in the equilibrium urban configuration i, it is not difficult to show that $B_i(U_i) = TNR_i$
where $B_i(U_i)$ is the value of (16) under $U = U_i$. Choose any utility level, $\bar{U}$, as the reference. Then, by definition (16) $B_i(\bar{U}) = B_i(U_i) + NZ(U_i) - NZ(\bar{U})$.
Therefore, for any pair of equilibrium configurations i and j,

$$B_i(\bar{U}) - B_j(\bar{U}) = (TNR_i + NZ(U_i)) - (TNR_j + NZ(U_j)),$$

which justifies the use of measure (21) as the criterion of the optimal urban configuration (among all equilibrium configurations).

externalities among households, this extension would be relatively easy. A more difficult extension is to generalize the fixed-coefficient production function of business firms to a general production function, f (S, L), for which inputs S and L are substituable. In this case, because of spatial externalities among firms, we must solve a certain integral equation to obtain the explicit solution of the model.

Distribution of Firm Output to Households

It is assumed in our model that firm output is either exported from the city or distributed among households at 0 transport cost. If we were to consider (positive) transport costs for the distribution of outputs, we would also have to specify whether shopping trips and working trips are independent or joint, and whether shopping trip cost depends on the amount purchased. In addition, the labor supply area for a production center may be different from the demand area for outputs of that production center. This would further complicate the problem.

Application to Shopping Center Model

By slightly modifying our model, a shopping-center model can be obtained. All we have to do is replace business firm in our model by retail stores, neglect the commuting costs of retail store workers, and assume a fixed income for each household. To pursue this extension, we would have to take into account a recent paper by Papageorgiou and Thisse [1982]. In addition, when different types of retail stores (or retail stores with choice of goods supplied) are introduced, savings associated with joint shopping must be addressed. For this, the work by Stahl [1981] would be consulted.

Different Types of Spatial Externalities

Thus far, our model considers agglomeration economies only among firms. On the other hand, Bechmann [1976] considers agglomeration economies only among households. It seems reasonable to try and take into account both types of agglomeration economies simultaneously. More generally, we could consider agglomeration economies and diseconomies within and between both household and production sectors.

Different Types of Firms and Households

A completely general equilibrium model would include three types of firms : business firms, retail stores and manufacturing industries. Furthermore manufacturing industries could be disaggregated into low-skill and high-skill industries. Similarly, different types of households (or labor) could be introduced : high-income and low-income households, high-skill and low-skill labor. With this level of refinement, a model might be able to explain various kinds of non-monocentric phenomena such as household and employment suburbanization, formation of subcenters, and clustering of the different types of households.

Public Sectors and Public Goods

Another shortcoming of our work up to this point is that we assume the transport network of the city is given a priori. To consider the simulaneous determination of (private) land use and the transport network, a transport sector would have to be introduced as a behavioral unit. It may be assumed that both private and transport sectors are *followers* ; i.e., each firm (household) maximizes its profit (utility), while the transport network evolves according to cost-benefit criteria. Alternatively, it could be assumed that the transport sector is the *leader* and that the private sectors are followers ; i.e., the transport sector determines an optimal transport network, which maximizes some social welfare function by taking into account reactions in private sectors. To be completely general, we could introduce other public sectors and local public goods such as utilities and green parks.

Two-Dimensional City

Another important extension would be to characterize equilibrium and optimal spatial configurations in a two-dimensional city. The formal extension of our model into a two-dimensional city is straightforward. However, since the *shape* of the city becomes the central issue, the determination of urban configuration becomes analytically very difficult. Ogawa and Fujita [1979] reports some initial efforts in this direction. Since actual cities are always two-dimensional, this extension is quite important with regards to practical applications.

Stability Analysis

As noted before, in the case of multiple equilibria, stability analysis becomes necessary both to determine the actual equilibrium configuration and to conduct a more complete analysis of the structural transition of urban configurations. In pursuing a stability analysis, we would first have to specify the behavior of the system when it is out of equilibrium ; we would then be able to obtain the time-path of the actual change. Related studies can be found in Smith and Papageorgiou [1982] and Papageorgiou and Thisse [1982]. It may also be useful to apply Thom-type catastrophe theory to this problem.

Urban Spatial Growth with Externalities

So far, the treatment of the problem is static in the sense that durability of urban infrastructures, adjustment cost of land use and expectations of agents are completely neglected. However, in determing actual urban patterns, these neglected elements are undoubtedly important.

As a first step, we could combine the model here with those found in Fujita [1976] and Fujita - Kashiwadani [1982]. The formulation of optimal urban spatial growth (or equilibrium growth) under perfect foresight is rather straighforward ; however, the problem would be analytically quite complicated.

Type C (Non-competitive Equilibrium with Scale Economies)

Since little work has been done on this subject, we will only enumerate conceivable models and give brief comments.

The factory town

The simplest class of Type C-model, as noted before, is the so called the factory town model, which depicts the land use in a city with a monopolistic firm and its workers. If the firm is free to choose location and if the agricultural land market is competitive, the firm can rent the entire areas extent of the city at the agricultural rent. Hence, the land use in the city is determined by the firm so as to maximize the total net revenue issuing from urban development. If there are no externalities between the firm and households or among households, the residential land use of the factory town coincides with that determined by the competitive market.

Many monopolistic firms producing different (or, differentiated) goods

The extension of the factory town model to the case of *many firms producing the same product* does not provide an interesting result ; each firm will just form its own city independently of others. Therefore, let us assume first that there are two independent firms, and that each produces a different (or, differentiated) good.

Let us also assume that each firm sells its product to its own workers and workers for the other firm (and possibly, to the rest of the economy). If the two firms locate closely, the (total spatial) demand for each firm tends to increase ; however, land rents and wages also increase. Therefore, there will be equilibrium location for each firm (and for both sets of workers) which altogether constitute the urban configuration. Here, of course, the firms are not price-takers ; each perceives the effects of its location on both output and factor markets. If we consider input-output linkages between the two firms, the tendency for concentration further increases.

If we consider two different goods, each of which can be produced by more than one firm (or more than one plant) there are two different cases : all the firms located within a city or a system of cities, or they separate into more than one city. If there are many different goods, each of which is produced by a monopolistic firm (or firms), then a big city possibly emerges at the equilibrium. In this case, the equilibrium configuration of the city may not be unique.

Spatial competition with land market

Consider a residential area with a fixed amount of land where two independent supermarkets are planning to locate. Households are assumed to be free to choose their locations in the area. The location of two supermarkets affects the distribution of households and land rents in the area. Both supermarkets are assumed to appropriately perceive these effects. The competition for customers tends to force the two supermarkets towards the center ; however, the perceived effects on land rents tend to keep them apart. This is a Hotelling-problem with a land market [29].

Of course, the consideration of both monopolistic producers for different goods and monopolistic retail stores make model more interesting, but, the analysis of the problem becomes more complex.

Finally, we can generate a variety of interesting problems by appropriately fusing different models in the three categories, A, B, and C. However, it would be wise to thoroughly study each *pure* model first.

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APPENDIX

THE OPTIMALITY CONDITIONS

Since $N = \int h(x)\,dx$ and $N/L_b = \int b(x)\,dx$, the objective function, (16), can be rewritten as follows :

$$(16') \qquad NR(U) = \int \{(a + k\,A(x) - R_a S_b)\, b(x) - (Z(U) + R_a S_b)\, h(x) - t\,|\,D(x) - S(x)\,|\}\, dx$$

Optimal land use is a pattern which consists of two non-negative distribution, $h(x)$ and $b(x)$, that together maximize (16′) subject to (17) and (18). The optimality conditions for this problem can be obtained by applying the Maximum Principle in optimal control theory (for details, see Ogawa (1980)). Define.

$$(22) \qquad \begin{aligned} \Psi(x, U) &= (W(x) - Z(U))/S_h \\ \hat{\Phi}(x) &= \Phi(x) + k\,A(x)/S_b \\ &= (a + 2\,k\,A(x) - W(x)\,L_b)/S_b \end{aligned}$$

29. Of course, the actual equilibrium locations depend on further assumptions such as the price behavior of each supermarket. Regarding this, a warning should be made ; in the original Hotelling-problem, when both locations and prices are variable, the problem may not have an equilibrium solution (see D'Aspremont, et al. [1979]).

The optimality conditions can be summarized as follows :

(i) land market equilibrium conditions at each x :

(i.1) $\quad R(x) = \max \{\Psi(x, U), \hat{\Phi}(x), R_a\}$

(i.2) $\quad R(x) = \Psi(x, U)$ *if* $h(x) > 0$

(i.3) $\quad R(x) = \hat{\Phi}(x)$ *if* $b(x) > 0$

(i.4) $\quad S_h\, h(x) + S_b\, b(x) \leqq 1$

(i.5) $\quad S_h\, h(x) + S_b\, b(x) = 1$ *if* $R(x) > R_a$

(ii) labor market equilibrium conditions

(ii.1) $$\dot{W}(x) = \begin{cases} t & \text{if } D(x) < S(x) \\ \in (-t, t) & \text{if } D(x) = S(x) \\ -t & \text{if } D(x) > S(x) \end{cases}$$

(ii.2) $$D(x) = \int_{-\infty}^{x} L_b\, b(y)\, dy, \quad S(x) = \int_{-\infty}^{x} h(x)\, dy$$

(ii.3) $$\int h(x)\, dx = N, \quad \int b(y)\, dx = N/L_b$$

It is not difficult to observe that the set of market equilibrium conditions, (ii.1) and (ii.2), with the definition of $\Psi(x, U)$ given by (10) is equivalent to the set of optimality conditions, (ii.1) and (ii.2) with definition of $\Psi(x, U)$ given by (22). Therefore, the only difference between the equilibrium conditions and the optimality conditions is the difference between $\Phi(x)$ and $\hat{\Phi}(x)$.

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